Study of charm fragmentation in e^+e^- annihilation and ep scattering

- Summer Student Report -

by

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Abstract

This report studies the fragmentation of charm-quarks to $D^{*\pm}$ mesons in e^+e^- annihilation via the channel $e^+e^- \rightarrow c\bar{c} \rightarrow D^{*\pm}X$, where X may be any final state and the $D^{*\pm}$ contribution coming from gluon splitting to heavy quarks is subtracted. The data of ALEPH ($\sqrt{s} = 91.2$ GeV), Belle ($\sqrt{s} = 10.52$ GeV) and CLEO ($\sqrt{s} = 10.56$ GeV) corrected for acceptance and detector effects are analysed using the Monte Carlo event generator PYTHIA in the H1 framework with Peterson fragmentation function for heavy quarks. Estimating the best value for the fragmentation parameter ϵ_c one obtains $\epsilon_c = 0.0316 \pm 0.0043$ for Belle and CLEO data sample and $\epsilon_c = 0.042 \pm 0.013$ for ALEPH data sample. This values agree well with the one obtained in ep scattering at H1 $\epsilon_c = 0.030^{+0.006}_{-0.005}$. Since all evaluations were done under the same conditions, this is a confirmation of the hypotheses of fragmentation function's universality.

Contents

1 Introduction

1.1 Fragmentation functions

In the studied events pairs of opposite (electromagnetic and colour) charged charm anticharm quarks are created from e^+e^- annihilation. Since no free, colour charged particles exist, the quarks have to pass into hadrons, which are colourless. Each produced hadron carries a part of the energy of the initial quark. For each hadron h the distribution of the quantity $z = \frac{(E+p_L)_{hadron}}{(E+p)_{quark}}$ [1] is described by the fragmentation function $D_c^h(z)$. Since it is not possible to calculate the fragmentation in perturbative QCD, phenomenological approaches were made to describe it. In the case of Peterson fragmentation parametrization the fragmentation is given by the following formula

$$D_c^h(z) = \frac{N}{z[1 - 1/z - \epsilon_c/(1-z)]^2}, \tag{1}$$

with N normalizing the total probability. Although ϵ_c in principle is a fixed parameter related to the quark masses, it will be regarded as free, since the quark masses are not known well.[2] By varying its value in the Monte Carlo (MC) event generation and comparing the simulated data to experimental results it is possible to extract the best value for ϵ_c. Since the fragmentation is believed to be universal one should within the errors obtain the same values in e^+e^- annihilation and ep scattering. A comparison of those values will be given in the end of this report.

1.2 Monte Carlo event generators

To generate the events, the MC generator PYTHIA 6.2 was used, which already existed in the H1 framework, but was modified to produce only the desired events $e^+e^- \to c\bar{c} \to D^{*\pm}X$. This modification was done by a former summer student (see [6]). To steer PYTHIA there exist hundreds of switches. All of them have a default value (D=...), but can be changed and passed to the program via the steering. For this work mainly parameter settings used by ALEPH (they will be called ALEPH settings in the following) and default parameter settings were used. Their values can be seen in table 1.2. Despite the switches listed there only the switches 'FRAM'(1,'CMS'), 'WIN'(1) (both concerning the center of mass energy) and in case of ALEPH data 'MSTP'(125) (concerning the event record) were changed from their H1 default values.

[1]this definition is used in Lund String Model
[2]compare to [7]

parameter	ALEPH setting	default setting	description
'MSTJ(11)'	3	3	choice of fragmentation function: Peterson fragmentation (for c, b)
'MSTJ(12)'	2	2	baryon model option
'MSTJ(46)'	0	3	parton shower azimut. corr.
'MSTJ(51)'	0	0	BEC off
'PARJ(1)'	0.108	0.100	P(qq)/P(q)
'PARJ(2)'	0.286	0.300	P(s)/P(u)
'PARJ(3)'	0.690	0.400	P(us)/P(ud)/P(s)/P(d)
'PARJ(4)'	0.050	0.050	(1/3)P(ud_1)/P(ud_0)
'PARJ(11)'	0.553	0.500	P(S=1)u,d
'PARJ(12)'	0.470	0.600	P(S=1)s
'PARJ(13)'	0.650	0.750	P(S=1)c,b
'PARJ(14)'	0.120	0.000	P(S=0,L=1,J=1)AXIAL
'PARJ(15)'	0.040	0.000	P(S=1,L=1,J=0)SCALAR
'PARJ(16)'	0.120	0.000	P(S=0,L=1,J=1)AXIAL
'PARJ(17)'	0.200	0.000	P(S=1,L=1,J=2)TENSOR
'PARJ(19)'	0.550	1.000	extra Baryon Suppression
'PARJ(21)'	0.366	0.360	σ_q
'PARJ(25)'	1.000	1.000	extra η suppression
'PARJ(26)'	0.276	0.400	extra η' suppression
'PARJ(41)'	0.400	0.300	Lund. symm. fragm.: a
'PARJ(42)'	0.885	0.580	Lund. symm. fragm.: b
'PARJ(54)'	-0.040	-0.050	ϵ_c
'PARJ(55)'	-0.002	-0.050	ϵ_b
'PARJ(82)'	1.390	1.000	Q_0
'PARP(72)'	0.250	0.250	Λ for α_s in time like parton showers

Table 1.2: PYTHIA parameter settings

2 Experimental data

Since one can measure neither the momentum nor the energy of the quark directly, it is not possible to use the definition of the variable z given in section 1.1 as experimental observable. Hence other variables sensitive to z are used. In e^+e^- annihilation this are the

$$\text{reduced momentum } x_p = \frac{p_{D^{*\pm}}}{p_{max}} \text{ and} \tag{2}$$

$$\text{the reduced energy } x_E = \frac{2E_{D^{*\pm}}}{E_{cms}}, \tag{3}$$

where $p_{max} = \sqrt{\frac{s}{4} - m^2_{D^{*\pm}}}$ is the maximum attainable $D^{*\pm}$ momentum at the relevant beam energy.

In the case of ep scattering an effective center of mass energy $\sqrt{\hat{s}}$ exists. This is in contrast to e^+e^- annihilation not known precisely. Hence the definition of the observable differs from the definition of x_p and x_E. The most common are the definitions used in hemisphere and jet method: In the hemisphere method the event is divided into two hemispheres assuming one containing mainly the fragmentation products of the charm quark, the other of the anti-charm quark. Thus one defines

$$z_{hem} = \frac{(E + p_L)_{D^{*\pm}}}{\sum_{hem}(E + p)} \tag{4}$$

In the jet method the energy and direction of the charm quark is approximated by reconstructing the jet which contains the $D^{*\pm}$ meson. One defines:

$$z_{jet} = \frac{(E + p_L)_{D^{*\pm}}}{(E + p)_{jet}} \tag{5}$$

More information is given in [7] and [4].

2.1 Belle and CLEO

In the experiments done by Belle [2] and CLEO [3] the used observable was the reduced momentum x_p. The naive limits of x_p are $0 < x_p < 1$. Due to detector resolution and kinematic effects the data contain nevertheless some events with $x_p > 1$, but do not contain bins near zero.
Belle presented their data in histograms with a range from 0 to 1.2 divided into 60 bins of size 0.02 (the data can be seen in table 7.1).
CLEO had less statistics and therefore chose only 16 bins with an extension of 0.05 in a x_p range from 0.20 to 1.00 (the data can be seen in table 7.2).
Both experiments corrected their data for detector efficiency, background and branching ratios hence we can treat their data as describing nature within their errors.

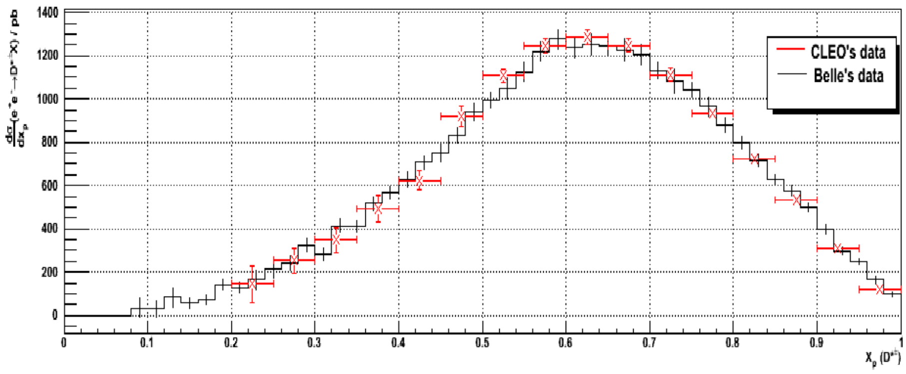

Figure 1: *The $D^{*\pm}$ cross sections, measured by CLEO (red points) and Belle (black, connected lines) collaboration: Within their errors they match perfectly.*

Since Belle and CLEO measured their spectra at nearly the same centre of mass energy $\sqrt{s} \approx 10.52$ GeV, their spectra should be very similar. Therefore we first compare their data by drawing them (normalised to each other) in the same histogram. As can be seen in figure 1 the data match within their errors perfectly.

As Belle has much more statistics and therefore smaller errors, we can restrict our study in the following to Belle's data.

We take the data collected in the continuum region at $\sqrt{s} = 10.52$ Gev using the $D^{*\pm}$ decay channel $D^{*\pm} \to D^0\pi^\pm$, which are listed in table 7.1. In contrast to the data at the Y(4S) resonance they have no $b\bar{b}$ background, thus we do not need to correct for it.

2.2 ALEPH's data

ALEPH's data was collected at the Z resonance at $\sqrt{s} = 91.2$ Gev. There the $b\bar{b}$ background and moreover the the background coming from gluon splitting to heavy quarks are present. The data used in this evaluation have been corrected for both backgrounds and can be seen in table 7.3.
Also in the generated MC events occurs a non negligible distribution of $D^{*\pm}$ coming from gluon splitting. Thus we extended our program to correct for it. (Since our MC generator is restricted to $e^+e^- \to c\bar{c} \to D^{*\pm}X$ the $b\bar{b}$ background is not present in our MC events.) Furthermore the MC event record had to be extended by using the switch 'MSTP'(125)=2, which renders a complete documentation of intermediate steps of parton-shower evolution.

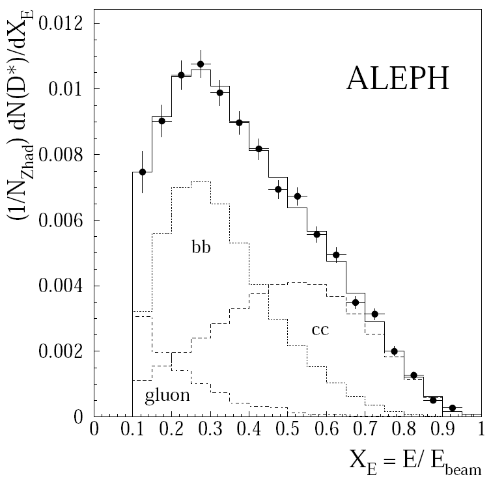

Figure 2: *The three contributions to the distribution of X_E for the $D^{*\pm}$ in ALEPH data: $b\bar{b}$ (dotted line), $c\bar{c}$ (dashed line) and gluon splitting (dashed-dotted)* [3]

Comparing our $D^{*\pm}$ spectra from gluon splitting and from $c\bar{c}$ source (see figure 3 [3]) to the corresponding spectra ALEPH generated (see figure 2) one recognizes the shape of each spectrum fits well to the shape of the corresponding spectrum, but the ratio of the area of the two sources differs by a factor of ten. On first sight this seems alarming, but taking into account the different quark couplings

[3]picture was taken from [1]

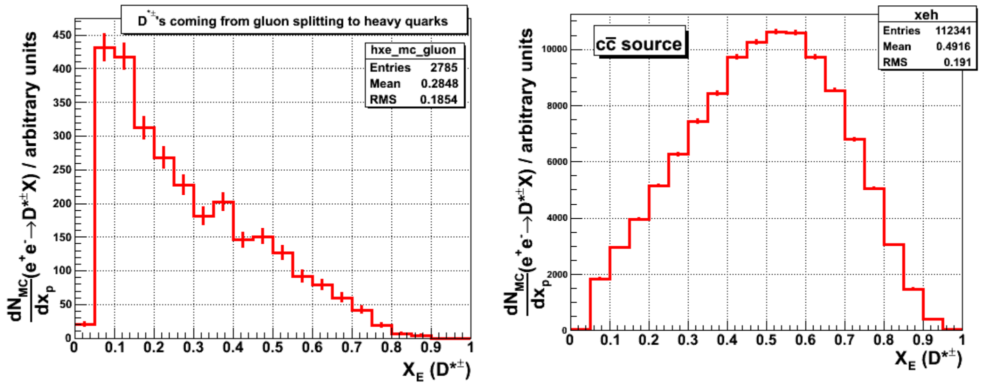

Figure 3: *Contributions of the $c\bar{c}$ channel (right) and the gluon channel (left) to the $D^{*\pm}$ spectrum in the generated MC events*

to electroweak interaction and the fact, that in contrast to ALEPH, which was using and generating all events $e^+e^- \to q\bar{q}|_{q\in\{u,d,c,s,b\}} \to D^{*\pm}X$, we restricted our MC data to events coming from $e^+e^- \to c\bar{c} \to D^{*\pm}X$, the factor can be explained satisfyingly.

As observable the fractional energy $x_E = \frac{2E}{\sqrt{s}}$ was used. The measured distribution is presented in histograms with a range from 0.1 to 1.0 consisting of 18 bins, each with an extension of 0.05. The background corrected data are listed in table 7.3 and can be seen in figure 5, where furthermore you see results from MC generation.

3 Data evaluation procedure

3.1 Belle data

As already explained in the section 1.2 we generate MC events with PYTHIA using different parameter settings (compare to table 1.2). We evaluate this events by C^{++} programs using ROOT and H1oo.
For the MC spectrum of x_p we use the same bin size Belle did and a range from zero to one with 50 equidistant bins. The program loops over all events searching for $D^{*\pm}$s. Having found a $D^{*\pm}$ it calculates the reduced momentum and fills it in the MC histogram. Having looped over all events the program determines the errors of the bins as the square root of their contents, for the errors are statistical. In the next step it scales the MC histogram to the area of the data histogram using the bins between 0.08 and 1.0. [4] Proceeding this way is appropriate as we are mainly interested in the shape of the spectrum and not in its normalization. As example the distribution of x_p using $\epsilon_c = 0.032$ is shown in figure 4.

[4]The content and the errorbars of the bins of the data histogram smaller than 0.08 are all equal to zero. Since the reason of the missing content is most probable the detector resolution, we doubt errors equal to zero to be correct and thus do not consider those bins in the evaluation.

The two histograms are drawn in the same plot. As quantity to specify their agreement χ^2 is determined via the equation:

$$\chi^2 = \sum_{\text{all bins with errors}\neq 0} \frac{(\text{data bin content} - \text{MC bin content})^2}{(\text{error of data bin})^2 + (\text{error of MC bin})^2} \tag{6}$$

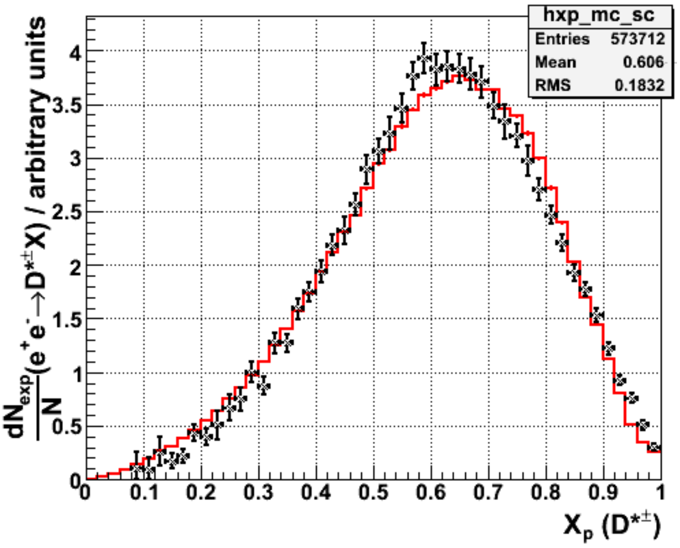

Figure 4: *Distribution of x_p for Belle data (black crosses) and for the generated MC event sample (red line) using* $\epsilon_c = 0.032$

Having calculated χ^2 for a given ϵ_c we repeat this procedure with a different value of ϵ_c. In this way we collect several pairs of values of ϵ_c and the associated $\chi^2(\epsilon_c)$, which is regarded as a function of - and thus plotted versus - ϵ_c. At ϵ_{best} - the value of ϵ_c describing data best - the minimal χ^2 is situated, to which we refer in the following as χ^2_{min}. We Taylor expand the function $\chi^2(\epsilon_c)$ around ϵ_{best} as:

$$\chi^2(\epsilon_c) = \chi^2_{min} + \left.\frac{d\chi^2}{d\epsilon_c}\right|_{\epsilon_{best}} (\epsilon_c - \epsilon_{best}) + \left.\frac{d^2\chi^2}{d\epsilon_c^2}\right|_{\epsilon_{best}} (\epsilon_c - \epsilon_{best})^2 + o((\epsilon_c - \epsilon_{best})^3) \tag{7}$$

Since $\chi^2(\epsilon_c)$ has a minimum at ϵ_{best} the first derivation vanishes and we can parametrize $\chi^2(\epsilon_c)$ around

$$\chi^2(\epsilon_c) = k(\epsilon_c - \epsilon_{best})^2 + \chi^2_{min} \tag{8}$$

Thus to determine ϵ_{best} from the collected pairs (ϵ_c, $\chi^2(\epsilon_c)$) we fit a second order polynomial to this points. Since the fit routine is not optimized for this way of parametrization, which can lead to inaccurate error matrices, we use an other equivalent parametrization:

$$\chi^2(\epsilon_c) = a_2\epsilon_c^2 + a_1\epsilon_c + a_0 \tag{9}$$

With the parametrization (9) we get rid of those problems. Due to the equivalence of the parametrizations we can acquire the parameters in (8) from a_2, a_1 and a_0.

$$\text{Since} \quad a_2\epsilon_c^2 + a_1\epsilon_c + a_0 = a_2(x + \frac{a_1}{2a_2})^2 - \frac{a_1^2}{4a_2} + a_0 \text{ , we have}$$
$$\epsilon_{best} = -\frac{a_1}{2a_2} \text{ ,}$$
$$\chi^2_{min} = a_0 - \frac{a_1^2}{4a_2}, k = a_2$$
$$\text{and} \quad \sigma_{\epsilon_{best}} = \frac{1}{\sqrt{a_2}}$$

In the last equation $\sigma_{\epsilon_{best}}$ is defined as $|\epsilon_{best} - \epsilon_1|$, where $\chi^2(\epsilon_1) = \chi^2_{min} + 1$.

It should be mentioned, that the approximation of $\chi^2(\epsilon_c)$ as a second order polynomial holds only in a narrow region around the minimum ϵ_{best}. Thus we first determine ϵ_{best} roughly and then measure many points close to the minimum. With this points we do the proper fit.

3.2 ALEPH data

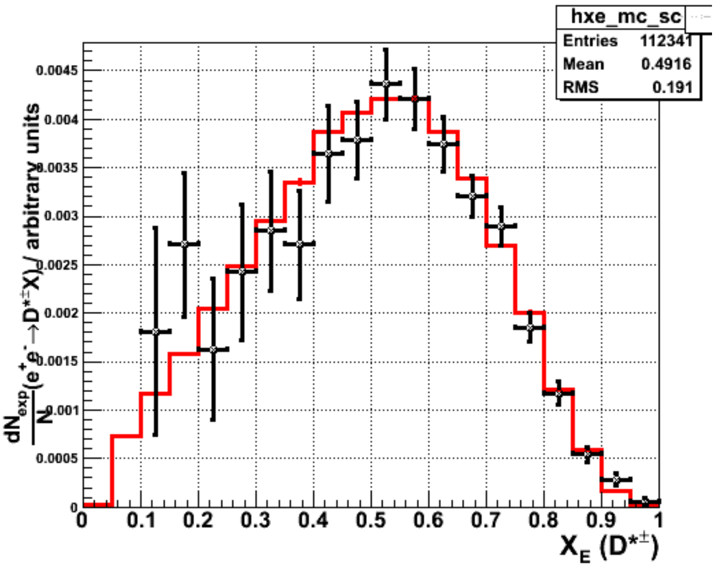

Figure 5: *The (corrected) data of ALEPH's experiment (black, points) and a MC generation (red, connected lines) using ALEPH settings and* $\epsilon_c = 0.042$ *are shown. Only* $D^{*\pm}$*s coming from* $c\bar{c}$ *source are considered. Within their errors the data sample and the MC sample match perfectly.*

ALEPH data we analyse in a simmilar way as Belle data. Instead of the x_p histogram we produce a MC x_E histogram with a bin size of 0.05 and for normalization and χ^2 determination took the bins from 0.1 to 1.0 into account. As an example the data and generated histogram with $\epsilon_c = 0.042$ is plotted in figure 5.

4 Results

Using the procedure explained in the last section ϵ_{best} was extracted for different settings and data samples.

4.1 ALEPH settings

To produce the MC events evaluated in this subsection ALEPH settings, which are listed in the second column of table 1.2, were used to fit the data samples.

4.1.1 Belle data

For Belle data eight MC event samples at $\sqrt{s} = 10.52$ GeV were generated, each with a different ϵ_c between 0.026 and 0.040. Due to the high number of bins and the small errors of the data points $5 \cdot 10^5$ events per sample were generated to ensure the MC error to be much smaller than the experimental error. We evaluated:

ϵ_c	0.026	0.028	0.030	0.032	0.034	0.036	0.038	0.040
χ^2_{belle}	5.18	3.93	3.55	3.08	3.81	4.67	5.52	7.00

Fitting the values from ϵ_c=0.026 to 0.038 to a second order polynomial

$$\epsilon_{best}^{belle} = 0.0316 \pm 0.0042 \tag{10}$$

and $\chi^2_{min} = 3.31$ were obtained. The plot is shown in figure 6

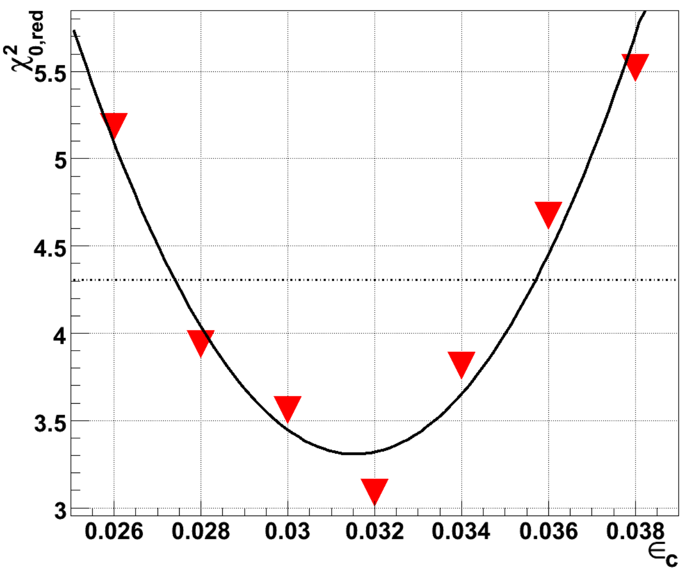

Figure 6: χ^2 *fit for Belle data and ALEPH settings*

4.1.2 ALEPH data

For ALEPH data nine MC event samples at $\sqrt{s} = 91.2$ GeV were generated using values of ϵ_c between 0.034 and 0.050. Since ALEPH data has fewer bins and larger errors than Belle data, only 10^5 MC events were generated per sample. With the different ϵ_c values following χ^2 values were calculated:

ϵ_c	0.034	0.036	0.038	0.040	0.042	0.044	0.046	0.048	0.050
χ^2_{aleph}	1.19	0.969	0.789	0.698	0.737	0.684	0.803	0.862	0.910

Fitting the values from 0.036 to 0.048 to a second order polynomial (see figure 7)

$$\epsilon_{best}^{aleph} = 0.042 \pm 0.013 \tag{11}$$

was obtained. The more precise value of Belle is located 0.84σ apart from this value and is therefore consistent with it.

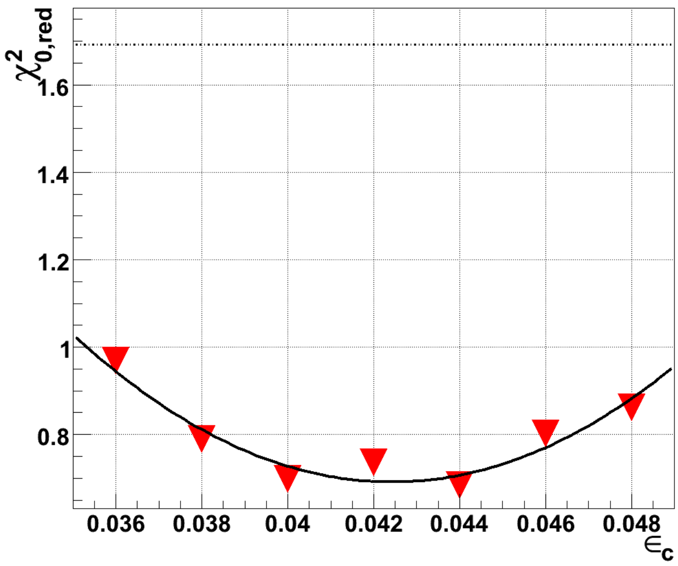

Figure 7: χ^2 *fit for ALEPH data and ALEPH settings*

4.2 Default settings

To produce the MC events evaluated in this subsection default settings, which are listed in the third column of table 1.2, were used to fit the data samples.

4.2.1 Belle data

Again MC events were generated at $\sqrt{s} = 10.52$ GeV. Seven samples with ϵ_c between 0.0375 and 0.0525 were produced. The spectrum for $\epsilon_c = 0.045$ is displayed as an example in figure 8. Evaluation led to:

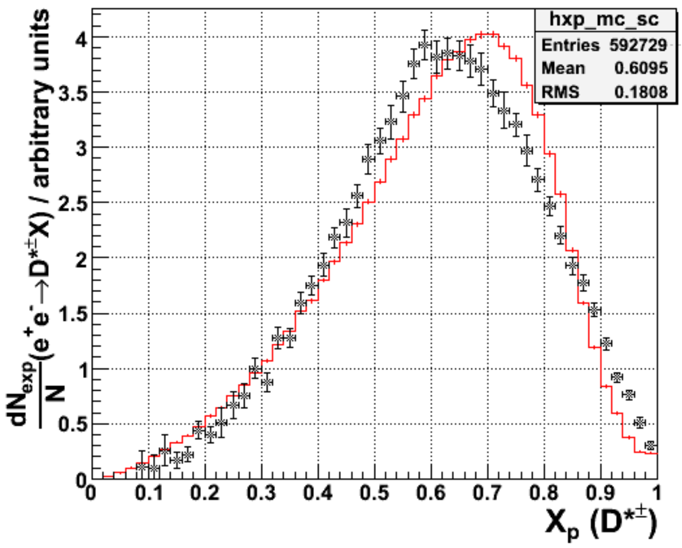

Figure 8: *Distribution of* x_p *for Belle data (black crosses) and for the generated MC event sample (red line) using default settings and* $\epsilon_c = 0.045$

ϵ_c	0.0375	0.040	0.0425	0.045	0.0475	0.050	0.0525
χ^2_{belle}	13.54	12.09	11.76	11.69	11.76	12.46	12.66

The fit of a second order polynomial resulted in

$$\epsilon^{belle}_{best,\,default} = 0.0455 \pm 0.006 \ \text{ and } \ \chi^2_{min} = 11.6 \tag{12}$$

Possessing a χ^2_{min} much bigger than one this result should be considered less reliable.

4.3 Results in ep scattering

H1 collaboration did experiments at HERA colliding positrons of energy 27.6 GeV with protons at 920 Gev. As observables they used z_{hem} and z_{jet}. More information is given in [7] and [4]. In both references values of ϵ_c for both observables were extracted using the same framework (H1oo) we did use. Since the value in [4] is more recent and precise, we will refer to this.
Using the MC generators RAPGAP 3.1 and PYTHIA 6.2 with ALEPH settings they extracted the following value with the jet method:

$$\epsilon^{H1}_{best} = 0.030^{+0.006}_{-0.005} \tag{13}$$

5 Conclusion

All results were obtained within the same framework (H1oo) and moreover with the same settings [ALEPH] (besides the values from default settings, which we do not consider, since their MC events are in bad agreement with data). Thus it is possible to compare the values of ϵ_c, which have been:

experiment	ALEPH	Belle/CLEO	H1
ϵ_{best}	0.042	0.0316	0.030
$\sigma_{\epsilon_{best}}$	0.013	0.0042	$^{+0.006}_{-0.005}$

Having the smallest error the value from Belle data should be the most precise one. It is located 0.84σ apart from the value extracted from ALEPH data [5] and only 0.26σ apart from the value published by the H1 group. Furthermore ϵ_{best}^{H1} is situated in the 1σ interval of ϵ_{best}^{aleph}. Thus - despite the fact of different center of mass energies and different types of particles - the values match within their errors well and in the limits of the errors

the fragmentation function is confirmed to be universal.

But nevertheless we want to remind you the relative errors are about 20% and one cannot be sure, if a more precise determination of ϵ_c with smaller errors, will hold this conformation.

6 Acknowledgments

We thank the members of the MPIM group at H1 and the organizers of the Summer Student Program at DESY to permit this work. Special thanks are rendered to Andrej Liptaj, Günter Grindhammer and Juraj Bracinik, who supported me during this work and were open to many questions.

[5] we consider the bigger error

7 Appendix

Experimental data used for this evaluation

Table 7.1 : Data from Belle at $\sqrt{s} = 10.52GeV$ in the channel $D^{*\pm} \to D^0\pi^\pm$

$x_{p,1}$	$x_{p,2}$	$\frac{d\sigma}{dx_p}/a.u.$	$\sigma_{\frac{d\sigma}{dx_p}}/a.u.$
0.00	0.02	0.0000	0.0000
0.02	0.04	0.0000	0.0000
0.04	0.06	0.0000	0.0000
0.06	0.08	0.0000	0.0000
0.08	0.10	0.1086	0.1442
0.10	0.12	0.0920	0.1202
0.12	0.14	0.2600	0.1363
0.14	0.16	0.1734	0.0721
0.16	0.18	0.2212	0.0650
0.18	0.20	0.4413	0.0767
0.20	0.22	0.3998	0.0764
0.22	0.24	0.5073	0.1346
0.24	0.26	0.6654	0.1206
0.26	0.28	0.7496	0.1081
0.28	0.30	1.0015	0.0933
0.30	0.32	0.8738	0.0857
0.32	0.34	1.2712	0.0971
0.34	0.36	1.2758	0.0825
0.36	0.38	1.5916	0.0949
0.38	0.40	1.7495	0.0845
0.40	0.42	1.9353	0.1019
0.42	0.44	2.1832	0.0943
0.44	0.46	2.3189	0.1261
0.46	0.48	2.5587	0.1023
0.48	0.50	2.8917	0.1308
0.50	0.52	3.0588	0.1117
0.52	0.54	3.2292	0.1532
0.54	0.56	3.4595	0.1372
0.56	0.58	3.7583	0.1305
0.58	0.60	3.9273	0.1360
0.60	0.62	3.8202	0.1456
0.62	0.64	3.8493	0.1424
0.64	0.66	3.8297	0.1332
0.66	0.68	3.7796	0.1490
0.68	0.70	3.7061	0.1468
0.70	0.72	3.4861	0.1198
0.72	0.74	3.3356	0.1634
0.74	0.76	3.2040	0.1704
0.76	0.78	2.9714	0.1405
0.78	0.80	2.7053	0.1021
0.80	0.82	2.4643	0.0853
0.82	0.84	2.2032	0.0776
0.84	0.86	1.9259	0.0766
0.86	0.88	1.7746	0.0688
0.88	0.90	1.5298	0.0615
0.90	0.92	1.2213	0.0597
0.92	0.94	0.9208	0.0444
0.94	0.96	0.7591	0.0402
0.96	0.98	0.5077	0.0448
0.98	1.00	0.3008	0.0362
1.00	1.02	0.0752	0.0233
1.02	1.04	0.0000	0.0000
1.04	1.06	0.0007	0.0009
1.06	1.08	0.0003	0.0004
1.08	1.10	0.0003	0.0003
1.10	1.12	0.0000	0.0000
1.12	1.14	0.0000	0.0002
1.14	1.16	0.0004	0.0003
1.16	1.18	0.0007	0.0008
1.18	1.20	0.0000	0.0000

Table 7.2 : Data from CLEO at $\sqrt{s} = 10.56GeV$ in the channel $D^{*\pm} \to D^0\pi^\pm$

$x_{p,1}$	$x_{p,2}$	$\frac{d\sigma}{dx_p}/pb$	$\sigma_{\frac{d\sigma}{dx_p}}/pb$
0.20	0.25	146	86
0.25	0.30	253	60
0.30	0.35	348	60
0.35	0.40	494	60
0.40	0.45	624	46
0.45	0.50	920	50
0.50	0.55	1108	32
0.55	0.60	1244	33
0.60	0.65	1286	32
0.65	0.70	1248	31
0.70	0.75	1113	29
0.75	0.80	932	25
0.80	0.85	723	21
0.85	0.90	531	17
0.90	0.95	310	12
0.95	1.00	119	7

Table 7.3 : Data from ALEPH at $\sqrt{s} = 91.2GeV$; $D^{*\pm}s$ from the $c\bar{c}$ source

$x_{E,1}$	$x_{E,2}$	$\frac{dN\cdot 10^{-5}}{N_{Zhad}\cdot dx_E}$	$\sigma \cdot 10^{-5}$
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0.10	0.15	181	107
0.15	0.20	271	74.2
0.20	0.25	163	72.5
0.25	0.30	243	70.0
0.30	0.35	285	61.7
0.35	0.40	271	55.8
0.40	0.45	364	49.2
0.45	0.50	378	40.0
0.50	0.55	436	36.7
0.55	0.60	421	31.7
0.60	0.65	374	28.3
0.65	0.70	321	20.8
0.70	0.75	289	20.0
0.75	0.80	185	15.0
0.80	0.85	118	12.5
0.85	0.90	54.2	7.50
0.90	0.95	28.3	6.25
0.95	1.00	5.83	4.17

8 References

1. [ALEPH Collaboration] Study of charm production in Z decays, Eur. Phys. J. C 16, 597-611 (2000)

2. [Belle Collaboration] Charm hadrons from fragmentation and B decays in e^+e^- annihilation at $\sqrt{s} = 10.6$ GeV, PHYSICAL REVIEW D 73, 032002 (2006)

3. [CLEO Collaboration] Charm meson spectra in in e^+e^- annihilation at 10.5 GeV center of mass energy, PHYSICAL REVIEW D 70, 112001 (2004)

4. [H1 Collaboration] Determination of the Charm Fragmentation Function in Deep-Inelastic Scattering at HERA, XXII International Symposium on Lepton-Photon Interactins at High Energy, June 30, 2005, Uppsala

5. [Particle Data Group] REVIEW OF PARTICLE PHYSICS, 2006

6. T. Leddig, Universality of Fragmentation Functions, DESY Summerstudentprogram 2006

7. Z. Rurikova [H1 Collaboration], DESY-THESIS 2006-009