# DESY Summer Student Program

# Rolling Tachyon in Open String Field Theory

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**Abstract**

During the Summer Student Program I worked in the String Theory Group under the supervision of Yuji Okawa. For the first weeks of my stay I learned some basic concepts and methods of string theory, such as conformal invariance, operator product expansions, the Virasoro algebra and vertex operators. Then I went on to work on the specific problem of the rolling tachyon, in the framework of string field theory. The report is divided in two main sections. The first one is a short introduction to string field theory, a relatively intuitive formulation of string theory. The second one concentrates on the solution of the equation of motion of the rolling tachyon and the understanding of its behaviour, starting with a review of previous work on the subject and then outlining my own results. At the end I give some conclusions and some ideas on which way could be taken in the future to study this problem further.

# 1 Heuristic introduction to String Field Theory

Even though the formal description of string theory is very complicated, its conceptual understanding can be easily carried out by using the concepts of fields and excitation modes of a string, in a formulation that is called string field theory [1].

In particle mechanics, one starts with the description of a point particle that has a definite trajectory in D space-time dimensions, through a D-vector $X^\mu(\tau)$ which depends on a parameter $\tau$ such as the proper time of the particle or its time coordinate in a fixed reference frame. Then one goes on to describe particles by fields $\phi(x^\mu)$, $A^\nu(x^\mu)$, $\Psi^\rho(x^\mu)$, etc. which take different values at every spacetime point. Note here that the parameter $\tau$ has disappeared since there no longer exists a trajectory which the particles follow.

In order to formulate string theory, one can start with the description of a one dimensional classical object. This will also be represented by a D-vector $X^\mu(\tau, \sigma)$, which will depend on a parameter $\tau$ analog to that of the point particle, but also on $\sigma$, which parametrizes the string. This could be regarded as an infinite number of point particles (one for each value of $\sigma$, with $0 \leq \sigma \leq L$), with some constraints that keep them together, and one could then promote each of these to a field $\phi(x^\mu(\sigma))$ to get a description of the string as an infinite number of fields. The $\tau$ dependence of the fields has again disappeared, but we still have the parameter $\sigma$, which could still represent, in a certain sense, the longitudinal direction on the string. Now, just as a classical string can be described in the Fourier space in terms of its excitation modes (instead of its spacetime vibrating amplitudes), we could also describe the fields of strings through its Fourier modes. If we impose boundary conditions at the ends of the string, these modes are infinite but discrete, and it turns out that the coefficient that accompanies each mode is a particle-field analogous to those of normal field theory, and of course independent of $\sigma$. That is, each excitation state of the string represents a type of particle, and the knowledge of their behaviour implies the knowledge of the behaviour of the string.

With this description, string theory reduces to a field theory with infinitely many fields. However, it is much more constrained than QFT, and the action of the string, and therefore the interactions between the fields are completely determined by the background in which the string lives. The calculations on string field theory are very complicated and the dynamics of the string are still not well understood. One of the main challenges of the theory right now is solving the equations of motion and understanding the dynamics of the string in spacetime.

## 2 Solving the equation of motion for the rolling tachyon

The state of a string is written in string field theory in a 26 dimensional flat Minkowskian spacetime, as an expansion of its vibrating modes as

$$\Psi = \int \frac{d^{26}k}{(2\pi)^{26}} \Big[ T(k) c_1|0;k\rangle + A_\mu(k)\alpha^\mu_{-1} c_1 |0;k\rangle \dots \Big] \tag{2.1}$$

where the state $|0;k\rangle$ is the ground state with momentum $k^\mu$, and the $\alpha^\mu_{-1}$ and $c_1$ are creation operators of bosonic matter and fermionic ghosts respectively. The coefficients $T(k)$, $A_{\mu(k)}$,... are the fields that represent the different kind of particles:

$T(k)$: Tachyon. Mass $m^2 < 0$, spin 0

$A_{\mu(k)}$: Bosonic, massless, spin 1.

... higher spin fields.

We are interested in analyzing the time-dependence of a homogeneous string state, so for our purposes, the only interesting variable will be $x^0$ (or equivalently $k^0$). Now, the easiest start point for understanding the time dependence of the string is to analyze the tachyonic excitation mode $T(x^0)$ which can be expanded as

$$T(x^0) = \mp e^{\frac{1}{\sqrt{\alpha'}}x^0} + \sum_{n=2}^{\infty} (\mp)^n \beta_n \, e^{\frac{1}{\sqrt{\alpha'}}nx^0} \, . \tag{2.2}$$

The top sign gives us the physical solution: the tachyon oscillates around the tachyon vacuum. Our goal is to calculate the coefficients $\beta_n$ and analyze the behaviour of the solution for all times. This was done in [2] and independently in [3] using the solution of the string motion in the Schnabl gauge. The time dependence of the tachyon field they found is

$$\begin{aligned} T(x^0) = \mp\, e^{\frac{1}{\sqrt{\alpha'}}\,x^0} + \; 0.15206 \; e^{\frac{1}{\sqrt{\alpha'}}\,2x^0} \quad & \mp 2.148 \cdot 10^{-3}\, e^{\frac{1}{\sqrt{\alpha'}}\,3x^0} \\ + 2.619 \cdot 10^{-6}\, e^{\frac{1}{\sqrt{\alpha'}}\,4x^0} \; & \mp 2.791 \cdot 10^{-10}\, e^{\frac{1}{\sqrt{\alpha'}}\,5x^0} \\ + 2.801 \cdot 10^{-15}\, e^{\frac{1}{\sqrt{\alpha'}}\,6x^0} \; & \mp 2.729 \cdot 10^{-21}\, e^{\frac{1}{\sqrt{\alpha'}}\,7x^0} + \dots \end{aligned} \tag{2.3}$$

The rapidly decreasing behaviour of the coefficients suggests that the series is absolutely convergent, and implies a wildly oscillatory motion of the tachyon, with ever growing amplitude, analogous to the one found in [4] using level truncation (an approximation in which only the first excitation modes of the string are taken into account, neglecting higher spin modes). This anomalous behaviour could be due to a bad definition of the fields or a bad gauge fixing condition, and might be repaired with some manipulations of the field, leading to the expected convergence of the tachyon to its ground state as $t \to \infty$. However, the solutions in [2] and [3] had to be calculated numerically, which makes such manipulations complicated.

A further step towards the understanding of the behaviour of the tachyon was given in [5] by Y. Okawa and M. Kiermaier. Using a different gauge, they found an analytic expression for the coefficients $\beta_n$ of the solution of the equation of motion.

## 3 Analytical solutions

The equation of motion of the string field $\Psi^{(n)}$ is written as

$$Q_B \Psi^{(n)} = -\sum_{i=1}^{n-1} \Psi^{(n-i)} * \Psi^{(i)} \tag{3.1}$$

where $*$ indicates the star product of the fields (see [1]) and $Q_B$ is the charge associated with the BRST symmetry transformation. The solutions of this equation can be expanded in terms of the BPZ inner products of the states $|\Psi^{(n)}\rangle$ with the dual states of an orthonormal basis which expands the whole Hilbert space of states $\langle\phi|$. We will only study the tachyonic field, so we concentrate in those basis vectors which single out this excitation mode from higher spin fields in (2.1) . With a correct choice of the basis we can relate the coefficients $\beta_n$ of the expansion (2.2) with the BPZ inner products:

$$\langle\, \phi, \Psi^{(n)}\, \rangle = \beta_n. \tag{3.2}$$

### 3.1 Left-handed solutions

One solution of the equation of motion (3.1) is given by

$$\begin{aligned}
\beta_n &= \langle\, \phi\,, \Psi_L^{(n)}\, \rangle \\
&= \Big\langle\, f \circ \phi(0)\ cV(1) \int_1^2 dt_1 \int_{t_1}^3 dt_2 \int_{t_2}^4 dt_3\, \ldots \int_{t_{n-2}}^n dt_{n-1}\, V(t_1)\, V(t_2)\, V(t_3)\, \ldots\, V(t_{n-1}) \Big\rangle_{\mathcal{W}_n} \\
&= \Big\langle\, f \circ \phi(0)\ cV(1) \prod_{j=1}^{n-1} \int_{t_{j-1}}^{j+1} dt_j\, V(t_j)\, \Big\rangle_{\mathcal{W}_n} \qquad \text{with} \quad t_0 \equiv 1\,.
\end{aligned} \tag{3.3}$$

Here $f$ represents the conformal transformation used to express the BPZ inner product on the left as an expectation value on the surface $\mathcal{W}_n$ (see figure 1). The explicit expression for $f$ is

$$f(z) = \frac{2}{\pi} \arctan z. \tag{3.4}$$

The operators $V(z)$ represent the exactly marginal deformations of the conformal field theory for the string background. For the case of the rolling tachyon problem, these operators are

$$V(z) = e^{\frac{1}{\sqrt{\alpha'}}\, x^0(z)}. \tag{3.5}$$

In order to extract the tachyonic field from this solution, the basis vectors we have to use are related by the state-operator correspondence to

$$\phi(z) = -c\partial c\, e^{\frac{-n}{\sqrt{\alpha'}}\, x^0}\,(z). \tag{3.6}$$

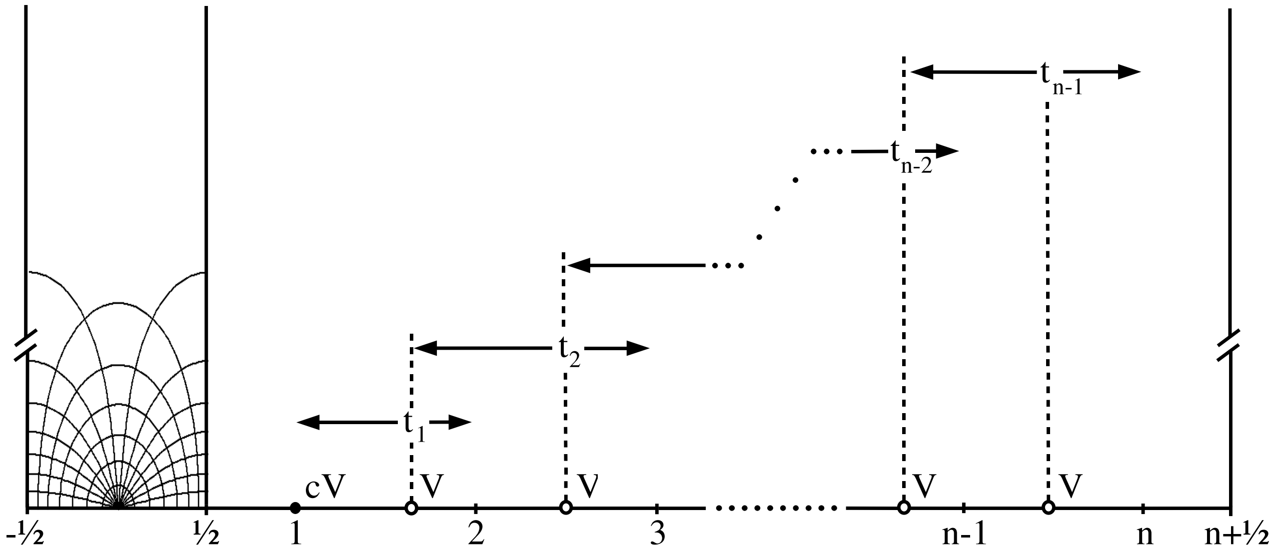


Figure 1: Illustration of $\Psi_L^{(n)}$. The solid dot represents the $cV$ insertion, and the circles represent the $V$ insertions. The integration region of $t_j$ is from $t_{j-1}$ to $j+1$.

These are tensor operators with weight $n^2-1$ which transform under conformal transformations $z \to f(z)$ as

$$f\circ\phi(z_0) = \Big|\frac{\partial f}{\partial z}\Big|_{z=z_0}^{n^2-1} \phi(f(z_0)). \tag{3.7}$$

The only thing that is left is to calculate the values of the coefficients $\beta_n$ from equation (3.3). For this purpose it will be useful to make a new conformal transformation $g(z) = \tan\left(\frac{\pi z}{n+1} - \alpha\right)$ which maps the surface $\mathcal{W}_n$ to the whole upper half plane (UHP). In this step we have to take care of all of the conformal transformation factors. The whole operator inside the expectation value is also a tensor operator of weight $n^2-1$, since the combinations $cV(z)$ and $dtV(z)$ are weightless. Therefore we can write

$$\begin{aligned}
\beta_n &= \langle\, \phi\,, \Psi_L^{(n)}\,\rangle \\
&= \Big\langle\, gf\circ\phi(0)\ cV(g(1)) \int_{g(1)}^{g(2)} dt_1 \int_{t_1}^{g(3)} dt_2\, \ldots \int_{t_{n-2}}^{g(n)} dt_{n-1}\, V(t_1)\, V(t_2)\, \ldots\, V(t_{n-1}) \Big\rangle_{UHP} \\
&= \Big|\frac{\partial g\circ f}{\partial z}\Big|_{z=g(0)}^{n^2-1} \Big\langle\, \phi(g(0))\ cV(g(1)) \prod_{j=1}^{n-1} \int_{t_{j-1}}^{g(j+1)} dt_j\, V(t_j)\, \Big\rangle_{UHP} \qquad \text{with} \quad t_0 \equiv g(1)\,.
\end{aligned} \tag{3.8}$$

We recall the following results, which are useful for the calculations

$$-\langle c\partial c(z_1)c(z_2)\rangle_{uhp} = (z_1 - z_2)^2 \tag{3.9}$$

$$\langle e^{\frac{-n}{\sqrt{\alpha'}}x^0(z_1)} e^{\frac{1}{\sqrt{\alpha'}}x^0(z_2)}\rangle_{uhp} = (z_1 - z_2)^{-2n} \tag{3.10}$$

$$\langle e^{\frac{-n}{\sqrt{\alpha'}}x^0(z_1)} e^{\frac{1}{\sqrt{\alpha'}}x^0(z_2)} e^{\frac{1}{\sqrt{\alpha'}}x^0(z_3)}\rangle_{uhp} = (z_1 - z_2)^{-2n}(z_1 - z_3)^{-2n}(z_2 - z_3)^2 \tag{3.11}$$

Using these results in (3.8), and taking the limit $\alpha \to \frac{\pi}{2}$ to simplify the expression (the value of $\beta_n$ is independent of $\alpha$), we obtain

$$\boxed{\beta_n = \Big(\frac{2}{n+1}\Big)^{n^2-1} \int_{t_0}^{g(2)} dt_1 \int_{t_1}^{g(3)} dt_2 \ldots \int_{t_{n-2}}^{g(n)} dt_{n-1} \prod_{0\leq i<j<n} (t_i - t_j)^2} \tag{3.12}$$

where we have defined $t_0 = g(1) = -\cot\frac{\pi}{n+1}$. These integrals can be analytically claculated for any $n$. The first few terms of this solution for the equation of motion of the rolling tachyon are

$$\begin{aligned} T(x^0) = \mp\, e^{\frac{1}{\sqrt{\alpha'}}x^0} + \ 0.15206\ e^{\frac{1}{\sqrt{\alpha'}}2x^0} \quad &\mp 2.588\cdot 10^{-3}\, e^{\frac{1}{\sqrt{\alpha'}}3x^0} \\ + 3.725\cdot 10^{-6}\, e^{\frac{1}{\sqrt{\alpha'}}4x^0} \ &\mp 3.569\cdot 10^{-10}\, e^{\frac{1}{\sqrt{\alpha'}}5x^0} \\ + 1.851\cdot 10^{-15}\, e^{\frac{1}{\sqrt{\alpha'}}6x^0} \ &+ \ldots \end{aligned} \tag{3.13}$$

We can see that the strange oscillating behaviour of the tachyon appears again, but this time we have an analytical calculation of the coefficients, and we will see in section 4 how to manipulate them and try to find the expected solution. But first, there is another problem with the solution $\Psi_L$ which has to be solved.

## 3.2 Solutions satisfying the reality condition

Not all higher spin fields are real in $\Psi_L$. Of course this does not make sense in a theory which is to represent physical entities, and the solution has to be modified. This was also done in [5] and their result can be written as

$$\Psi = \frac{1}{\sqrt{U}} * \Psi_L * \sqrt{U} + \frac{1}{\sqrt{U}} * Q_B\sqrt{U}\,. \tag{3.14}$$

Here $U$ represents the gauge parameter which relates $\Psi_L$ to its conjugate $\Psi_R$, which is itself another solution for the equation of motion. It can be expanded in terms of the parameter $\lambda$, and its explicit

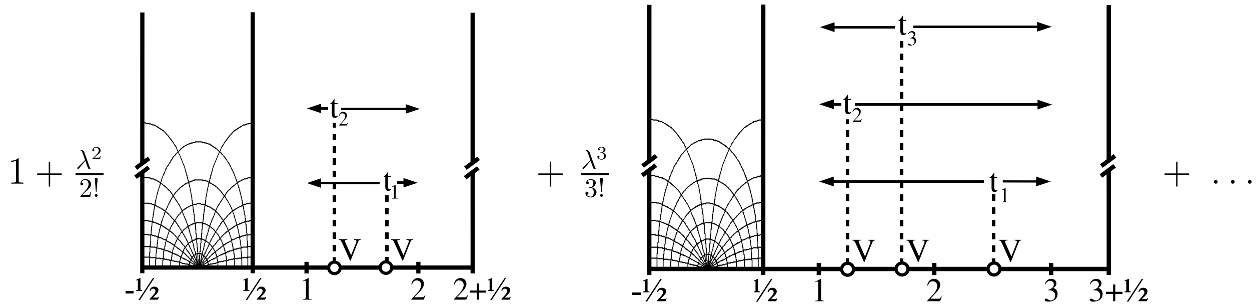


Figure 2: Illustration of the expansion $U = 1 + \lambda^2\, U^{(2)} + \lambda^3\, U^{(3)} + \mathcal{O}(\lambda^4)$.

expression takes the form

$$\begin{aligned} U &= 1 + \sum_{n=2}^{\infty} \lambda^n\, U^{(n)}\,, \\ \langle\, \phi\,, U^{(n)}\,\rangle &= \langle\, f\circ\phi(0)\, V^{(n)}(1,n)\,\rangle_{\mathcal{W}_n} \\ &= \frac{1}{n!}\int_1^n dt_1 \int_1^n dt_2 \ldots \int_1^n dt_n\, \langle\, f\circ\phi(0)\, V(t_1)\, V(t_2)\, \ldots V(t_n)\,\rangle_{\mathcal{W}_n}\,. \end{aligned} \tag{3.15}$$

This is represented in figure 2. It is interesting by itself to study the behaviour of the tachyonic modes of the state $U$, and this can be done in an analog manner as we did with $\Psi_L$. However, in this case the operator insertions are different, so the basis states we have to use to single out these modes now are

$$\phi(z) = -\frac{1}{2} c\ \partial c\ \partial^2 c\ e^{\frac{-n}{\sqrt{\alpha'}}\, x^0}\ (z). \tag{3.16}$$

Taking into account that $\langle c\ \partial c\ \partial^2 c\ (z)\rangle_{uhp} = -2$, and that the operator inside the expectation value of (3.15) behaves this time as a $n^2$ weighted tensor, we can use the same conformal transformation and the same tricks as in last section to obtain

$$\boxed{\langle \phi, U^{(n)}\rangle = \Big(\frac{2}{n+1}\Big)^{n^2} \frac{1}{n!} (g(n) - g(1))^{n^2} \int_0^1 dt_1 \int_0^1 dt_2\, \ldots \int_0^1 dt_{n-1} \prod_{1\le i<j\le n} (t_i - t_j)^2} \tag{3.17}$$

This can again be easily calculated for any $n$. The first few values are represented in Table 1.

We can see that these values have the same behaviour as those of $\beta_n$, but the integrals on (3.17) are much simpler than those on (3.8). This is the main reason for their interest, it will be much easier to manipulate the results of $\langle \phi, U^{(n)}\rangle$ than those of $\beta_n$ and those of the real solutions, and they are expected to have the same behaviour.

| $n$ | $\langle\phi, U^{(n)}\rangle$ |
|---|---|
| 2 | $\frac{64}{2187} \approx 0.0292638$ |
| 3 | $\frac{1}{2160} \approx 4.62963 \cdot 10^{-4}$ |
| 4 | $7.72061 \cdot 10^{-7}$ |
| 5 | $1.36673 \cdot 10^{-10}$ |

Table 1: Numerical values of the tachyon profile coefficients for $U^{(n)}$

Now we are ready to calculate explicitly the solutions (3.14) satisfying the reality condition. The calculations are completely analog to those for $\Psi_L$, with the same conformal transformations, the same conformal factors and the same basis states. However, first of all we have to expand $\frac{1}{\sqrt{U}} * \Psi_L * \sqrt{U}$ and $\frac{1}{\sqrt{U}} * Q_B\sqrt{U}$ in terms of $\lambda$. Once we have done this we realise that the first correction to the left-handed solution comes in the order $O(\lambda^4)$. The new solution takes now the form

$$T(x^0) = \mp e^{\frac{1}{\sqrt{\alpha'}}x^0} + 0.15206\ e^{\frac{1}{\sqrt{\alpha'}}2x^0} \mp 2.588 \cdot 10^{-3}\, e^{\frac{1}{\sqrt{\alpha'}}3x^0} + 4.6889 \cdot 10^{-6}\, e^{\frac{1}{\sqrt{\alpha'}}4x^0} \mp 9.089 \cdot 10^{-10}\, e^{\frac{1}{\sqrt{\alpha'}}5x^0} + \dots \tag{3.18}$$

The behaviour of the solution is still wildly oscillating. Therefore we can analyze more deeply the results $\langle\phi, \Psi_L\rangle$ and $\langle\phi, U\rangle$, which have a much easier expression, and expect them to behave as the real solution. We will do this in the next section.

## 4 $\beta$-deformations

As we already mentioned, the behaviour of the tachyon motion in the form expressed by equations (2.3), (3.13) and (3.18) exhibit a wild oscillation. However, we can manipulate these results by applying what is called a $\beta$-deformation (see [6]), which is a kind of gauge transformation of the fields.

The effect of $\beta$-deformations can be seen as a transformation of the surfaces $\mathcal{W}_n$, which takes every point $n$ on the boundary to the point $\frac{1}{2} + (n - \frac{1}{2})e^{\beta}$ (except for the region $[-\frac{1}{2}, \frac{1}{2}]$, that stays invariant). Of course, the total surface is dilated and $\mathcal{W}_n$ goes to $\mathcal{W}_{1+ne^{\beta}}$.

We will study the effects of such a transformation on the solution $\Psi_L$ which has an easy analytical expression. The coefficients $\beta_n$ (3.3) now depend on the factor $\beta$ of the transformation. Our main objective is to find a value of $\beta$ for which this series shows a smooth behaviour, which could happen if the perturbation theory breaks down for some time t.

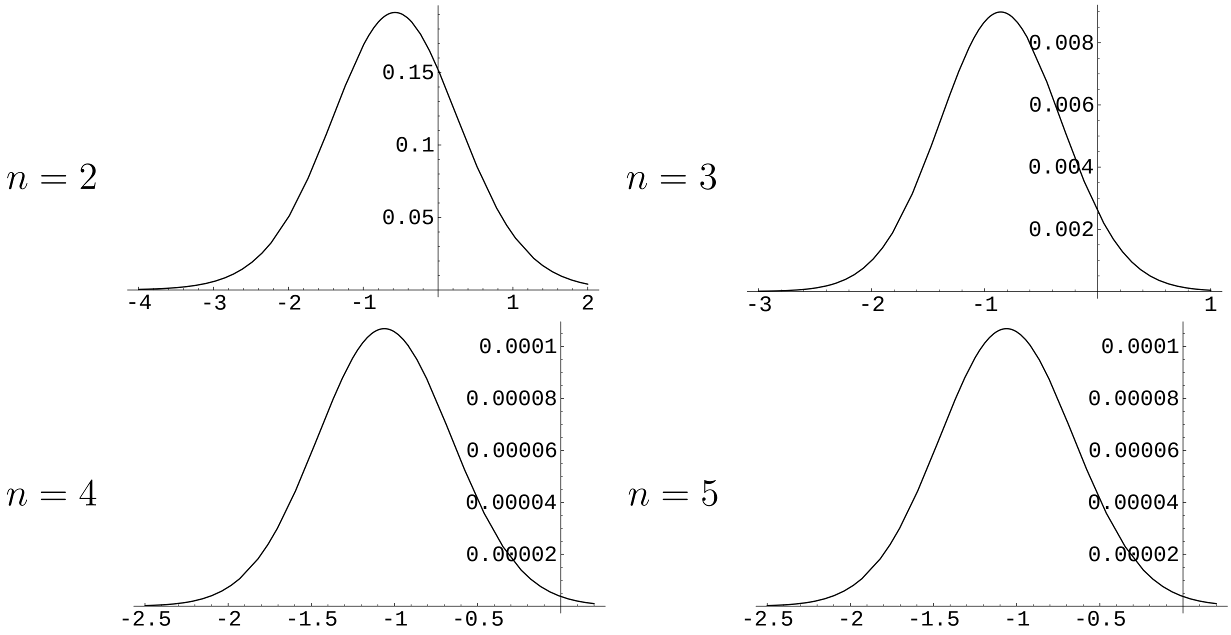


Figure 3: $\beta$ dependence of the coefficients $\langle\,\phi\,,\Psi_L^{(n)}(\beta)\,\rangle$ for $n = 2, 3, 4, 5$.

The expression for the coefficients of the series will now be (compare with (3.3)):

$$\langle\,\phi\,,\Psi_L^{(n)}(\beta)\,\rangle = \Big\langle\, f\circ\phi(0)\ cV(t_0)\prod_{j=1}^{n-1}\int_{t_{j-1}}^{\frac{1}{2}+(j+\frac{1}{2})e^{\beta}} dt_j\, V(t_j)\,\Big\rangle_{\mathcal{W}_{1+ne^{\beta}}} \qquad \text{with} \quad t_0 \equiv \frac{1}{2}+\frac{1}{2}e^{\beta}\,. \quad (4.1)$$

This is computed in the same way as we did for $\beta_n$, using now the conformal transformation

$$g(z) = \tan\Big(\frac{\pi z}{1+ne^{\beta}} - \alpha\Big) \qquad (4.2)$$

which takes the surface $\mathcal{W}_{1+ne^{\beta}}$ to the upper half plane. Again, taking the limit $\alpha \to \frac{\pi}{2}$ simplifies the calculations. The final expression is

$$\boxed{\langle\,\phi\,,\Psi_L^{(n)}(\beta)\,\rangle = \Big(\frac{2}{1+ne^{\beta}}\Big)^{n^2-1}\int_{t_0}^{g(\frac{1}{2}+\frac{3}{2}e\beta)} dt_1\int_{t_1}^{g(\frac{1}{2}+\frac{5}{2}e\beta)} dt_2\,\ldots\int_{t_{n-2}}^{g(\frac{1}{2}+\frac{2n-1}{2}e\beta)} dt_{n-1}\prod_{0\le i<j<n}(t_i-t_j)^2} \qquad (4.3)$$

These integrals have been calculated for $n = 2, 3, 4, 5$ and their $\beta$-dependences have been plotted on figure 3. However what is most important for us is the ratio of the coefficients of order $n$ and $n+1$, since it determines the behaviour of the series that describes the equation of motion of the tachyon. If, for any $\beta$ it is satisfied to all orders

$$\frac{\langle\,\phi\,,\Psi_L^{(n+1)}(\beta)\,\rangle}{\langle\,\phi\,,\Psi_L^{(n)}(\beta)\,\rangle} \approx \frac{\langle\,\phi\,,\Psi_L^{(n)}(\beta)\,\rangle}{\langle\,\phi\,,\Psi_L^{(n-1)}(\beta)\,\rangle} \quad \to \quad \frac{(\langle\,\phi\,,\Psi_L^{(n)}(\beta)\,\rangle)^2}{\langle\,\phi\,,\Psi_L^{(n-1)}(\beta)\,\rangle\langle\,\phi\,,\Psi_L^{(n+1)}(\beta)\,\rangle} \approx 1 \qquad (4.4)$$

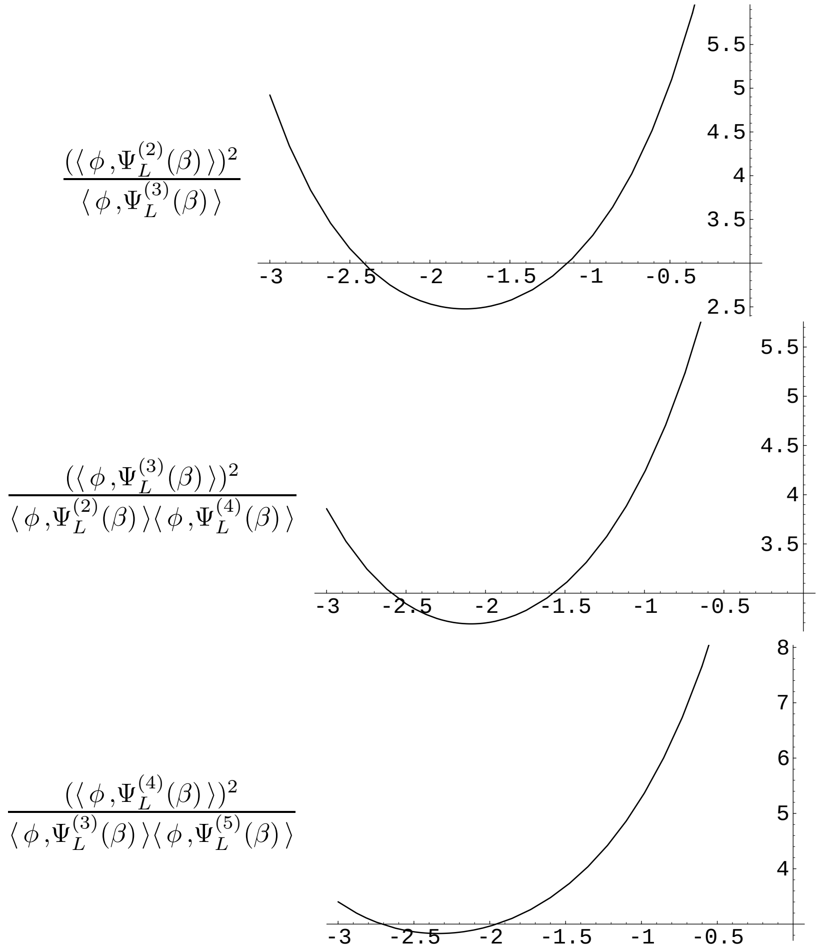


Figure 4: Ratios of the coefficients $\langle \phi , \Psi_L^{(n)}(\beta) \rangle$ for $n = 2, 3, 4, 5$.

the series breaks down for some $t$, and the behaviour of the tachyon could change qualitatively.

Figure 4 shows the second quantity of equation ( 4.4) for the values of $n$ represented in figure 3. We can see that for $\beta \approx -2$, this quantities are of order one, which was our main objective. It rests to know if this situation is kept for higher values of $n$, and what is the motion of the string $\Psi_L^{(n)}(\beta \approx -2)$, but this is left for future studies.

## 5 Conclusions

The motion of the tachyon, and therefore of the whole string remains totally mysterious to us. On one hand boundary string field theory (BSFT) indicates that the tachyon should roll towards its ground state and approach it as $t \to \infty$. This is the behaviour that would be expected from a physical viewpoint. However, BSFT is not as well defined as cubic open string field theory (CSFT), because its formulation can only be carried out on a level truncated way. On the other hand CSFT is more well defined, in the sense that it includes all the vibrating modes in its formulation. But the description it makes of the dynamics of the tachyon seems quite unphysical, with an oscillation with ever growing amplitude. We have given in this report a first step towards what might be the solution of the problem: a $\beta$-deformation that brings the solution series to break down. However, this method still has to be further studied.

## References


[1] E. Witten, "Noncommutative Geometry And String Field Theory," Nucl. Phys. B **268**, 253 (1986).

[2] M. Kiermaier, Y. Okawa, L. Rastelli and B. Zwiebach, "Analytic solutions for marginal deformations in open string field theory," arXiv:hep-th/0701249.

[3] M. Schnabl, "Comments on marginal deformations in open string field theory," arXiv:hep-th/0701248.

[4] E. Coletti, I. Sigalov and W. Taylor, "Taming the tachyon in cubic string field theory," JHEP **0508**, 104 (2005) [arXiv:hep-th/0505031].

[5] M. Kiermaier and Y. Okawa, "Exact marginality in open string field theory: a general framework," arXiv:0707.4472 [hep-th].

[6] Y. Okawa, L. Rastelli and B. Zwiebach, "Analytic solutions for tachyon condensation with general projectors," arXiv:hep-th/0611110.

[7] J. Polchinski, "String theory. Vol. 1: An Introduction to the Bosonic String," *Cambridge, UK: Univ. Pr. (1998) 402 p*