Additional U(1)-boson in string phenomenology

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ABSTRACT: Embeddings of the SM in type II string theory typically contain many additional $U(1)$ gauge factors. We can imagine that not only one of these bosons, corresponding to weak hypercharge, is massless. As we know from experiments, the standard model must be neutral under additional massless $U(1)$s, therefore the latter must belong to the so-called hidden sector. However, the exchange of heavy messengers can kinetically mix a photon and this new boson which is testable with near future experiments. In this report it is shown, how the kinetic mixing can be derived from the underlying type II compactification. For this purpose we use conformal field theory techniques and then develop a more general supergravity approach that allows us to study the phenomenon in more general backgrounds. We also discuss some simple examples of models with kinetic mixing and speak about phenomenological consequences of experiments at the low-energy frontier, searching for signatures of light or even massless new $U(1)$ gauge bosons and minicharged particles.

1. Introduction String theory is one of the most promising theories that could explain many phenomena in our world. Today we know 5 consistent superstring theories:

*) the type I theory of open and unoriented strings with gauge group $SO(32)$

*) the type IIA(B) theories of closed oriented strings where the right- and the left-moving modes, transforming under separate spacetime supersymmetries, have an opposite (the same) chirality

*) the heterotic string theories with different constraint algebras acting on right- and left-moving fields; it admits gauge groups $E_8 \times E_8$ or $SO(32)$.

In the so-called 'second string revolution' physicists realized that all these theories are linked through 'dualities' - for example, the strong coupling limit of one theory corresponds to the weak coupling limit of another (S-duality) or the big radii of compactification tori of one theory correspond to the small radii of another (T-duality). For this and some other reasons, scientists hope that all these theories are in fact just manifestations of one big theory. They call it M-theory.

To relate all these complicated models to the real world, we must reduce them in low-energy limit to the standard model, which works very well in the case. We can embed the SM in $E_8 \times E_8$ heterotic closed string theory as well as in type I, IIA or IIB open string theories with branes. It turns out that many of them for consistency and for proper supersymmetry breaking should have so-called hidden sectors - additional particle multiplets that we cannot observe directly. So how one can prove the existing of such hidden sectors?

A unique window there is provided by hidden Abelian gauge bosons. In fact, hidden sectors' gauge groups often contain additional $U(1)$ factors which mix kinetically with the hypercharge

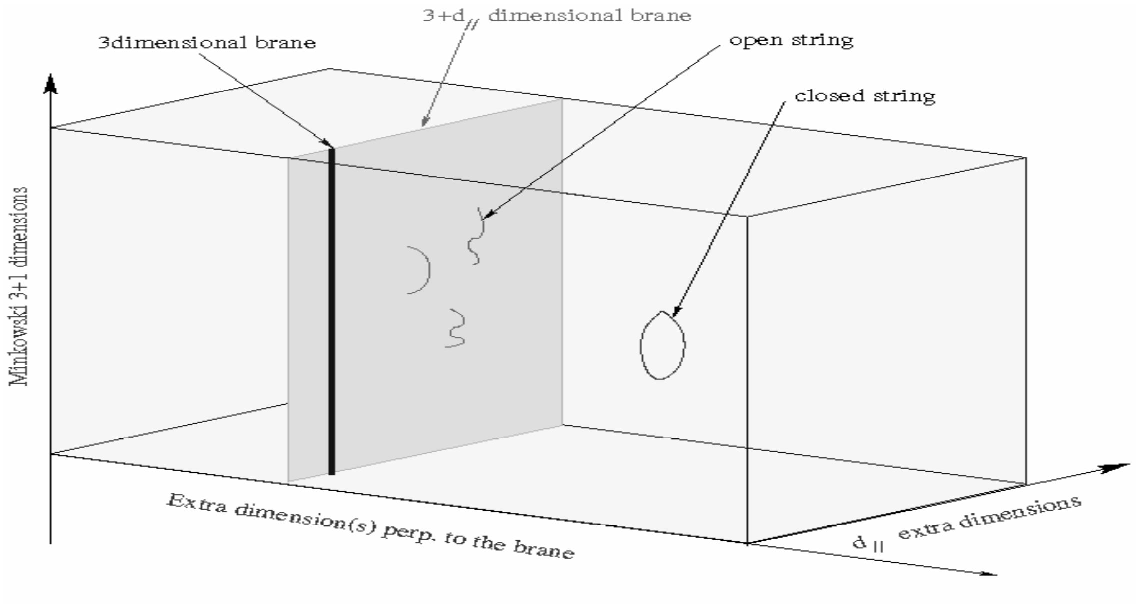

Figure 1: D-brane setup with $d_{\|}$ parallel and $d_{\perp}$transverse internal directions (from Ref. [20]). SM-brane, responsible for the visible sector, is grey-colored.

$U(1)$ from the visible sector and give rise to following terms in the low-energy effective Lagrangian:

$$L \supset -\frac{1}{4g_a^2} F_{\mu\nu}^{(a)} F_{(a)}^{\mu\nu} - \frac{1}{4g_b^2} F_{\mu\nu}^{(b)} F_{(b)}^{\mu\nu} + \frac{\chi_{ab}}{2g_a g_b} F_{\mu\nu}^{(a)} F^{(b)\mu\nu} + m_{ab}^2 A_\mu^{(a)} A^{(b)\mu}, \tag{1}$$

where $a(b)$ labels the visible (hidden) $U(1)$, with field strength $F_{\mu\nu}^{(a(b))}$ and gauge coupling $g_{a(b)}$. The dimensionless kinetic mixing parameter χ_{ab}, appearing in front of the effective renormalizable operator in (1), can be generated at an arbitrarily high energy scale and does not suffer from any kind of mass suppression from the mass messengers that induce it. Its measurement could provide us with clues to physics at energies that we never access with colliders. In present report we will consider the effect and the generation of this term. Actually, we consider even more particular question - kinetic mixing in compactifications of type II string models, because a systematic and rigorous study of the subject is still lacking [1]. Type II string models are also considered in [2, 3]. However, an exhaustive study of the predicted size of kinetic mixing in realistic compactifications of heterotic string theory has been performed in [4].

In the type II compactification is usually performed using some special hypersurfaces, D-branes, that confine some of our particles. Here hidden $U(1)$s arise as D-branes in the bulk (see Fig. 1) that have no intersection with the branes responsible for the visible sector. To obtain the kinetic mixing of the form (1) we must integrate out massive modes coupling to different $U(1)$s. These modes correspond to open strings stretched between the visible and hidden stacks of branes. In closed string channel this also can be understood as mediation by light or massless closed string modes. Since in type II string compactifications there are usually many hidden $U(1)$s, we can hope that any of them can be non-anomalous and therefore massless. Furthermore, the Ramond sector on intersecting D-branes always yields the massless charged matter fermions that could make the kinetic mixing detectable due to the quantum corrections generated at loop level.

We can then subdivide the type II models into two classes, each of which we consider in the present report. First there are models in which the compact space plays the role of a large quasi-flat bulk volume. These include the D-brane of singularities models and, due to the simplicity, models in which intersecting $D6$-branes wrap 3-cycles on toroidal backgrounds, despite the volumes in this cases being restricted rather small. Here we will consider both supersymmetric and non-supersymmetric set-ups and demonstrate by explicit example that kinetic mixing can occur without Stückelberg mass mixing (i.e. when the last term in (1) vanishes). Here we will use conformal field theory (CFT) techniques. The second class are those models in which the compact volume is significantly warped and Randall-Sundrum like. In this class the SM branes are typically assumed to be located at the bottom of a warped throat. To explore these models we develop a supergravity approach which allows to examine the kinetic mixing in more general backgrounds, than the CFT.

How could we find an additional $U(1)$ gauge boson γ' experimentally? The best way to search depends primarily on its hitherto undetermined mass. For a mass in the range $m_Z \approx 100\ GeV \lesssim m_{\gamma'} \lesssim 1\ TeV$, precision electroweak tests can be used to set an upper limit $\chi \lesssim few \times 10^{-2}$,[5], which will be only mildly improved by future measurements at the LHC and ILC. For smaller masses, the best limits arise from searches for $\gamma \leftrightarrow \gamma'$ oscillations[6] and for deviations from Coulomb's law[7]. But if the hidden sector photon is massless, then in the absence of light hidden matter there is no limit on its mixing with hypercharge, because the effect can be reabsorbed by the redefinition of hypercharge coupling constant.

Things are different if the hidden sector contains also light matter particles which are charged under the hidden-sector $U(1)$ gauge symmetry. Thus, if we have such a fermion h with a bare coupling to $A_\mu^{(b)}$ given by

$$L \supset \bar{h}(\gamma A^{(b)})h, \tag{2}$$

its electric charge will be proportional to the gauge kinetic mixing parameter (therefore it is called electrically minicharged particle, MCP). Indeed, upon diagonalizing of the gauge kinetic term in (1) by the shift

$$A_\mu^{(b)} \to \tilde{A}_\mu^{(b)} + \chi A_\mu^{(a)}, \tag{3}$$

the coupling term gives rise to a coupling with visible gauge field $A_\mu^{(a)}$,

$$\bar{h}(\gamma A^{(b)})h \rightarrow \bar{h}(\gamma \tilde{A^{(b)}})h + \chi \bar{h}(\gamma A^{(a)})h, \tag{4}$$

corresponding to a possibly small, non-integer charge with respect to the visible sector $U(1)$:

$$Q_h^{(a)} = \chi g_b \equiv \epsilon e. \tag{5}$$

For low MCP masses, $m_\epsilon \lesssim 0.1\ eV$, the best current laboratory limit on the electric charge, $\epsilon \lesssim few \times 10^{-7}$, are obtained from laser polarization experiments, where linearly polarized laser light is sent through a transverse magnetic field, and changes in the polarization state are searched for[8]. Similar bounds are obtained from a light-shining-through-a-wall experiments[9] and from the non-observation of an excessive energy loss due to Schwinger pair production of minicharged particles in the strong electric fields in superconducting accelerator cavities[10]. In the mass range from eV up to the electron mass, the best laboratory limits, $\epsilon \lesssim 3 \times 10^{-5}$, arise from searches for the invisible decay of orthopositronium[11], while in the higher mass range the accelerator limits dominate - these are however rather loose. Bounds involving cosmology or astrophysics are much better[12], notably in the sub-electron mass region.

2. CFT computation of kinetic mixing At the beginning we consider the simplest way to compute kinetic mixing that can be used in flat backgrounds - the CFT approach. Typically we have several stacks of branes, and anomaly free $U(1)$ is linear combination of the $U(1)$-factors (labelled as a, b) coming from different stacks (labelled as i, j). If orientifold planes are present in our model, we consider the image as different stacks. Vertex operator for $U(1)^a$ is:

$$V^a = \sum_i c_i^a V_i^a, \tag{6}$$

where we choose constants c_i^a so that the corresponding $U(1)$ is anomaly free and the individual vertex operator is

$$V_i^a = \lambda_i^a \epsilon_\mu (\partial X^\mu + 2\alpha'(k\psi)\psi^\mu) e^{ikX}. \tag{7}$$

Here ϵ_μ is a polarization vector, ψ^μ and X^μ are the worldsheet fermions and bosons and λ_i^a are the Chan-Paton matrices. Condition that $U(1)$ is anomaly free implies:

$$\sum_i c_i^a tr_i(\lambda_i^a) = 0. \tag{8}$$

We can calculate amplitude in the closed string channel as:

$$\langle V_i^a V_j^b \rangle = 4(\alpha')^2 tr_i(\lambda_i^a) tr_j(\lambda_j^b) \epsilon_\mu \epsilon_\nu (g^{\mu\nu} k^2 - k^\mu k^\nu) \int_0^\infty dl \int_0^1 dx e^{k^\mu k^\nu G_{\mu\nu}} \times$$
$$\times \sum_\nu \left[\frac{\theta_4''(x)}{\theta_4(x)} - \frac{\theta_4''(0)}{\theta_4(0)} \right] \frac{1}{(8\pi^2\alpha')^2} \frac{\theta_\nu(0)}{\eta^3(il)} Z_\nu^{ij}(il), \quad (9)$$

with Green function on the annulus, given by

$$G_{\mu\nu}(x) = -2\alpha' g_{\mu\nu} \log \left| \frac{1}{l} \frac{\theta_4(x)}{\eta^3(il)} \right|, \quad (10)$$

where θ and η are the elliptic theta and Dedekind eta functions[2] and Z_ν^{ij} is the model-dependent partition function with spin structure ν. In the open string channel this is a non-planar diagram.

In the low-energy limit $k^2 \to 0$ the amplitude should read as follows:

$$\langle V^a V^b \rangle = m_{ab}^2 A_\mu^a A_b^\mu + \frac{\chi_{ab}}{g_a g_b} k^2 A_\mu^a A_b^\mu, \quad (11)$$

and we must sum over all the relevant stacks i, j contributing to the hypercharge A_μ^a and to the hidden anomaly-free $U(1)$-boson A_μ^b. Comparing this with (9), we see that the contribution to the mass comes from the $1/k^2$ pole of the integral, as the amplitude has the structure $k^2 g^{\mu\nu} - k^\mu k^\nu$. To make the pole structure manifest we can do some approximations, namely we take the large l limit, so that

$$e^{k^\mu k^\nu G_{\mu\nu}} = e^{-\frac{1}{4} 2\pi\alpha' k^2 l}, \quad (12)$$

and the rest of the integrand one expands as

$$\sum_\nu \left[\frac{\theta_4''(x)}{\theta_4(x)} - \frac{\theta_4''(0)}{\theta_4(0)} \right] \frac{1}{(8\pi^2\alpha')^2} \frac{\theta_\nu(0)}{\eta^3(il)} Z_\nu^{ij}(il) \propto 1 + \sum_{\beta_{ij}>0} N(\beta_{ij}) e^{-\pi\beta_{ij} l}, \quad (13)$$

with $N(\beta)$ counting the multiplicity of closed-string models at level β. The first term corresponding to massless closed string states generates the mass term for the gauge fields and the second contributes to χ_{ab}.

As an example take the simple model containing a single $D3$ brane and single $\bar{D}3$ brane on a T^6 factorized into 3 complex 2-tori numbered by $\kappa = 1, 2, 3$:

$$Z_\nu^{ij}(il) = \frac{1}{2} \delta_\nu' \frac{(\alpha')^3 (2\pi)^6}{8V_6} \frac{\theta_\nu^3(0)}{\eta^9(il)} \prod_\kappa \sum_{q^\kappa, p^\kappa} exp \left[-\frac{\pi\alpha' l}{2T_2^\kappa U_2^\kappa} |q^\kappa + \bar{U}^\kappa p^\kappa|^2 - \frac{2\pi i}{U_2^\kappa} Im(z_{ij}^\kappa (\bar{U}^\kappa p^\kappa + q^\kappa)) \right], \quad (14)$$

where U^κ, T^κ are the complex and Kähler moduli and z^κ_{ij} is the distance between the branes i and j, multiplied by $\frac{1}{2\pi}\sqrt{\frac{U_2^\kappa}{T_2^\kappa}}$. The amplitude at large l is then:

$$\langle V_i^a V_j^b \rangle = tr_i(\lambda_i^a) tr_j(\lambda_j^b)\epsilon_\mu\epsilon_\nu(g^{\mu\nu}k^2 - k^\mu k^\nu)\int_0^\infty dl \frac{(2\pi\alpha')^4}{4\alpha' V_6} e^{-\frac{1}{4}2\pi\alpha' k^2 l}$$
$$\left\{1 + \prod_\kappa \sum_{q^\kappa, p^\kappa \neq 0} exp\left[-\frac{\pi\alpha' l}{2T_2^\kappa U_2^\kappa}|q^\kappa + \bar{U}^\kappa p^\kappa|^2 - \frac{2\pi i}{U_2^\kappa} Im(z_{ij}^\kappa(\bar{U}^\kappa p^\kappa + q^\kappa))\right]\right\}(1\text{+string mass terms}), \tag{15}$$

where we explicitly wrote the massless mode that gives a z_{ij} independent contribution and neglected terms of order the string mass because of exponential damping of massive modes beyond their wavelength. Performing the integration we obtain the following expression for kinetic mixing:

$$\chi_{ab} \approx g_a g_b \frac{(2\pi\alpha')^3}{V_6} \sum_{q^\kappa, p^\kappa \neq 0} \frac{exp\left[\sum_\kappa -\frac{2\pi i}{U_2^\kappa} Im(z^\kappa \bar{U}^\kappa p^\kappa + q^\kappa z^\kappa)\right]}{\frac{\alpha'}{T_2^\kappa U_2^\kappa}|q^\kappa + \bar{U}^\kappa p^\kappa|^2}, \tag{16}$$

where z^κ is the displacement between the brane and antibrane in the k'th complex 2-torus. It is interesting to note that the kinetic-mixing term produced by the string amplitude, $\chi_{ab}/g_a g_b$, contains no gauge couplings (because the vertex operators don't depend on them).

3. Supersymmetric models In order to confirm that kinetic mixing can occur between anomaly free and massless $U(1)$s we will now examine self-consistent global configurations that have non-vanishing kinetic mixing between mutually supersymmetric branes. A convenient framework in which to construct supersymmetric models consists of a simple orientifold with $D6$ branes and $O6$ planes in type IIA string theory, as reviewed in [13]. We will construct the $N = 2$ supersymmetric model, because typically the $D6$ branes wrap all the internal cycles and therefore almost always intersect - so it is difficult to construct $N = 1$ models with any hidden sector. In any case, on a very symmetric toroidal orientifold that we will use preserving of $N = 1$ supersymmetry would imply that kinetic mixing have no dependence on the separation (therefore we couldn't separate mass mixing from the kinetic mixing) and in the case of preserving of $N = 4$ supersymmetry the amplitude would cancel. So, dealing with $D6$-branes it is much more convenient to build models with $N = 2$ SUSY.

The configuration is as follows: our space-time will be $R^{3,1} \times T^2 \times (T^2 \times T^2)/Z_2$, where the tori are taken to be rectangular. Denoting the complex coordinates on the compact space as $z^i \in T_i^2$, the orbifold involution acts as $\theta : (z_2, z_3) \to (-z_2, -z_3)$. The orientifold involution

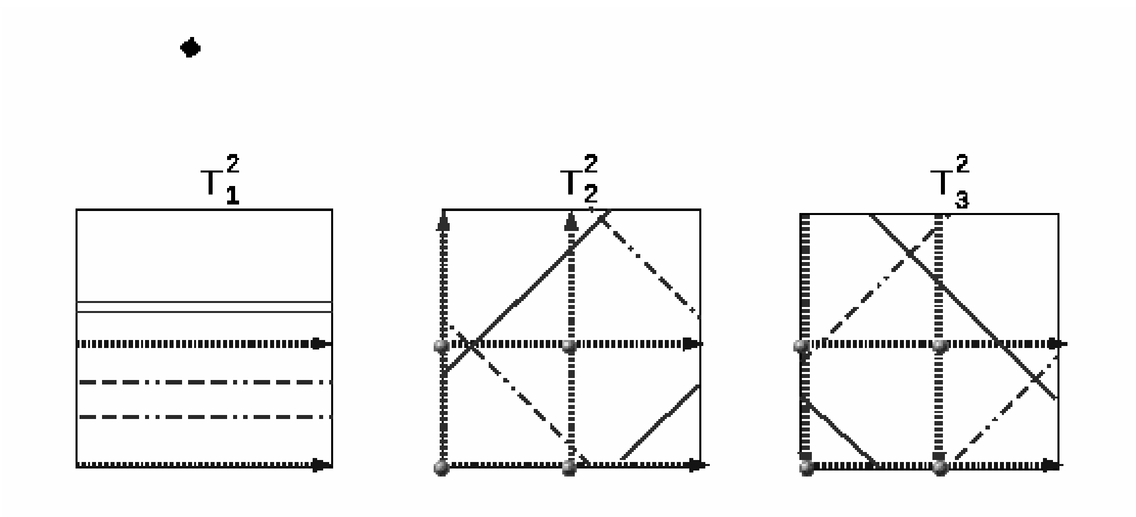

Figure 2: Supersymmetric configuration corresponding to our simple model. Solid lines denote A stacks and dashed-dotted lines represent B stacks. Each of these stacks is separated into A_1, A_2 and B_1, B_2 in the first torus only. The orientifold planes are represented by the dashed lines with arrows. In the first torus, the two sets of orientifold planes are coincident. Finally the dots on each of the last two tori show the orbifold fixed points (to be more precise - planes).

consists then of world sheet parity transformation Ω coupled with a non-holomorphic reflection R in the internal complex coordinates, $R : z^k \to \bar{z}^k$. The projections leave $4 \times 4 = 16$ fixed points of the Z_2 orbifold, and 16 orientifold fixed planes (O6-planes), 8 for each of the orientifold actions, $\Omega R, \Omega R\theta$.

In the orbifold case we may derive rather general expression for kinetic mixing. The only assumption we need is that the two massless gauge groups $U(1)_a$ and $U(1)_b$ come from two parallel stacks of branes each, labelled A_1, A_2 and B_1, B_2 (see Fig. 2). In order not to intersect they must be parallel to the orientifold plane in torus 1, but not lie upon it. We denote separations from the $O6$-plane in the torus 1 y_{A_i}, and write $\delta_{ij} \equiv y_{A_i} - y_{B_j}$. The charges for the massless combinations are given by $Q_a = \frac{1}{N_{A_1}} Q_{A_1} - \frac{1}{N_{A_2}} Q_{A_2} = \sum_i \frac{c_i^a}{N_{A_i}} Q_{A_i}$, and similarly for Q_b, where N_{A_i} is the number of branes in stack A_i and $Q_{A_i}, Q_{B_i} = \pm 1$. The kinetic mixing then is

$$\chi = \sum_{ij} c_i^a c_j^b Q_{A_i} Q_{B_j} \chi_{ij} = \chi_{11} - \chi_{12} - \chi_{21} + \chi_{22}, \tag{17}$$

where

$$\chi_{ij} = \frac{g_a g_b}{4\pi^2} I_{AB} \left[log \left| \frac{\theta_1(\frac{i\delta_{ij} L_1}{2\pi^2 \alpha'}, \frac{iT_1^2}{\alpha'})}{\eta(\frac{iT_1^2}{\alpha'})} \right|^2 - \frac{\delta_{ij}^2}{2\pi^3 \alpha'} \frac{(L_1)^2}{T_2^1} \right]. \tag{18}$$

Here χ_{ij} is the kinetic mixing between A_i and B_j, I_{AB} is the number of intersections between the branes in the non-parallel directions, L_1 is the length of both branes on the torus 1, in which they are parallel, and T_2^1 is the Kähler modulus of torus 1, in case of rectangular tori it is proportional to the product of radii. It is interesting that the above result can be calculated exactly by the effective supergravity techniques that we consider in the next section, since supersymmetry ensures that all of the string mass excitations don't contribute.

To verify that such model will be consistent, one must impose conditions of supersymmetry preserving and of R-R tadpole cancellation (supersymmetry then ensures the cancellation of NS-NS tadpoles automatically). It is shown in article that for two explicit models we can satisfy these conditions by selecting matching wrapping numbers. So we can see that kinetic mixing between massless anomaly-free gauge bosons can exist in consistent models.

4. The supergravity calculation of kinetic mixing Let's see how one can obtain CFT results of section 2 using only the effective field theory. Take the action of the brane and the supergravity fields in the following form [14, 15]. Define the Dirac-Born-Infeld action:

$$S_{DBI} = \mu_p \int d^{p+1}x\, e^{-\Phi} \sqrt{-det\, g + 2\pi\alpha' F + B}$$
$$\approx \int d^{p+1}x\, \mu_p\, e^{-\Phi}\sqrt{-g} - \frac{1}{4}\mu_p e^{-\Phi}\sqrt{-g}\left((2\pi\alpha')^2 F_{\mu\nu}F^{\mu\nu} + 2(2\pi\alpha')F_{\mu\nu}B^{\mu\nu} + B_{\mu\nu}B^{\mu\nu}\right), \quad (19)$$

the D-brane action:

$$S_{WZ} = \mu_p \int_{Dp} \sum_q C_q \wedge tr\, exp(2\pi\alpha' F + B) \wedge \sqrt{\frac{\hat{A}(4\pi^2\alpha' R_T)}{\hat{A}(4\pi^2\alpha' R_N)}}, \quad (20)$$

and the usual actions of low-energy type IIB supergravity:

$$S_R = -\frac{1}{4\kappa_{10}^2} \int d^{10}x(-det\, G)^{1/2} \left(|F_1|^2 + |\tilde{F}_3|^2 + \frac{1}{2}|\tilde{F}_5|^2\right), \quad (21)$$

$$S_{NS} = -\frac{1}{4\kappa_{10}^2} \int d^{10}x(-det\, G)^{1/2} e^{-2\Phi}|H_3|^2, \quad (22)$$

where A_μ is a gauge field, C_q are the R-R forms, B_2 is the NS-NS 2-form. The latter terms mean that if one makes a mode decomposition of those forms, their right- and left-moving parts turn to be fermionic in the R-R case and bosonic in the NS-NS case.

The field-strengths are defined as

$$F = dA,$$
$$F_{q+1} = dC_q,$$
$$H_3 = dB_2,$$
$$\tilde{F}_3 = F_3 - C_0 \wedge H_3,$$
$$\tilde{F}_5 = F_5 - \frac{1}{2}C_2 \wedge H_3 + \frac{1}{2}B_2 \wedge F_3$$
$$*_{10}\tilde{F}_5 = \tilde{F}_5.$$

Also $\mu_p = \sqrt{2\pi}(4\pi^2\alpha')^{-\frac{1+p}{2}}$ is the brane tension, and $2\kappa_{10}^2 = (\alpha')^4(2\pi)^7$.

We are now going to obtain the results of previous paragraph using supergravity approach based on Dirac-Born-Infeld action. As we could see from the actions, mediator of kinetic mixing is the B-field. Additionally, for Dp-branes, a $p-1$-form C_{p-1} couples to the gauge fields, which can mediate too, but only between the branes of the same dimensionality. From the DBI-action we read vertex for the antisymmetric $B_{\mu\nu}$ and A_ρ:

$$\frac{1}{2}2\pi\alpha'\mu_p g_s^{-1}(k_\mu g_{\nu\rho} - k_\nu g_{\mu\rho})\delta(\Sigma_p), \tag{23}$$

on a p-brane of worldvolume Σ_p and a propagator (we need only diagonal part) for $B_{\mu\nu}$:

$$G_{\mu\nu;\rho\sigma}(k_4, y_0, y_1) = \delta_{\mu\rho}\delta_{\nu\sigma}\frac{2g_s^2\kappa_{10}^2}{V_6}\sum_{k_6}\frac{exp[ik_6(y_1 - y_0)]}{|k_4|^2 + |k_6|^2}. \tag{24}$$

Here $\mu, \nu \in \{0, 1, 2, 3\}$, k_4 and k_6 are the 4-dimensional and the transverse 6-dimensional momenta, and $y_1 - y_0$ is the 6D distance vector in the transverse space.

The B-field contribution to the 2-point function of gauge fields is:

$$\langle A^{a_1}_{\mu_1} A^{b_1}_{\nu_1}\rangle_B = \frac{\delta}{\delta A^{a_1}_{\mu_1}}\frac{\delta}{\delta A^{b_1}_{\nu_1}} tr_1\lambda_a tr_2\lambda_b \frac{1}{2}\frac{1}{\alpha'}\frac{(2\pi\alpha')^3}{V_6}(4\pi^2\alpha')^{\frac{3-p_a}{2}}(4\pi^2\alpha')^{\frac{3-p_b}{2}} \times$$
$$\times \left[A^a_\mu A^\mu_b V_{Dp_a} V_{Dp_b} + (k_4^2 A^a_\mu A^\mu_b - k_4\cdot A^a k_4\cdot A^b)\int d^{p_a-3}y_a d^{p_b-3}y_b \sum_{k_6}\frac{exp[ik_6\cdot(y_b - y_a)]}{|k_6|^2}\right]. \tag{25}$$

On the torus there will be a contribution from C_{p-1}-forms, but only if $p_a = p_b$. Firstly, consider rectangular untwisted tori. In this case one can show that for brane-brane mixing the C-form contribution cancels the B-term, while for brane-antibrane mixing the B-contribution just multiplies by factor 2. Also, in Neumann-Dirichlet directions for brane-antibrane case the integrals in the above become delta-functions, and for $p \neq q$ we obtain

$$\chi_{ab} = g_a g_b tr_1\lambda_a tr_2\lambda_b \frac{1}{2\pi}\frac{l_s^6}{V_6}\frac{V_a V_b}{l_s^{p_a+p_b-6}}\sum_{n_i}\frac{\prod_{i=1}^{N_{DD}} exp\left[2\pi i\frac{n_i}{R_i}(y^i_b - y^i_a)\right]}{\sum_{i=1}^{N_{DD}} n_i^2 l_s^2/R_i^2}, \tag{26}$$

where $l_s^2 = 2\pi\alpha'$, and N_{DD} is the number of Dirichlet-Dirichlet directions. For $p = q = 3$ this agrees with our earlier CFT-derived mixing (in the context of an untwisted toroidal background).

Now we can generalize this to models with any compact manifold, not necessarily rectangular torus. Consider the action for the single component of $B_{\mu\nu}$ which we denote ϕ and neglect the transverse modes:

$$S = \frac{1}{2\kappa_{10}^2}\int\frac{d^4x}{(2\pi)^4}\int_{M^6} e^{-2\Phi}\left(\frac{1}{2}k_4^2\phi^2 + \frac{1}{2}d^{(6)}\phi\wedge_{*6} d^{(6)}\phi\right). \tag{27}$$

For a constant dilaton e^{Φ} is just a coupling constant g_s, and the Green functions therefore obey an usual free equation

$$(k_4^2 + \Delta_6)G_{\mu\nu;\rho\sigma}(y_0, y_1) = \delta_{\mu\rho}\delta\nu\sigma 2\kappa_{10}^2 g_s^2 \delta(y_1 - y_0), \tag{28}$$

where the 6D Laplacian is $\Delta_6 = d_{*6}d + (dd_{*6})$. If our 6-manifold admits Hermitian metric, Laplacian is Hermitian operator, and therefore we can write the general solution of the above equation in terms of orthonormal eigenfunctions basis $\{\phi_n\}$ with eigenvalues α_n:

$$G_{\mu\nu;\rho\sigma} = \delta_{\mu\rho}\delta_{\nu\sigma}2\kappa_{10}^2 g_s^2 \sum_n \frac{b_n(y_1)\phi_n^*(y_1)}{\alpha_n + k_4^2}\phi_n(y_0), \tag{29}$$

where $b_n(y_1)$ is the weight function. From the pole structure it is obvious that a contribution to the mass term occurs only when $\alpha_n = 0$, and in all the other contributions to obtain the mixing one can imply $k_4^2 = 0$ in the denominator.

Before we give an explicit example of the warped model with background fluxes we must solve the last problem. As far as we know, in type IIB model compactifications it is usually required to include vacuum expectation values for the three-form fluxes in order to stabilize the moduli. So we need to take this into account in our calculation of kinetic mixing.

The effect of the fluxes in some of the most popular models [16, 17] is that the metric is warped near the SM-branes:

$$ds^2 = e^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{-2A(y)}g_{mn}dy^m dy^n. \tag{30}$$

If we would consider the $D3$-branes (antibranes) then we see from our actions that the coupling of the gauge fields to the antisymmetric tensor and the R-R two-form is classically conformal. Therefore kinetic mixing cannot depend upon the warp factor and all the modification of our method must affect only Green function.

The idea is that we split both two-forms into $B_2 = B_2^{(4)} + B_2^{(6)} + B_2^{(46)}$ and similarly for the C_2, where the superscripts denote spacetime-spacetime, compact-compact and spacetime-compact indices correspondingly. We would like to give vevs to components $B_2^{(6)}, C_2^{(6)}$, that can't mediate kinetic mixing. The role of these components is that their vevs give masses to the two-form fields $B_2^{(4)}, C_2^{(4)}$. It can be easily seen, if we just write explicitly actions for these fields[1]. Thus, fluxes generate masses for the two-form fields; from the string point of view we have stabilized the moduli. Ignoring the non-compact dimensions' kinetic terms, for a component ϕ of $C_{\mu\nu}$ we have

$$L = \frac{e^{-2A}\sqrt{g}}{2\kappa_{10}^2}\left[g^{mn}\partial_m\phi\partial_n\phi + \frac{1}{8}|B_2^{(6)} \wedge d^{(6)}\phi|^2 + \frac{1}{8}|H_3^{(6)}|^2\phi^2\right], \tag{31}$$

where the last term is a mass of ϕ.

To estimate the effective ϕ-mass consider that H_3 and F_3 are defined as fluxes threading three-cycles[17],

$$\frac{1}{(2\pi)^2\alpha'}\int_{A_K} H_3 = m^K, \tag{32}$$

$$\frac{1}{(2\pi)^2\alpha'}\int_{B^K} F_3 = e_K, \tag{33}$$

where m^K, e_K are integers and $K = 1, \ldots h^3$. Thus we can estimate that

$$H_3, F_3 \sim n l_s^2 / V_3, \tag{34}$$

for some integer n and different three-cycle volumes V_3. Provided that the cycles threaded by the flux are larger than the string scale, we could neglect the second term in ϕ-Lagrangian and see that ϕ should behave as a massive scalar with characteristic length $L \sim V_3/(n l_s^2)$. It implies that the Green function for two-form fields behaves like

$$G_{\mu\nu;\rho\sigma}(y) \propto \delta_{\mu\rho}\delta_{\nu\sigma} e^{-y n l_s^2/V_3}. \tag{35}$$

Obviously, it is a 'Yukawa type'-interaction - exponential form is because of the massive mediating scalar.

5. Randall-Sundrum model Randall-Sundrum models[18, 19] deal with branes embedded in a slice of AdS_5. We consider a string-inspired scenario, in which matter fields are confined to branes, and introduce a string-inspired mediating B-field. The metric is taken to be:

$$ds^2 = e^{-2k|y|}\eta_{\mu\nu} dx^\mu dx^\nu + dy^2, \tag{36}$$

with k a parameter of the order of the Planck scale.

We consider the SM-brane to be a $D3$-brane at a position $y = 0$ in the fifth dimension, and a hidden brane - at some position $y_1 = \pi R$. The Lagrangian will have a form:

$$\begin{gathered} L = L_{bulk} + L_{D3} + L_{\bar{D}3}, \\ L_{bulk} = \frac{M_5^3}{2g^4}\int \frac{-1}{2} dB \wedge_{*5} dB + \frac{1}{2} m^2 B \wedge_{*5} B, \\ L_{D3} = \frac{1}{4g^2}\int_{D3} \frac{1}{2\pi\alpha'} F \wedge_{*4} B + \frac{1}{(2\pi\alpha')^2} B \wedge_{*4} B, \\ L_{\bar{D}3} = -L_{D3}. \end{gathered} \tag{37}$$

The coupling of the B-field to the gauge field is specified by the DBI action, but we need to introduce in this model 3 parameters: the coupling of the kinetic term M_5, the mass-like parameter m and the string mass. If we imagine our model to be derived from an underlying string theory, we could expect M_5 to be related to Planck's constant and the volume of the compactification, and m to be determined by the fluxes; the string scale however generally exists as a free parameter to be determined by experiment.

To calculate the mixing, we require the Green function, and thus we derive the simple equation of motion:

$$\left[e^{2k|y|}\eta^{\alpha\beta}\partial_\alpha\partial_\beta + \partial_5\partial_5 - m^2\right] B^{(4)}_{\mu\nu} = 0. \tag{38}$$

From the above action we also find boundary conditions for the B-field at the brane:

$$\partial_y B^{(4)}_{\mu\nu} - \frac{M_2^4}{M_5^3} B^{(4)}_{\mu\nu}|_{y=0,\pi R} = 0. \tag{39}$$

The idea of finding the Green function in fifth dimension in this case (the residual of Green function is usual), defined as

$$\Delta G(y, y') = \delta(y - y'), \tag{40}$$

is to decompose it into 'advanced' and 'retarded' components that satisfy the homogenous equations and to impose matching conditions at $y = y'$.Then for each component we separate the variables and obtain the result (in case of massive RS-action the equations can be solved exactly):

$$G(y_0, y_1) = \frac{4g^2}{M_5^3 m} \frac{1}{sinh m\pi R} \frac{1}{(1 - \frac{M_s^8}{M_5^6 m^2})}, \tag{41}$$

where y_0, y_1 are coordinates of the branes. This gives mixing:

$$\chi = g_a g_b \frac{32 M_s^4}{M_5^3 m} \frac{1}{sinh m\pi R} \frac{1}{(1 - \frac{M_s^8}{M_5^6} m^2)}. \tag{42}$$

To estimate χ one could try to identify M_5 with M, composed from the existing RS parameters, where $e^{-k\pi R} = M_{SUSY}/M_{PL}$, (so $\pi k R \sim 37$), $M_{Pl}^2 \approx M^3/k$. We also assume the quantity $M_s^8/(M_5^6 m^2)$ to be small and take an intermediate string mass of $\sqrt{M_{SUSY} M_{Pl}}$. This gives

$$\chi \approx g_a g_b \times \frac{M_{SUSY}^2 \pi R}{m} \frac{1}{sinh \pi m R}.$$

In the limit $mR \ll 1$, one has

$$\chi \sim g_a g_b \times \frac{M_{SUSY}^2}{m^2}. \tag{43}$$

We see that for gauge couplings of order unity values of $m \sim 10^4 M_{SUSY}$ leads to a mixing that can be observable in the near future.

In the opposite limit, $mR \gg 1$ one gets the expected exponential suppression because of non-zero mass:

$$\chi \sim g_a g_b \times \frac{M_{SUSY}^2}{m^2}(m\pi R)e^{-m\pi R}. \tag{44}$$

Thus, in RS-backgrounds the kinetic mixing can take any value between zero and the experimental limits, depending on the configuration. In more realistic warped models with background fluxes, for example Klebanov-Tseytlin throat or some different Calabi-Yau manifolds with explicitly known metric, the kinetic mixing demonstrates analogous behavior [1], being rather large in the $mR \ll 1$ limit and being exponentially damped due to the backreaction of fluxes in opposite limit .

6. Conclusion It was shown that models with massless hidden $U(1)$s can be found by compactifications of string theory, and that they are natural for certain classes of backgrounds. Nevertheless, this hidden sector can have observable experimental effects because they will typically mix with photon via so-called kinetic mixing term. These effects were calculated by using conformal field theory and supergravity techniques. The latter method is rather general and can be used even when fluxes are included to stabilize the moduli on the compactification manifolds. Also it was demonstrated that in general kinetic mixing is non-zero even if all the $U(1)$s involved are anomaly free. This provides extremely sensitive tests in many low-energy experiments. The size of kinetic mixing is model-dependent. Yet, for wide range of parameter values it is often within reach of the current and near future experiments. The discovery of such effects could become the first indirect confirmation of the existing of extra-dimensions.

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