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SASE FEL simulation for quadropole-free optics

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Introduction

In this report the results of numerical simulation studies for the European XFEL project are presented. The horizontal and vertical beta variations along the SASE1 beamline are calculated. In this report we present the results of a study for the case without external focusing quadropoles. The saturation length and saturation power are calculated for different lattice arrangements and for different value of initial Twiss parameters (α and β). The calculations are made using MAD and SIMPLEX codes. All simulation are performed for SASE 1.

The basic principle of FEL

The basic principle of the free electron laser can be described within the standard picture for the generation of synchrotron radiation: while travelling with relativistic velocity ($v \simeq c$) through the undulator, the electrons are accelerated in the direction transverse to their propagation due to the Lorentz force introduced by the magnetic field. They propagate along a sinusoidal path and emit synchrotron radiation in a narrow cone in the forward direction. The typical opening angle of the wavelength integrated radiation is [3]

$$\frac{1}{\gamma} = \frac{m_e c^2}{E_e} \quad , \tag{1}$$

where m_e is the electron mass (511keV/c^2) and E_e the electron energy. In the undulator, the deflection of the electrons from the forward direction is comparable to the opening angle of the synchrotron radiation cone. Thus the radiation generated by the electrons while travelling along the individual magnetic periods overlaps. This interference effect is reflected in the formula for the wavelength λ_{ph} of the first harmonic of the spontaneous, on-axis undulator emission [1]

$$\lambda_{ph} = \frac{\lambda_u}{2\gamma^2}(1 + K_{rms}^2) \tag{2}$$

where λ_u is the length of the magnetic period of the undulator and K_{rms} is the undulator parameter

$$K_{rms} = \frac{eB_u \lambda_u}{2\pi m_e c} \tag{3}$$

which gives the ratio between the average deflection angle of the electrons and the typical opening cone of the synchrotron radiation. B_u is the rms magnetic field of the undulator and e the electron charge.

The interference condition basically means that, while travelling along one period of the undulator, the electrons slip by one radiation wavelength with respect to the (faster) electromagnetic field. This is one of the prerequisites for the SASE process of the FEL. To obtain an exponential amplification of the spontaneous emission present in any undulator, some additional criteria have to be met: One has to guarantee a good electron beam quality and a sufficient overlap between radiation pulse and electron bunch along the undulator. To achieve that, one needs a low emittance, low energy spread electron beam with an extremely high charge density in conjunction with a very precise magnetic field and accurate beam steering through a long undulator.

Oscillating through the undulator, the electron bunch then interacts with its own electromagnetic field created via spontaneous emission. Depending on the relative phase between radiation and electron oscillation, electrons experience either a deceleration or acceleration: Electrons that are in phase with the electromagnetic wave are retarded while the ones with opposite phase gain energy. Through this interaction a longitudinal fine structure, the so called micro-bunching, is established which amplifies the electromagnetic field (Fig. 1) [1].

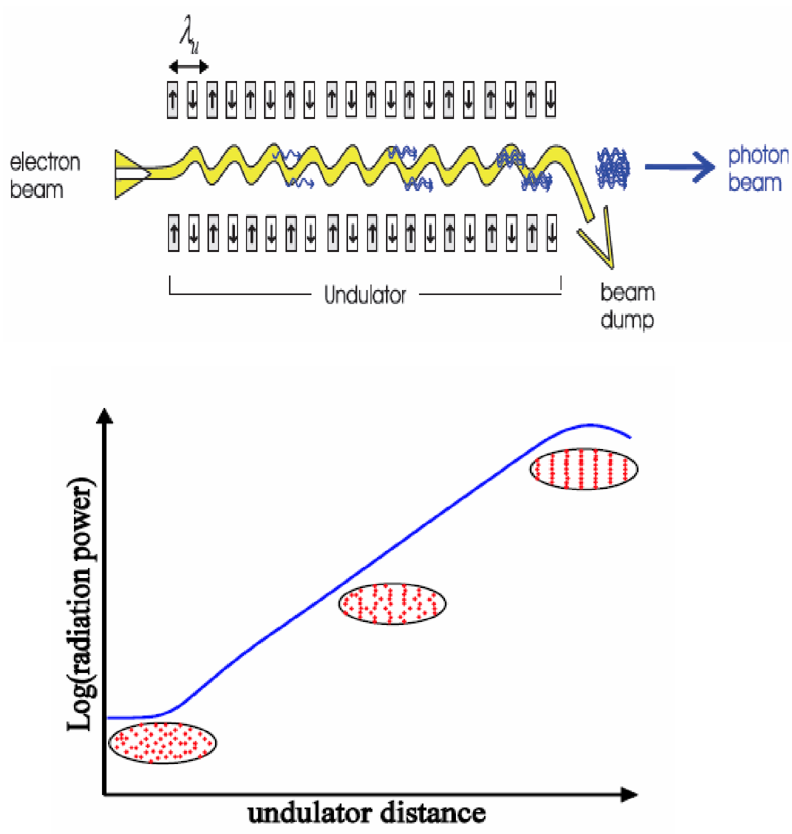


Figure 1:*In the lower part of the figure the longitudinal density modulation (micro-bunching) of the electron bunch is shown together with the resulting exponential growth of the radiation power along the undulator.*

The longitudinal distribution of electrons in the bunch is "cut" into equidistant slices with a separation corresponding to the wavelength λ_{ph} of the emitted radiation which causes the modulation. More and more electrons begin to radiate in phase, which results in an increasingly coherent superposition of the radiation emitted from the micro-bunched electrons. The more intense the electromagnetic field gets, the more pronounced the longitudinal density modulation of the electron bunch and vice versa.
In the beginning - without micro-bunching - all the N_e electrons in a bunch ($N_e \geq 10^9$) can be treated as individually radiating charges with the power of the spontaneous emission $\approx N_e$. With complete micro-bunching, all electrons radiate almost in phase. This leads to a radiation power $\approx N_e^2$ and thus an amplification of many orders of magnitude with respect to the spontaneous emission of the undulator [3].
Due to the progressing micro-bunching, the radiation power P(z) of such a SASE FEL grows exponentially with the distance z along the undulator:

$$P(z) = AP_{in} \exp(\frac{2z}{L_g}) \tag{4}$$

where L_g is the field gain length, P_{in} the "effective" input power, and A the input coupling factor. A is equal to 1/9 in one-dimensional FEL theory with an ideal electron beam. The exponential growth takes place until the electron beam is completely bunched after which it is overmodulated resulting in saturation.
The main properties of the FEL radiation can be simply estimated in terms of the FEL parameter ρ: the field gain length is $L_g \approx \frac{\lambda_u}{4\pi\rho}$, the FEL amplifier bandwidth $\Delta\omega/\omega$ and saturation efficiency (ratio of the output radiation power to the electron beam power) are about ρ . This parameter depends on the parameters of electron beam and undulator [1].

SASE1 Parameters

In the European XFEL intense coherent radiation will be produce at wavelength down to 0.1 nm in long undulator section. The maximum energy of the electron beam from the driving linac is 17.5 GeV with a normalized emittance of 1.4 nm. Five photon beams will be deliver the X-ray pulses to the experimental stations (Fig. 2.) [2].

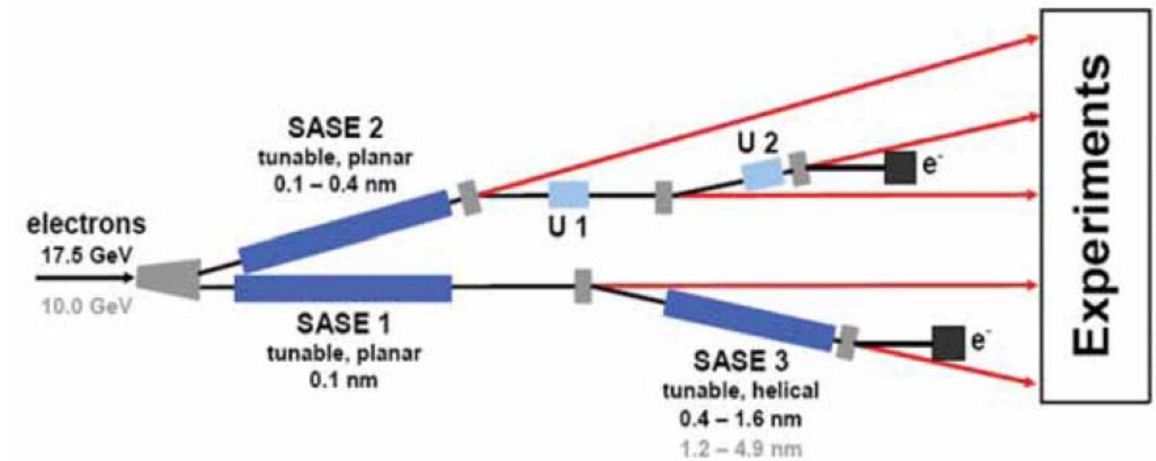


Fig.2. *Schematic overview of the electron beam distribution between accelerator and experimental holl.*

The European XFEL undulator system consists of 5 m long segments and the shortest FODO lattice period is obtained when one quadropole magnet is installed in each inter-segment gap. External alternating-gradient quadropole magnet inserted into the undulator to provide sufficient focusing of the electron beam.

The major parameters list for SASE 1 is presented in Table 1 [2].

Table 1: Parameters for SASE 1.

Radiation wavelength [nm]	0.1
K value	3.3
Period length [cm]	3.56
Segment length [m]	5
Total length [m]	201
Number of FODO cells	17
Average beta [m]	32
Phase advance per cell [degree]	22.4

SIMPLEX Simulation

SIMPLEX is an application software to investigate phenomena in free electron lasers (FELs), such as the power growth, electron motion in the phase space, evolution of angular and spatial profile of radiation field.

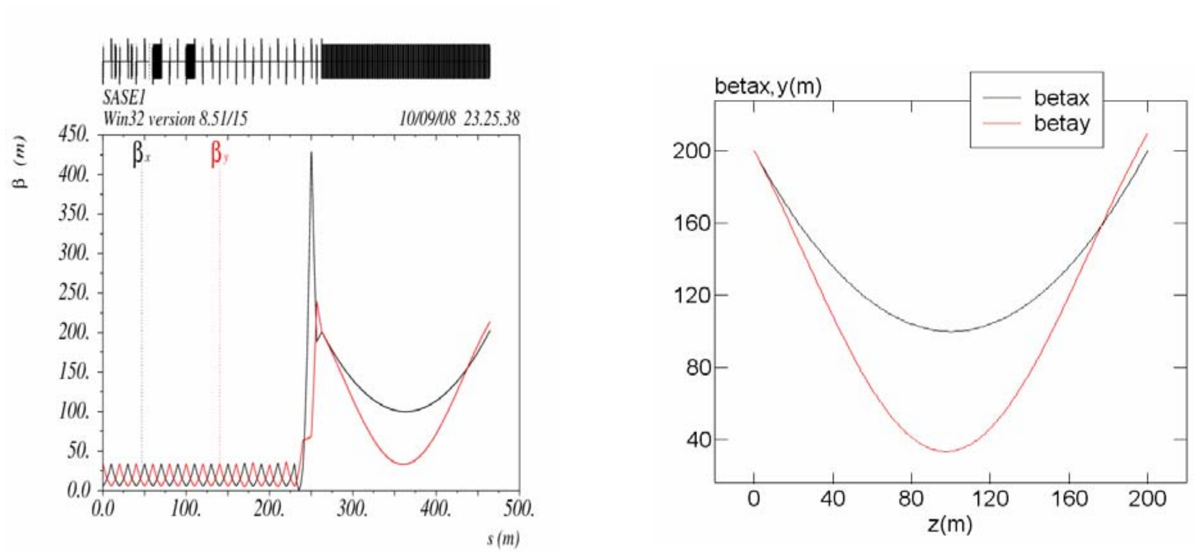
The main code for numerical computation in SIMPLEX is written in the standard C language, which solves the equations describing FEL process in the 3-dimensional (3-D) form. The simulation can be divided into two types according to how to deal with the slippage between electrons and radiation emitted by them. In order to investigate the FEL amplification process, it is necessary to solve three equations: equation of motion, energy equation, and wave equation. The main function of an FEL simulator is to numerical solve the FEL equations.

Assuming that the electron beam is infinitely long with performances such as the emittance, energy, current, and offset (angular and positional) being constant, and the radiation is completely monochromatic, the partial differentiation with respect to time can be dropped from the wave equation. In such a case, the simulation algorithm is considerably simplified and the simulation is called "steady-state". If the above assumption cannot be applied, the wave equation should be solved with the time dependence being taken into account. In practice, the time dependence in the wave equation should be solved with the time dependence in the wave equation causes slippage between the electron and radiation. The FEL simulation that takes into account the slippage effect is called "time-dependent" [4].

Results

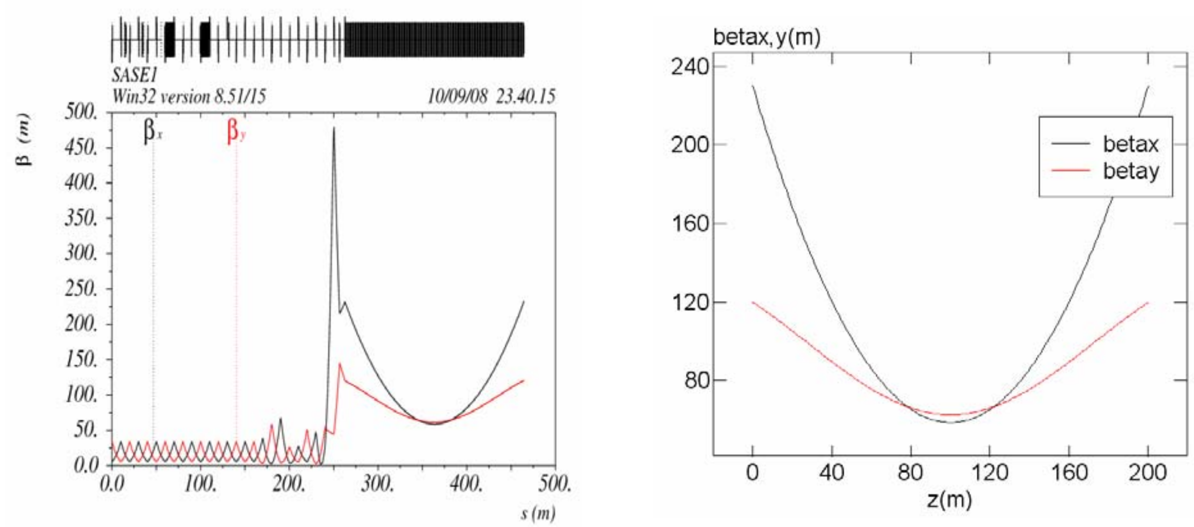
Though it is common knowledge that for an FEL operating in the X-ray region focusing by external lattice is necessary, the problem of the FEL performance operating with no strong regular focusing in the undulator is of great interest. In the commissioning stage, if all quadropoles are turned off, only so called natural focusing by the undulator field takes place and gets FEL radiation.

The horizontal and vertical beta variations along the SASE1 beamline are calculated by MAD and SIMPLEX for different initial values of Twiss parameters. MAD and SIMPLEX give the similar results for beta variations. In the Fig.1 presented the horizontal and vertical beta variations when $\beta_{x,y,in} = 200, \alpha_{x,y,in} = 1$ (initial Twiss parameters). In the Fig.2 presented the variations of beta when the initial Twiss parameters are $\beta_{x,in} = 230, \beta_{y,in} = 119.68, \alpha_{x,in} = 1.71, \alpha_{y,in} = 0.32$. The same initial Twiss parameters we can get by MAD using matching. The insertion matching mode is initiated by the MATCH command. The initial values for the optical function are taken from the periodic solution of another beam line (T2 beam line).



a) *b)*

Fig.1. *The horizontal and vertical beta calculated a) by MAD, b) by SIMPLEX*



a) *b)*

Fig.2. *The horizontal and vertical beta calculated a) by MAD, b) by SIMPLEX*

In the Table 2 presented the simulation results by SIMPLEX for this two cases and for initial lattice.

Table 2:

$\beta_{x,0}$	$\beta_{y,0}$	$\alpha_{x,0}$	$\alpha_{y,0}$	P_{max}[GeV]	L_{sat}[m]
200	200	1	1	12.42	201.79
230	119.68	1.71	0.32	14.01	191.52
Initial Lattice				21.88	194.6

The steady-state simulation indicate that SASE1 operating at the 0.1 nm wavelength the saturation power decreases by about 35 - 45 %.

References

[1]. Z.Huang, K.Kim, Review of x-ray free-electron laser theory, Phys. Rev, 0348201, 2007.
[2]. The European XFEL, TDR, DESY 2006-097, 2006.
[3]. J.Rossbach, Linac based free-electron laser, TESLA-FEL Report 2004-08
[4]. T.Tanaka, Reference manual for SIMPLEX, (2005).