Heavy quark effective theory

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Main problems of HEP

- Phenomenon of generations
- Origin of baryogenesis
- Connections between flavor physics and TeV-scale physics

Necessity of precision calculations of the SM parameters.

Main ideas of HQET

- Heavy hadron contains a charm or a bottom quark
- Exchange of soft gluons
- Perturbation in $\frac{\Lambda_{QCD}}{m_Q} << 1$

First approximation: the HQ moves with hadron's velocity

Deviations of the behavior of the system from the ideal case are described by $\left(\frac{1}{m_Q}\right)^n$ terms.

Structure of HQET

$$L_{QCD} = -\frac{1}{4} G^{A}_{\mu\nu} G^{A\mu\nu} + \overline{q}\left(i\hat{D} - m_Q\right) q + counterterms$$

$$L_{QCD}\left(m_Q \rightarrow \infty\right) = \overline{Q}_v \left(iv \cdot D\right) Q_v$$

$$\left(\frac{1 + \hat{v}}{2}\right) Q_v = Q_v$$

$$L = L_0 + L_1 + \ldots$$

$$L_1 = -\overline{Q}_v \left(\frac{D_\perp^2}{2m_Q}\right) Q_v - a\left(\mu\right) g \overline{Q}_v \left(g_{\mu\nu} \frac{G^{\mu\nu}}{4m_Q}\right) Q_v$$

Structure of HQET

- However, we can not obtain several matrix elements, that are necessity for calculations, within the framework of HQET
- For example, these are matrix elements

$$\mu_\pi^2(\mu) \equiv \frac{1}{2M_B}\left\langle B\left|\bar{b}\left(i\vec{D}\right)^2 b\right|B\right\rangle_\mu, \qquad \mu_G^2(\mu) \equiv \frac{1}{2M_B}\left\langle B\left|\bar{b}\,\frac{i}{2}\sigma_{jk}G^{jk}\,b\right|B\right\rangle_\mu$$

$$\rho_D^3(\mu) \equiv \frac{1}{2M_B}\left\langle B\left|\bar{b}\left(-\frac{1}{2}\vec{D}\cdot\vec{E}\right)b\right|B\right\rangle_\mu, \qquad \rho_{LS}^3(\mu) \equiv \left\langle B\left|\bar{b}\left(\vec{\sigma}\cdot\vec{E}\times i\vec{D}\right)b\right|B\right\rangle_\mu .$$

which arise in the Lagrangian in high orders of the inverse quark mass.

- So, we should carry out experimental measurements or make non-perturbative calculations

B-meson decays

$$B \longrightarrow X_c l\nu_l \qquad\qquad B \longrightarrow X_s \gamma$$

- **Described by HQET**
- **Allow to obtain SM parameters:**

$$\Gamma_{sl}(b \mapsto c) = \frac{G_F^2\, m_b^5(\mu)}{192\pi^3}\left|V_{cb}\right|^2\left(1 + A_{ew}\right)\Bigg[z_0(r)\left[1 + A_3^{pert}(r,\mu)\right]\left(1 - \frac{\mu_\pi^2(\mu) - \mu_G^2(\mu) + \frac{\rho_D^3(\mu) + \rho_{LS}^3(\mu)}{m_b(\mu)}}{2m_b^2(\mu)}\right)$$
$$-\left(1 + A_5^{pert}(r,\mu)\right)2(1-r)^4\,\frac{\mu_G^2(\mu) - \frac{\rho_D^3(\mu) + \rho_{LS}^3(\mu)}{m_b^3(\mu)}}{m_b^2(\mu)} + \left(1 + A_D^{pert}\right)d(r)\,\frac{\rho_D^3(\mu)}{m_b^3(\mu)}$$
$$+ 32\pi^2\left(1 + A_{6c}^{pert}(r)\right)\left(1 - \sqrt{r}\right)^2\frac{H_c}{m_b^3(\mu)} + 32\pi^2\,\widetilde{A}_{6c}^{pert}(r)\left(1 - \sqrt{r}\right)^2\frac{\widetilde{H}_c}{m_b^3(\mu)}$$
$$+ 32\pi^2 A_{6q}^{pert}(r)\,\frac{F_q}{m_b^3(\mu)} + O\left(\frac{1}{m_b^4}\right)\Bigg].$$

Experimental approach

- Straight measurement of moments:

$$R_n\left(E_{cut},\mu\right)=\int_{E_{cut}}\left(V-\mu\right)^n\frac{\mathrm{d}\Gamma}{\mathrm{d}V}\,\mathrm{d}\,V$$

- Minimization of the χ^2 function:

$$\chi^2=\sum_{i,j}\left(\langle X\rangle_i^{meas}-\langle X\rangle_i^{pred}\right)cov_{ij}^{-1}\left(\langle X\rangle_j^{meas}-\langle X\rangle_j^{pred}\right).$$

Experimental results

$$\left|V_{cb}\right| = (41.93 \pm 0.65 \pm 0.07 \pm 0.63) \cdot 10^{-3}$$

$$B_{cl\nu} = (10.590 \pm 0.164 \pm 0.006)\ \%$$

$$m_b = (4.564 \pm 0.076 \pm 0.003)\ GeV$$

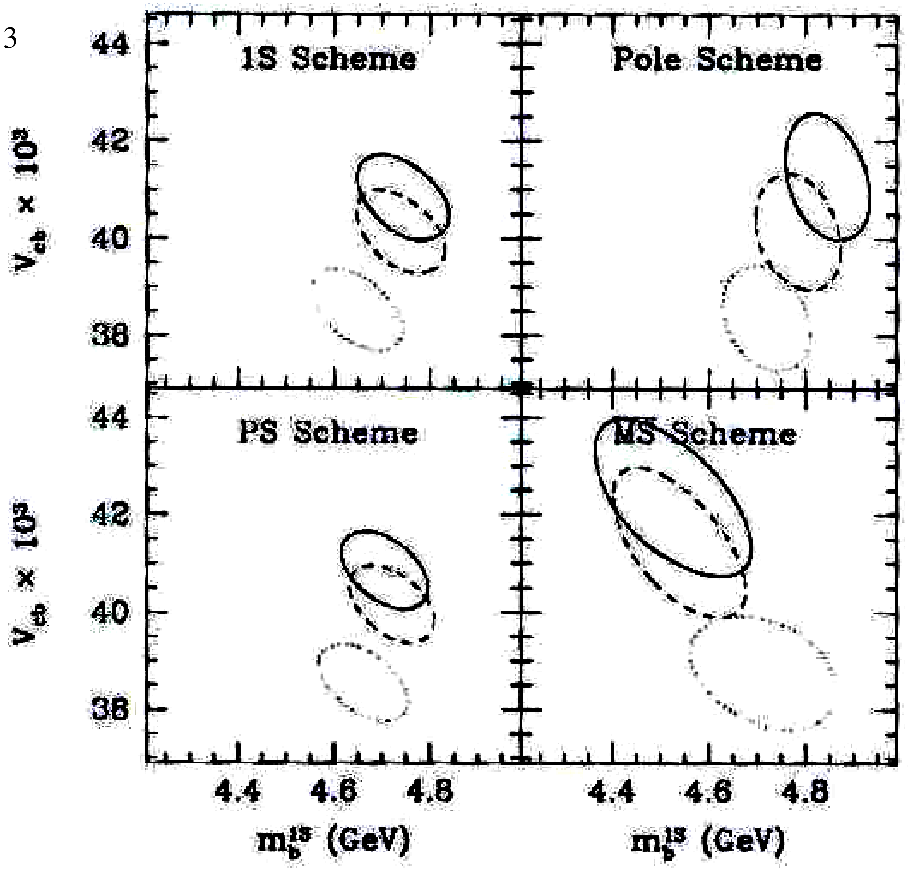
$$m_c = (1.105 \pm 0.116 + 0.005)\ GeV$$

$$\mu_\pi^2 = (0.557 \pm 0.091 \pm 0.013)\ GeV^2$$

$$\mu_G^2 = (0.358 \pm 0.060 \pm 0.003)\ GeV^2$$

$$\tilde{\rho}_D^3 = (0.162 \pm 0.053 \pm 0.008)\ GeV^3$$

$$\tilde{\rho}_{LS}^3 = (-0.174 \pm 0.098 \pm 0.003)\ GeV^3$$



Lattice calculations

- We can avoid experimental measurements and carry out numerical calculations of matrix elements, form-factors etc.
- A quark field is presented as a function on a discrete space-time lattice.
- Only interactions between neighboring vertices of the lattice are taken into account by exchange of gluons.

Lattice calculations

$$S_{LQCD} = \sum_{x,y} Q^{+}(x)\left(\delta_{x,y} - K_Q(x,y)\right) Q(y)$$

$$K_Q(x,y) \equiv \left(1 - \frac{aH_0}{2n}\right)^n_{t+1} \left(1 - \frac{a\delta H}{2}\right)_{t+1} \delta_4^{(-)} U_4^{+}(t) \left(1 - \frac{a\delta H}{2}\right)_t \left(1 - \frac{aH_0}{2n}\right)^n_t$$

$$\delta_4^{(-)} \equiv \delta_{x_4 - 1, y_4} \delta_{\vec{x}, \vec{y}}$$

$$H_0 \equiv -\frac{\Delta^{(2)}}{2m_Q},$$

$$\delta H \equiv -c_B \frac{g}{2m_Q} \vec{\sigma} \cdot \vec{B} \cdot$$

$$\Delta^{(2)} Q(x) = \sum_{i=1}^{3} \Delta_i^{(2)} Q(x) =$$

$$= \sum_{i=1}^{3} \left[U_i(x) Q\left(x + \hat{i}\right) + U_i^{+}\left(x - \hat{i}\right) Q\left(x - \hat{i}\right) - 2Q(x) \right]$$

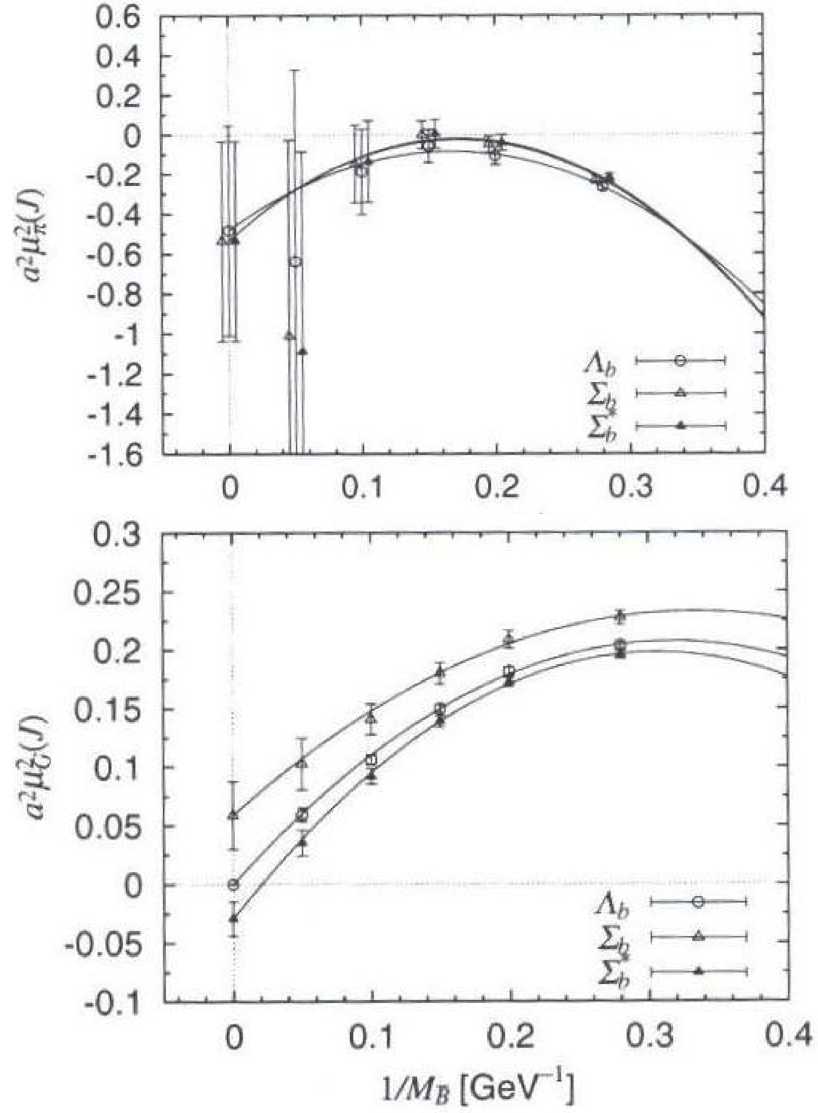
Results of lattice calculations

$$\left|V_{cb}\right| = (40.8 \pm 0.9)\times 10^{-3},$$

$$m_b = (4.74 \pm 0.10)\, GeV,$$

$$\overline{\Lambda} = 0.68^{+0.02}_{0.12}\, GeV,$$

$$\lambda_1 = -\mu_\pi^2 = -(0.45 \pm 0.12)\, GeV^2 \cdot$$



Conclusions

- Use of the HQET allows one to obtain several important parameters of the SM.
- These methods have allowed to improve the precision in the knowledge of several fundamental parameters in the Nature.
- A number of hadronic matrix elements are determined for the moment analysis of the semileptonic and radiative B-decays.
- Lattice calculations are very promising but not yet precise enough.