# Measurement of Electron Bunch Length at FLASH with Synchrotron Radiation Analyses

Alan WahLun Mak

Summer Student Programme 2007
Deutsches Elektronen-Synchrotron (DESY)
Notkestraße 85, 22607 Hamburg, Germany

Department of Physics / College of Creative Studies
University of California, Santa Barbara (UCSB)
Santa Barbara, CA 93106, United States

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## Abstract

This report serves to summarize the research project that the author has engaged in as a 2007 summer student at the Deutsches Elektronen-Synchrotron (DESY). The project involves various tasks at FLASH, the vacuum ultraviolet free-electron laser on the site of DESY. The main objective is to measure the electron bunch length at FLASH, by analyzing the synchrotron radiation extracted from the first bunch compressor. The study is divided into two major parts, which utilize coherent synchrotron radiation and incoherent synchrotron radiation, respectively.

## 1 Introduction

"FLASH" is an acronym, which stands for "**F**reie-Elektronen-**LAS**er in **H**amburg" in German, or "**F**ree-electron **LAS**er in **H**amburg" in English. It is a vacuum ultraviolet free-electron laser driven by a linear accelerator, producing an electron beam with short bunch length, and hence high peak current. The short bunch length is achieved by the two bunch compressors, BC2 and BC3 (Fig. 1). As the electron bunch is compressed to smaller than a picosecond long, the bunch length can no longer be measured by conventional methods, which have very limited resolution [1]. This suggests the need of alternative methods for bunch length measurement.

Out of the many alternative methods, synchrotron radiation has proven to be a useful tool for bunch length measurement. At the last dipole magnet of BC2, synchrotron radiation is extracted and led through a transfer line to TOSYLAB for this purpose (Fig. 1). "TOSYLAB" stands for "**T**erahertz and **O**ptical **SY**nchrotron Radiation **LAB**oratory". It is a laboratory located outside the accelerator tunnel, where experimental apparatuses are set up for various kinds of synchrotron radiation analysis (Fig. 2).

The research project discussed in this report is divided into two major parts, which utilize coherent synchrotron radiation and incoherent synchrotron radiation, respectively. The differences between coherent and incoherent synchrotron radiation will be discussed in the next section.

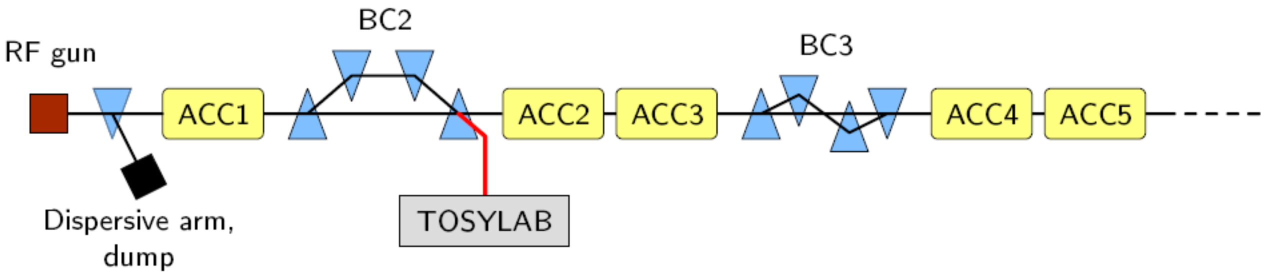


Figure 1: A sketch showing part of FLASH. The electron bunches are produced in the photocathode RF gun, accelerated in the modules ACC1 to ACC5, and compressed by the bunch compressors BC2 and BC3. At the last dipole magnet of BC2, synchrotron radiation is extracted and led through a transfer line to TOSYLAB for various kinds of analysis. (Figure drawn by L. Fröhlich)

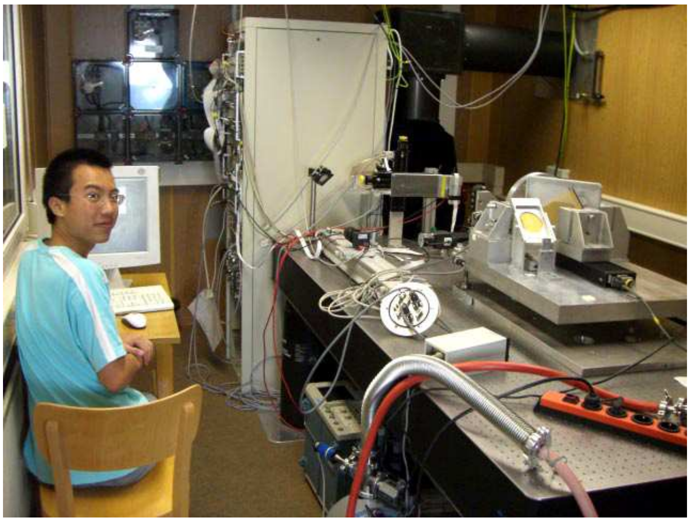

Figure 2: The interior of TOSYLAB. Various experimental apparatuses are set up to analyze the synchrotron radiation that comes out from the transfer line (upper-right). (Photograph taken by A. Willner)

# 2 Theoretical background

This section gives a brief discussion on the theory behind coherent and incoherent synchrotron radiation. Generally speaking, a relativistic bunch of electrons radiate under random phases. If the bunch contains $N$ electrons, the total energy radiated by the whole bunch is simply $N$ times the energy radiated by one single electrons. In this case, the radiation is *incoherent*. However, this is the case only when the wavelength $\lambda$ of the radiation is much shorter compared to the bunch length [1]. When the wavelength $\lambda$ of the radiation is much longer compared to the bunch length, and the total energy radiated by the whole bunch scales with $N^2$ times the energy radiated by one single electron. This is due to the *coherent* radiation by a large fraction of all electrons in the bunch [2].

These relations can also be seen mathematically. The radiation spectrum $I(\lambda)$ produced by a bunch of relativistic electrons is given by [2]

$$I(\lambda) = I_1(\lambda)(N + N(N-1)|F(\lambda)|^2), \tag{1}$$

where $I_1(\lambda)$ is the radiation spectrum of one single electron, and $N$ is the total number of electrons in the bunch. $F(\lambda)$ is the *longitudinal form factor*, defined as the Fourier transform of the normalized longitudinal charge density distribution $\rho(z)$, or mathematically [2]

$$F(\lambda) = \int_{-\infty}^{+\infty} \rho(z) e^{-\frac{2\pi i z}{\lambda}} dz. \tag{2}$$

It can be seen from equation (2) that for short wavelength $\lambda$, the exponential factor $e^{-\frac{2\pi i z}{\lambda}}$ tends to zero, and so the form factor $F(\lambda)$ vanishes. As a result, equation (1) reduces to

$$I(\lambda) = N I_1(\lambda). \tag{3}$$

This corresponds to *incoherent radiation*.

On the other hand, for sufficiently long wavelength $\lambda$, the exponential factor $e^{-\frac{2\pi i z}{\lambda}}$ in equation (2) tends to one, and so the form factor becomes

$$F(\lambda) \approx \int_{-\infty}^{+\infty} \rho(z) dz = 1. \tag{4}$$

As a result, equation (1) reduces to

$$I(\lambda) = N(N-1) I_1(\lambda) \approx N^2 I_1(\lambda). \tag{5}$$

This corresponds to *coherent radiation*.

# 3 Study with coherent radiation

The bunch length measurement with coherent synchrotron radiation involves the deployment of a *Martin-Puplett interferometer*, which is an apparatus for obtaining the radiation spectrum of an electron bunch. Provided that the total number of electrons in the bunch $N$ and the radiation spectrum of one single electron $I_1(\lambda)$ are known, the *magnitude* of the form factor, $|F(\lambda)|$, can be deduced from equation (1). After that, the *phase* of the form factor can be reconstructed by the *Kramers-Kronig relation*. After the whole form factor (both the magnitude and the phase) is known, the longitudinal charge density distribution $\rho(z)$ (and hence the bunch length) can be worked out by inverse Fourier transform, the reverse process of equation (2). These techniques are detailed in chapter 4 of reference [1] and in chapter 3 of reference [2].

However, owing to time constraint, it is infeasible to finish this whole series of tasks within the Summer Student Program. Within the framework of the Summer Student Program, the author has carried out the preliminary, but crucial, steps of the entire analysis, which involves setting up a new alignment laser, installing an additional focusing lens in the accelerator tunnel, and mounting the Martin-Puplett interferometer in TOSYLAB.

## 3.1 The alignment laser

A new laser source has been installed near the bunch compressor BC2 in the FLASH accelerator tunnel. This laser source emits visible laser light, which is then fed into the transfer line that leads to TOSYLAB. A very crucial task is to align this alignment laser with the synchrotron radiation that goes through the same transfer line to TOSYLAB.

The main purpose of having this alignment laser is to help keeping track of the synchrotron radiation, which can hardly been seen by bare eyes. By aligning the visible alignment laser to the synchrotron radiation, one can easily know the path in which the synchrotron radiation travels. This, in turn, makes it more convenient for the synchrotron radiation to be fed into the Martin-Puplett interferometer properly.

The mounting of the alignment laser source in the accelerator tunnel has four adjustment knobs: two for moving the laser source along two horizontal axes, and the other two for tilting the laser source from the vertical axis towards the two horizontal axes. The alignment is achieved by turning these four knobs, until the alignment laser overlaps with the center of the synchrotron radiation as they come out from the transfer line into TOSYLAB.

However, this is easier said than done. First, we have to find out the effect of turning each of the four adjustment knobs on the displacement of the alignment laser at the end of the transfer line (in TOSYLAB). Then, a radiation detector (Fig. 3), which is connected to the computer in TOSYLAB, is used to scan for the transverse distribution of the radiation intensity. After each scan, the computer generates a graph showing the transverse distribution of the radiation intensity, with different radiation intensities shown in different colors. From the graph, one can tell whether the alignment laser is aligned with the synchrotron radiation. If not, then one has to adjust the laser source in the tunnel with the four adjustment knobs, and then perform another scan in TOSYLAB. These processes have to be repeated until the alignment laser is brought into overlap with the center of the synchrotron radiation.

It should also be noted that in order to ensure proper alignment, the scanning in TOSYLAB has to be done at a minimum of two positions along the direction in which the two beams come out from the transfer line. This is because the alignment laser beam and the synchrotron radiation are essentially two straight lines, and the criterion for two straight lines to be aligned is that they must have at least two points of intersection.

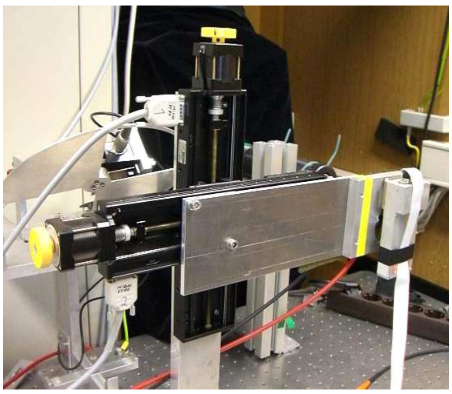

Figure 3: The radiation detector in TOSYLAB. The detector is mounted on a frame with two stepper motors, which are capable of moving the detector horizontally and vertically.

## 3.2 The additional focusing lens

Along the transfer line that leads to TOSYLAB, there are one parabolic mirror of focal length 0.81 m and three plane mirrors. The parabolic mirror is about 4 meters from the alignment laser source and about 11 meters from the radiation detector in TOSYLAB. Among these mirrors, the three plane mirrors are merely for changing the direction in which light travels; only the parabolic mirror has focusing power. Meanwhile, the parabolic mirror has essentially the same effect as the combination of a converging lens with the same focal length 0.81 m and a plane mirror. For the purpose of ray tracing, the transfer line can be regarded as a straight line, all the plane mirrors can be neglected, and the parabolic mirror can be represented by a converging lens L1 with the same focal length 0.81 m, as shown in Figure 4(a).

Experimentally, it is seen that when the alignment laser beam is captured with a screen at the end of the transfer line, it shows up as a relatively large spot, meaning that the alignment

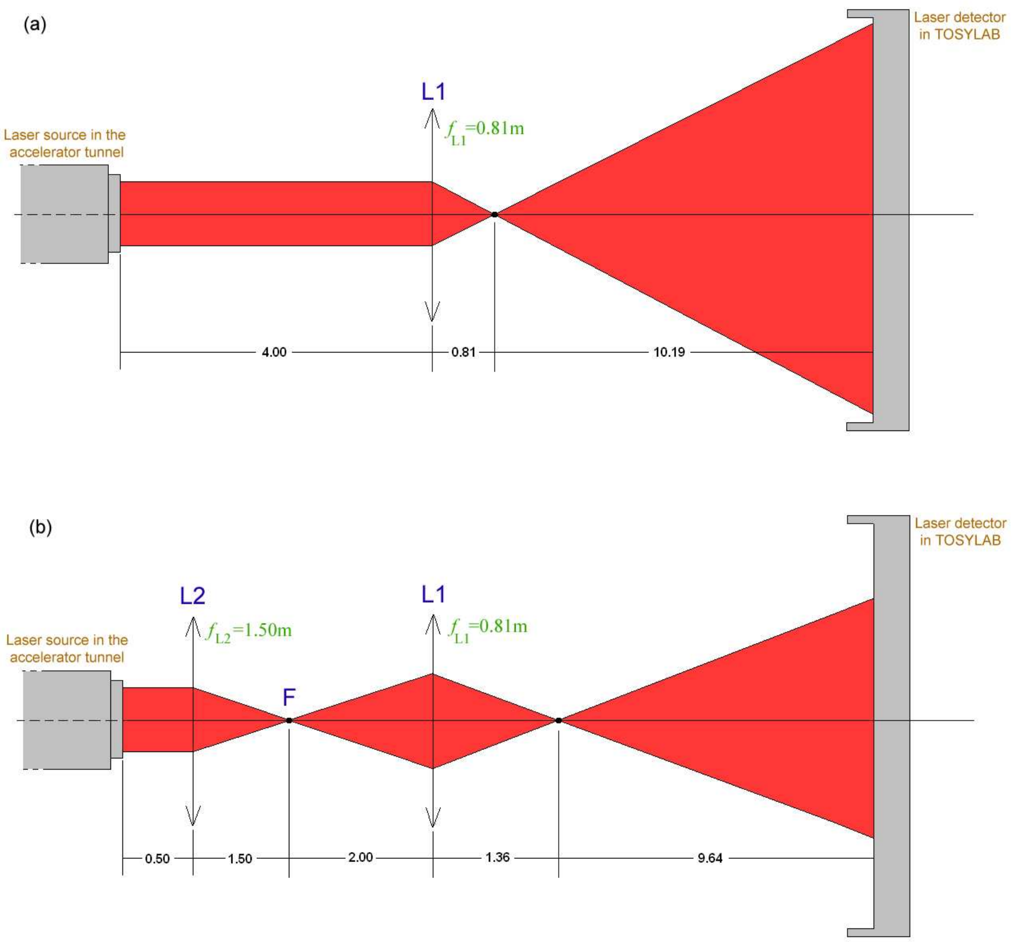


Figure 4: Ray diagrams representing the transfer line being regarded as a straight line, with all the plane mirrors neglected, and the parabolic mirror represented by a converging lens with the same focal length 0.81 m. (a) Before the additional focusing lens L2 is added (b) After the lens L2 is added (Note that the ray diagrams are not to scale)

laser beam has a relatively large diameter as it comes to the end of the transfer line. However, a large beam diameter is not desired, as that would make the power of the alignment laser more spreaded out, thus lowering the intensity of the beam.

To cope with this, an additional converging lens L2 of focal length 1.5 m is installed in the accelerator tunnel, at a position about 0.5 m in front of the alignment laser source, as shown in Figure 4(b).

By considering similar triangles in Figures 4(a), it can be seen that before the additional focusing lens L2 is added, the beam diameter at the end of the transfer line is about 12.58 times larger than that at the laser source. Similarly, by considering similar triangles in Figures 4(b), it can be seen that after L2 is added, the beam diameter at the end of the transfer line is only about 9.45 times larger than that at the laser source. The beam diameter at the end of the transfer line is quite significantly reduced.

# 4 Study with incoherent radiation

The study with incohrent synchrotron radiation involves a method known as *fluctuation anaylsis*, which is detailed in reference [3]. In brief, photons are collected with a photomultiplier at the end of the radiation transfer line (located in TOSYLAB), and the energy radiated by an electron bunch, $W$, is measured. A narrow-band filter is attached to the front of the photomultiplier (Fig. 5), to collect the photons of a narrow range of frequencies where no coherent emission is present. The measurement of $W$ is repeated for many electron bunches. Using the data obtained, the mean value $\langle W \rangle$ and the standard deviation $\sigma_W$ are calculated. However, the quantity of interest in this fluctuation analysis is actually the *relative RMS variation* of $W$, which is denoted by the symbol $\delta^2$, and is given by [3]

$$\delta^2 = \frac{\sigma_W^2}{\langle W \rangle^2}. \tag{6}$$

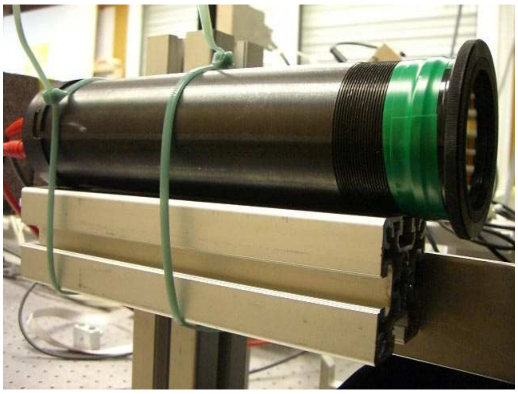

Figure 5: The photomultiplier used in the incoherent synchrotron radiation fluctuation analysis for the collection of photons at the end of the radiation transfer line. A narrow-band filter is attached on the front of the photomultiplier, to collect the photons of a narrow range of frequencies where no coherent emission is present.

It is worth pointing out that in the course of measurement using the photomultiplier, there are inevitably always some unwanted signals, which are not truely due to the energy radiated by the electron bunch. These unwanted signals are attributed to the background radiation that is present at the laboratory, the slight variation in total charge from bunch to bunch, and possibly some other origins. In order to enhance the accuracy of the measurements, these unwanted signals should be taken into account. The effect due to background radiation is found out by a control experiment that is carried out at the same time, while the charge variation is read in by the computer from the toroid 10DBC2 in the accelerator also at the same time.

Let $W$ denote the signal that is truely due to the energy radiated by the electron bunch, $W_S$ denote the signal that is measured by the photomultiplier, $W_B$ denote the unwanted signal due to background radiation, and $\Delta W_C$ denote the unwanted signal due to the deviation of the total charge in a bunch from its normal value. These quantities are related by

$$W_S = W + W_B + \Delta W_C, \tag{7}$$

or after rearranging the terms,

$$W = W_S - W_B - \Delta W_C. \tag{8}$$

As discussed in section 2, for incoherent synchrotron radiation, the energy radiated is directly proportional to the total number of electrons (and hence the total charge $Q$) in the electron bunch. Therefore, we may write

$$W = kQ \tag{9}$$

for some constant of proportionality, $k$. On the other hand, $\Delta W_C$ is descrepancy in energy measured by the photomultiplier due to the deviation of the total charge in a bunch from its normal value,

$$\Delta Q = Q - \langle Q \rangle. \tag{10}$$

Thus, we may also write

$$\Delta W_C = k\Delta Q = kQ - k\langle Q \rangle \tag{11}$$

with the same constant of proportionality, $k$. It then follows from equation (8) that

$$W = W_S - W_B - kQ + k\langle Q \rangle. \tag{12}$$

Taking the arithmetic mean on both sides of equation (12) yields

$$\begin{aligned} \langle W \rangle &= \langle W_S \rangle - \langle W_B \rangle - k\langle Q \rangle + k\langle\langle Q \rangle\rangle \\ &= \langle W_S \rangle - \langle W_B \rangle - k\langle Q \rangle + k\langle Q \rangle \\ &= \langle W_S \rangle - \langle W_B \rangle. \end{aligned} \tag{13}$$

Taking the artithmetic mean on both sides of equation (9) yields

$$\langle W \rangle = k\langle Q \rangle. \tag{14}$$

Equating equations (13) and (14) gives

$$\boxed{k = \frac{\langle W_S \rangle - \langle W_B \rangle}{\langle Q \rangle}} \tag{15}$$

This expression helps us determine the unknown constant of proportionality.

On the other hand, from equation (8), $W$ is a function of $W_S$, $W_B$ and $\Delta W_C$. By error propagation,

$$\begin{aligned} \sigma_W &= \sqrt{\left(\frac{\partial W}{\partial W_S}\right)^2 \sigma_{W_S}^2 + \left(\frac{\partial W}{\partial W_B}\right)^2 \sigma_{W_B}^2 + \left(\frac{\partial W}{\partial(\Delta W_C)}\right)^2 \sigma_{\Delta W_C}^2} \\ &= \sqrt{\sigma_{W_S}^2 + \sigma_{W_B}^2 + \sigma_{\Delta W_C}^2}. \end{aligned} \tag{16}$$

Furthermore, from equation (11), $\Delta W_C$ is a function of $Q$ and $\langle Q \rangle$. By error propagation,

$$\begin{aligned} \sigma_{\Delta W_C} &= \sqrt{\left(\frac{\partial W_C}{\partial Q}\right)^2 \sigma_Q^2 + \left(\frac{\partial W_C}{\partial \langle Q \rangle}\right)^2 \sigma_{\langle Q \rangle}^2} \\ &= \sqrt{k^2 \sigma_Q^2 + k^2 \sigma_{\langle Q \rangle}^2}. \end{aligned} \tag{17}$$

Here the term $\sigma_{\langle Q \rangle}$ is what is known in statics as the *standard deviation of the mean* of $Q$. It is related to the standard deviation of $Q$ itself (i.e. $\sigma_Q$) by

$$\sigma_{\langle Q \rangle} = \frac{\sigma_Q}{\sqrt{n}}, \tag{18}$$

where $n$ is the number of measurements taken. It follows that

$$\begin{aligned} \sigma_{\Delta W_C} &= \sqrt{k^2 \sigma_Q^2 + k^2 \left(\frac{\sigma_Q}{\sqrt{n}}\right)^2} \\ &= k \sigma_Q \sqrt{1 + \frac{1}{n}}. \end{aligned} \tag{19}$$

Putting this result into equation (16) yields

$$\sigma_W = \sqrt{\sigma_{W_S}^2 + \sigma_{W_B}^2 + k^2 \sigma_Q^2 \left(1 + \frac{1}{n}\right)}. \tag{20}$$

Now, we have an expression for the mean $\langle W \rangle$ (equation (12)) and an expression for the standard deviation $\sigma_W$ (equation (20)). The next step is to substitute these expressions into equation (6), so as to write the relative RMS variation $\delta^2$ in terms of the quantities that are actually measured:

$$\delta^2 = \frac{\sigma_{W_S}^2 + \sigma_{W_B}^2 + k^2 \sigma_Q^2 \left(1 + \frac{1}{n}\right)}{k^2 \langle Q \rangle^2}. \tag{21}$$

It turns out that for typical bunches that do not have microstructures with characteristic length much smaller than the bunch length, it suffices to approximate the charge distribution within the bunch by a Gaussian charge distribution [3]. For a Gaussian bunch with RMS length $\sigma_\tau$ (in time unit) and a Gaussian filter with RMS bandwidth $\sigma_\omega$, the relative RMS variation of $W$ can be calculated by the relation [3]

$$\delta^2 = \frac{1}{\sqrt{1+4\sigma_\tau^2\sigma_\omega^2}}. \tag{22}$$

Solving this equation for the bunch length $\sigma_\tau$ yields

$$\sigma_\tau = \frac{1}{2\sigma_\omega}\sqrt{\frac{1}{(\delta^2)^2} - 1}. \tag{23}$$

Using equation (21), we then get an expression for the bunch length in terms of the measured quantities:

$$\boxed{\sigma_\tau = \frac{1}{2\sigma_\omega}\sqrt{\frac{k^4\langle Q\rangle^4}{\left[\sigma_{W_S}^2 + \sigma_{W_B}^2 + k^2\sigma_Q^2\left(1+\frac{1}{n}\right)\right]^2} - 1}} \tag{24}$$

Now that we have obtained an expression from which we can calculate the bunch length simply by plugging in the things that we measured, the problem is already solved for the most part. Nevertheless, it is essential to calculate also the error $\varepsilon$ in the bunch length $\sigma_\tau$. From equation (23), when the bandwidth $\sigma_\omega$ of the filter is precisely known, the error in $\sigma_\tau$ should only depend on $\delta^2$. By error propagation,

$$\varepsilon = \sqrt{\left(\frac{\partial\sigma_\tau}{\partial\delta^2}\right)^2\sigma_{\delta^2}^2} = \left|\frac{\partial\sigma_\tau}{\partial\delta^2}\right|\sigma_{\delta^2}. \tag{25}$$

The derivative $\frac{\partial\sigma_\tau}{\partial\delta^2}$ can easily be found out by differentiating equation (23). It turns out that

$$\varepsilon = \frac{\sigma_{\delta^2}}{2\sigma_\omega(\delta^2)^2\sqrt{1-(\delta^2)^2}}. \tag{26}$$

Meanwhile, it can be shown that the error $\sigma_{\delta^2}$ in the relative RMS variation $\delta^2$ is related to $\delta^2$ itself by the ratio [3]

$$\frac{\sigma_{\delta^2}}{\delta^2} = \sqrt{\frac{2}{n}} \tag{27}$$

Using this relation, equation (26) can then be simplified as

$$\varepsilon = \frac{1}{\sqrt{2n}\sigma_\omega\delta^2\sqrt{1-(\delta^2)^2}} \tag{28}$$

Finally, we substitute in equation (21) to rewrite this in terms of the measured quantities:

$$\boxed{\varepsilon = \frac{k^4\langle Q\rangle^4}{\sqrt{2n}\sigma_\omega\left[\sigma_{W_S}^2 + \sigma_{W_B}^2 + k^2\sigma_Q^2\left(1+\frac{1}{n}\right)\right]\sqrt{k^4\langle Q\rangle^4 - \left[\sigma_{W_S}^2 + \sigma_{W_B}^2 + k^2\sigma_Q^2\left(1+\frac{1}{n}\right)\right]^2}}} \tag{29}$$

The three equations in boxes (i.e. equations (15), (24) and (29)) are all we need, as far as the bunch length calculation is concerned. The following table shows three sets of results obtained by these equations. The quantities in the table are the input voltage of the photomultiplier ($V_{in}$), the bunch length ($\sigma_\tau$) in picoseconds, the error in the bunch length ($\varepsilon$), and the percentage error ($\sigma_\tau/\varepsilon$). The filter that is attached to the front of the photomultiplier (Fig. 5) has $\sigma_\omega = 10^{13}$ s$^{-1}$.

| $V_{in}$ (V) | $k$ (mV/nC) | $\sigma_\tau$ (ps) | $\varepsilon$ (ps) | $\sigma_\tau/\varepsilon$ (%) |
|---|---|---|---|---|
| -1180 | -0.620 | 3.895 | 0.126 | 3.245 |
| -1251 | -0.946 | 4.134 | 0.134 | 3.245 |
| -1316 | -1.410 | 3.794 | 0.120 | 3.163 |

The values of the bunch length are fairly consistent, within the experimental error. The bunch length measurement was repeated on another day, and the following data were obtained:

| $V_{in}$ (V) | $k$ (mV/nC) | $\sigma_\tau$ (ps) | $\varepsilon$ (ps) | $\sigma_\tau/\varepsilon$ (%) |
|---|---|---|---|---|
| -999 | -0.161 | 8.695 | 0.278 | 3.203 |
| -1100 | -0.348 | 8.460 | 0.271 | 3.203 |
| -1390 | -2.041 | 8.886 | 0.292 | 3.288 |
| -1450 | -2.473 | 17.176 | 0.543 | 3.162 |
| -1800 | -2.219 | 16.746 | 0.583 | 3.482 |

It is seen that as the magnitude of the input voltage exceeds 1450V, there is a sudden increase in the bunch length obtained. The exact reason for this sudden increase is still unknown, and needed to be further investigated. However, it is possibly because the photomultiplier is saturated at such input voltage.

# 5 Conclusion

In this summer research project, the very crucial preliminary tasks for the study with conherent synchrotron radiation has been carried out. The additional focusing lens has been properly installed between the alignment laser and the parabolic mirror in the FLASH accelerator tunnel. The Martin-Puplett interferometer in TOSYLAB has been set up. Also, the alignment laser is properly aligned with the synchrotron radiation in the radiation transfer line that leads to TOSYLAB, and enables the path of the synchrotron radiation to be easily kept track of. In the coming months, the group envision to continue the diagnostics of electron bunches with coherent synchrotron radiation by obtaining actual data with the Martin-Puplett interferometer in TOSYLAB.

On the other hand, the measurement of electron bunch length with incoherent synchrotron radiation fluctuation analysis has already been in practice at the Advanced Light Source (ALS) of the Lawrence Berkeley National Laboratory (LBNL) in the United States [3]. Yet it has never been done at DESY before. This summer research project has made the first step of introducing this technique to DESY. This technique makes good use of the incoherent part of the synchrotron emission spectrum, and can be used as an alternative approach in the study of electron bunches at FLASH.

# Acknowledgement

I would like to express my gratitude to the Deutsches Elektronen-Synchrotron (DESY) for organizing the Summer Student Programme. The programme has enriched my experience by giving me the opportunity to be involved in the research on Accelerator Physics in a world-class research institute and the multicultural experience with students and reseachers from many different countries. I am also very thankful to DESY for the welcoming hospitality. In particular, I would like to thank Dr. Oliver Grimm and Mr. Arik Willner for their guidance and support throughout the programme. I would also like to thank Dr. Joachim Meyer for coordinating the entire Summer Student Programme, and for giving me the opportunity to host one of the student sessions.


# References


[1] L. Frölich, "Bunch Length Measurements using a Martin-Puplett Interferometer at the VUV-FEL," DESY-THESIS 2005-011 (April 2005)

[2] O. Grimm and P. Schmüser, "Principle of Longitudinal Beam Diagnostics with Coherent Radiation," TESLA-FEL 2006-03 (April 2006)

[3] F. Sannibale, M. S. Zolotorev, D. Filippetto and G. V. Stupakov, "Absolute Bunch Length Measurements at the ALS by Incoherent Synchrotron Radiation Fluctuation Analysis," Proceedings of PAC07, Albuquerque, New Mexico, United States