DESY Summer Student Program 2007

Work Report

Chameleon Field Theories and their Detection in Optical Experiments

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I. PARAPHOTONS, MINICHARGED PARTICLES AND LIGHT SHINING THROUGH WALLS EXPERIMENTS [1]

A. Introduction

Apart form the search of new heavy particles which is going on in present and future accelerators, there have also been of lately experiments with non-accelerator setups devoted to discover a hidden sector of particles in the meV region which couple weakly to standard-model ones.

On this behalf, there exists two kinds of experiments: laser polarisation ones, like BFRT, PVLAS and Q&A, where linearly polarised laser light propagates through a transverse external magnetic field and changes in rotation and ellipticity are looked for, and light shining through walls experiments, like ALPS, BMW, GammeV, LIPSS, OSQAR and PVLAS. In this case, linearly polarised light is shone on a wall and the presence of photons is searched for on the other side.

Light shining through walls experiments can be done in two ways: either with an external magnetic field, with which the incoming photons can oscillate into axion-like particles which penetrate the wall and are then reconverted into photons; or without one, in which it is the vacuum oscillation of photons into paraphotons the cause of light detection at the other side of the wall.

Currently, the ALPS experiment at DESY will be able to throw more light into the existence of such particles and others, like the chameleon.

B. General idea behind light shining through walls experiments

Paraphotons and minicharged particles arise in models with extra U(1) gauge degrees of freedom.

Here, I shall briefly comment on the simplest model that can give rise to the paraphoton and how kinetic mixing leads to the existence of minicharged particles.

The simplest model that includes extra U(1) gauge degrees of freedom is that with two U(1) gauge groups, one being the electromagnetic from QED, and the other that of the hidden sector. The Lagrangian for such a model is then:

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{4}B^{\mu\nu}B_{\mu\nu} - \frac{1}{2}\chi F^{\mu\nu}B_{\mu\nu} \tag{1}$$

where $F^{\mu\nu}$ is the field strength tensor for the electromagnetic gauge field A^{μ}, and $B^{\mu\nu}$ the field strength for the hidden sector field B^{μ} (the paraphoton). The first two terms are the normal kinetic terms for the two fields, while the third one is the so-called kinetic mixing. We now assume that we have a charge one hidden sector fermion under B^{μ}. It is easy to see by applying the shift $B^{\mu} \to \tilde{B}^{\mu} - \chi A^{\mu}$ that this particle would then have a charge of

$$\epsilon e = -\chi e_h. \tag{2}$$

Since χ is, in principle, an arbitrary number, the charge of the hidden sector fermion is thus not necessarily integer. For small values of χ, we have:

$$|\epsilon| \ll 1 \tag{3}$$

and our fermion becomes then a minicharged particle.

I will now briefly describe the two possible methods to realise a light shining through walls experiment.

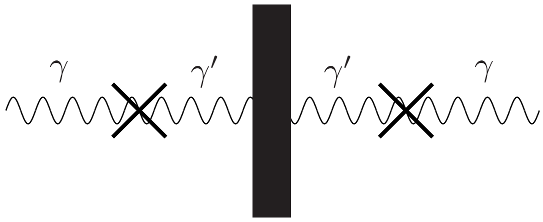

FIG. 1: Light shining through walls in the absence of a magnetic field. The crosses denote the conversion of a photon (γ) into a paraphoton (γ') and viceversa.

1. *Case I: Absence of external magnetic field*

Even in the absence of an external magnetic field there exists the possibility of a non-trivial propagation of a photon into a massive paraphotons. Since the paraphotons do not interact with standard model matter, they can easily go through a wall and then be reconverted into photons, as seen in figure 1. This process of reconversion of paraphotons much resembles neutrino oscillations. As in the case with neturinos, the propagation eigenstates are not the same as the interaction ones.

2. *Case II: $B \neq 0$*

In the actually existing light shining through walls experiments, light passes through a transverse magnetic field. The reason for this is that these experiments search for axions whose production requires such a field.

In our case, if we consider a purely minicharged particle model without paraphotons, we would observe no photon signal at the other end of the wall. This is because the minicharged particle-antiparticle pairs that would be produced would move away from each other in the presence of the magnetic field, as they have opposite charges, and, typically, also opposite momenta along the direction of the magnetic field. The pair will therefore not annihilate behind the wall and produce a visible photon.

If we include paraphotons, the photons can now convert into paraphotons that will go through the wall. This conversion with a magnetic field is also possible for massless paraphotons.

C. Conclusion

Light shining through walls experiments present both model and experimental parameter dependence. This fact can be used as an advantage to distinguish different theoretical models such as those involving ALPs or paraphotons. This difference can be seen in the polarisation of the laser beam. In the case of ALPs, a signal in a light shining through walls experiment is only expected in case of parallel or perpendicular laser polarisation with respect to the direction of the magnetic field, whereas in the paraphoton model a signal is expected for both polarisations. There are also other parameters which enable us to establish further distinguishing criteria, such as the laser frequency and the magnetic field strength.

To conclude: light shining through walls experiments offer the possibility to explore a field beyond the standard model as yet unknown to us.

II. CHAMELEON FIELD THEORIES

A. Introduction: What are chameleons and why do we need them?

Current cosmological observations and theories suggest that the Universe is undergoing an accelerated expansion. The responsible for this acceleration has been given the name of dark energy. Possible candidates for this dark energy are a cosmological constant, a slowly rolling scalar field and, of lately, chameleons.

The cosmological constant scenario with a spatially flat or a nearly spatially flat Universe implies an energy density of order $\rho_{critical} = 3H_0^2/8\pi G$ today. However, in this model, this energy density should be constant in time, giving rise to severe problems [2], making the explanation of dark energy with a cosmological constant seem quite unnatural.

A more general model is that of a slowly rolling scalar field, known as quintessence. That it is slow-rolling means that it has a negative pressure and therefore accelerates expansion. A candidate one could come up with for such a scalar field could be the dilaton from string theory, however, if this field couples strongly to matter, we would have already detected it as a "fifth force"in gravitational experiments. There must, therefore, exist a mechanism which suppresses the coupling of the field to matter, avoiding detection.

Recently, a new model was suggested by Khoury and Weltman [3] in which a scalar field with a thin-shell mechanism couples to matter with gravitational strength while remaining very light on cosmological scales. The crucial feature of this field is that the coupling gives it a mass depending on the local density of matter (and thus the name *chameleon*). In regions of high matter density, the field has a large mass and therefore its interaction with matter is small. In regions of low energy density, like the solar system, the mass is small and the interaction should be large and observable. The action of the field is in this case, however, suppressed by a so called "thin-shell mechanism", to be discussed later.

In opposition to slow rolling scalar field models, chameleons are not limited to quintessence. Chameleon behaviour can also be exhibited in, for example ϕ^4 theories.

B. Scalar-Tensor Theories [4]

Notation: For convenience, we use the reduced Planckian unit system, in which

$$8\pi G = c\hbar M_P^{-2} \tag{4}$$

where M_P is the Planck mass and has a value of 2.44×10^{18}GeV. In our case we choose $M_P = 1$

Einstein's relativity is a geometrical theory of space-time. It is sometimes called a "tensor theory"because its fundamental building block is the metric tensor, which describes the curvature of space-time. In spite of the great success of general relativity (now called the standard theory of gravitation), there have been many attempts to introduce alternative theories, one of which being the one which concerns us. This theory may seem, at first, to bring back the idea of scalar gravity but, in fact, it does not merely combine the two types of field, but is built on the basis of general relativity. The scalar field couples to the usual Einstein-Hilbert action through a nontrivial "nonminimal coupling term". The Lagrange function for chameleon fields is such as that proposed by C. Brans and R.H. Dicke [5] in their prototype model

$$\mathcal{L}_{BD} = \sqrt{-g}\left[\varphi R - \omega\frac{1}{\varphi}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi\right] + \mathcal{L}_{matter}(\psi) \tag{5}$$

where φ denotes the scalar field and ψ all other fields excepting gravitation. The dimensionless constant ω is just a parameter in the theory. Note that the scalar field does not couple to matter at the level of the Lagrangian. This assures us that the WEP is not violated.

Let us now take a closer look at the Lagrangian:

• *The nonminimal coupling term:* This term is the first term on the right hand side of equation (5). If we compare it to the standard Einstein-Hilbert term

$$\mathcal{L}_{EH} = \sqrt{-g}\frac{1}{16\pi G}R \tag{6}$$

we note that in the Brans-Dicke model there is no gravitational constant, but is characterised by an *effective gravitational constant* G_{eff} defined by

$$\frac{1}{16\pi G_{\text{eff}}} = \varphi \tag{7}$$

as long as the dynamical field φ varies sufficiently slowly. In particular we may expect that the field does not vary spatially, but only slowly with cosmic time.

• *The kinetic term:* We can notice that the second term on the right hand side of equation (5) resembles a kinetic term of the scalar field φ. However, the presence of a term in the form φ^{-1}, which seems to indicate a singularity, and that of a multiplying constant ω, can lead to confusion.

By defining new variables we can easily bring this term into the usual canonical form. For this purpose, we define a new field ϕ and a new dimensionless positive constant ξ as follows

$$\varphi = \frac{1}{2}\xi\phi^2 \tag{8}$$

and

$$\epsilon\xi^{-1} = 4\omega \tag{9}$$

Introducing these two new definitions into (5) we get

$$\begin{aligned}\mathcal{L} &= \sqrt{-g}\left[\frac{1}{2}\xi\phi^2 R - \omega\frac{2}{\xi\phi^2}g^{\mu\nu}\partial_\mu(\frac{1}{2}\xi\phi^2)\partial_\nu(\frac{1}{2}\xi\phi^2)\right] + \mathcal{L}_{matter} \\ &= \sqrt{-g}\left[\frac{1}{2}\xi\phi^2 R - \frac{\epsilon}{2\xi^2\phi^2}g^{\mu\nu}\xi^2\phi^2\partial_\mu\phi^2\partial_\nu\phi^2\right] + \mathcal{L}_{matter}\end{aligned}$$

$$\mathcal{L} = \sqrt{-g}\left[\frac{1}{2}\xi\phi^2 R - \epsilon\frac{1}{2}g^{\mu\nu}\partial_\mu\phi^2\partial_\nu\phi^2\right] + \mathcal{L}_{matter} \tag{10}$$

which now has a canonical kinetic term. As it can easily be seen, (10) describes something beyond the simple fact of adding a kinetic term to a Lagrangian with an Einstein-Hilbert action.

• *Conformal Transformations: Jordan frame and Einstein frame.* It can be shown that there always exists a conformal transformation

$$g_{\mu\nu} \rightarrow \tilde{g}_{\mu\nu} = A^2(x) g_{\mu\nu} \tag{11}$$

that eliminates the nonminimal term. In our case, the Lagrangian takes the form of a nonminimal coupling term in the "Jordan frame", while in the "Einstein frame" this term disappears after a conformal transformation, leaving only the usual Einstein-Hilbert term in the Lagrange function.

Our analysis on chameleon field theories will be done in the Einstein frame.

C. A Lagrangian for Chameleon Theories

As we have already seen, the coupling of the chameleon to both gravity and matter is given by an action of the type

$$S = \frac{1}{2\kappa_4^2} \int d^4x \sqrt{-g} \left[R - (\partial\phi)^2 - 2\kappa_4^2 V(\phi) \right] \tag{12}$$

Matter couples to both gravity and the scalar field according to

$$S_m(\psi, \tilde{g}_{\mu\nu}), \tag{13}$$

where ψ is a matter field and $\tilde{g}_{\mu\nu}$ is related to the metric in the Einstein frame by the following conformal transformation

$$\tilde{g}_{\mu\nu} = A^2(\phi) g_{\mu\nu} \tag{14}$$

in which A is an arbitrary function of ϕ.

It is now convenient to discuss some of the features of this action in a chameleon field theory.

1. *Chameleon to matter coupling*

When a scalar field ϕ couples to matter, the effect of that coupling is to make the mass, m, ϕ-dependent. This can either happen at the classical level (i.e. in the Lagrangian) or as a result of quantum corrections. We can parametrise this dependence by

$$m(\phi) = m_0 A\left(\frac{\beta\phi}{M_P}\right) \tag{15}$$

where m_0 is some constant with mass units whose definition depends on the choice of the function $A\left(\frac{\beta\phi}{M_P}\right)$. β defines the strength of the coupling of the scalar field ϕ. A mass that depends on ϕ will cause the rest-mass density to be also ϕ-dependent

$$\rho(\phi) = \rho_0 A\left(\frac{\beta\phi}{M_P}\right) \tag{16}$$

The coupling of ϕ to the local energy density is given by

$$\frac{\partial\rho(\phi)}{\partial\phi} = B'\left(\frac{\beta\phi}{M_P}\right)\frac{\beta\rho(\phi)}{M_P} \tag{17}$$

where $B(x) = \ln A(x)$. Cosmological bounds require that $|\beta B'(\beta\phi/M_P)\phi/M_P| < 0.1$ everywhere since nucleosynthesis. We use this fact it to linearise $B(\beta\phi/M_P)$:

$$B\left(\frac{\beta\phi}{M_P}\right) \approx B(0) + \frac{\beta B'(0)\phi}{M_P} \tag{18}$$

To be able to truncate the series at this point, we must require

$$\left(\frac{B''(0)}{B'(0)}\right)\frac{\beta\phi}{M_P} \ll 1. \tag{19}$$

So long as $|B''(0)| < 10|B'(0)|$, the cosmological bounds on ϕ will then ensure that the above truncation of the expansion of B is a valid one. The only forms of B that are excluded from this analysis are the ones where $|B''(0)| \gtrsim 10|B'(0)|$; we generally expect $B''(0) \sim \mathcal{O}(B'(0))$.

Provided $B'(0) \neq 0$, we can use the freedom in the definition of β to set $B'(0) = 1$. With this assumption, β quantifies the strength of the chameleon to matter coupling, as we wanted.

A particular choice for A that is usually used in the literature, [3, 6], is $A = e^{k\phi/M_P}$ for some positive k. It follows that $B = k\phi/M_P$, and so we choose $\beta = k$ which ensures $B'(0) = 1$, $B''(0) = 0$.

2. The equation of motion

The Klein–Gordon equation for the scalar field can be derived from the action using the Euler-Lagrange equations

$$\partial_\mu \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)} - \frac{\partial\mathcal{L}}{\partial\phi} = 0 \tag{20}$$

Then we have:

$$\begin{aligned}
\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)} &= -\frac{1}{2\kappa_4^2}g^{\mu\nu}(\delta^\alpha_\mu\partial_\nu\phi + \delta^\alpha_\nu\partial_\mu\phi) \\
&= -\frac{1}{2\kappa_4^2}(g^{\alpha\nu}\partial_\nu\phi + g^{\mu\alpha}\partial_\mu\phi) \\
&= -\frac{1}{2\kappa_4^2}(\partial^\alpha\phi + \partial^\alpha\phi) \\
&= -\frac{1}{\kappa_4^2}\partial^\mu\phi
\end{aligned}$$

And thus

$$\partial_\mu \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)} = -\frac{1}{\kappa_4^2}\partial_\mu\partial^\mu\phi = -\frac{1}{\kappa_4^2}\nabla_\mu\nabla^\mu\phi \tag{21}$$

On the other hand

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \phi} &= -\frac{\partial V(\phi)}{\partial \phi} - \frac{1}{\sqrt{-g}}\frac{\partial \mathcal{L}_m}{\partial \phi} \\
&= -\frac{\partial V(\phi)}{\partial \phi} - \frac{1}{\sqrt{-g}}\frac{\partial \mathcal{L}_m}{\partial \tilde{g}_{\mu\nu}}\frac{\partial \tilde{g}_{\mu\nu}}{\partial \phi} \\
&= -\frac{\partial V(\phi)}{\partial \phi} - \frac{1}{\sqrt{-g}} 2A(\phi) g_{\mu\nu}\frac{\partial \mathcal{L}_m}{\partial \tilde{g}_{\mu\nu}}\frac{\partial A}{\partial \phi} \\
&= -\frac{\partial V(\phi)}{\partial \phi} - A^2(\phi) g_{\mu\nu}\frac{2}{\sqrt{-g}}\frac{\partial \mathcal{L}_m}{\partial \tilde{g}_{\mu\nu}}\frac{\partial \ln A}{\partial \phi} \\
&= -\frac{\partial V(\phi)}{\partial \phi} - \tilde{g}_{\mu\nu}\frac{2}{\sqrt{-g}}\frac{\partial \ln A}{\partial \phi}\frac{\partial \mathcal{L}_m}{\partial \tilde{g}_{\mu\nu}} \\
&= -\frac{\partial V(\phi)}{\partial \phi} + T\frac{\partial \ln A}{\partial \phi}
\end{aligned}$$

$$\frac{\partial \mathcal{L}}{\partial \phi} = -\frac{\partial V(\phi)}{\partial \phi} + T\alpha_\phi \tag{22}$$

Where T is the trace of the energy-momentum tensor $T_{\mu\nu}$ and

$$\alpha_\phi = \frac{\partial \ln A}{\partial \phi} \tag{23}$$

Then, substracting equations (21) and (22) and applying (20), we obtain

$$\nabla_\mu \nabla^\mu \phi = \kappa_4^2 \frac{\partial V(\phi)}{\partial \phi} - \kappa_4^2 \alpha_\phi T \tag{24}$$

We can now assume, without loss of generality that we are in a Friedmann-Robertson-Walker universe which is both isotropic and homegeneous with the metric

$$g_{\mu\nu} = \mathrm{diag}(-1, \mathrm{a}^2, \mathrm{a}^2, \mathrm{a}^2)$$

The only non-vanishing Christoffel symbols for this metric are:

$$\begin{aligned}
\Gamma^1_{10} = \Gamma^2_{20} = \Gamma^3_{30} = a^{-1}\dot{a} \\
\Gamma^0_{11} = \Gamma^0_{22} = \Gamma^0_{33} = a\dot{a}
\end{aligned}$$

Also assuming that the potential is homogeneous, i.e. it doesn't depend on the spatial coordinates, we have

$$\begin{aligned}
\nabla^\mu \nabla_\mu \phi = g^{\mu\nu}\nabla_\mu \nabla_\nu \phi &= g^{\mu\nu}\partial_\mu \partial_\nu \phi - g^{\mu\nu}\Gamma^\rho_{\mu\nu}\partial_\rho \phi \\
&= g^{00}\partial_0 \partial_0 \phi - g^{ii}\Gamma^0_{ii}\partial_0 \phi \\
&= -\ddot{\phi} - 3a^{-2}a\dot{a}\dot{\phi} \\
&= -(\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi}) \\
&= -(\ddot{\phi} + 3H\dot{\phi})
\end{aligned}$$

Substituting in equation (24) we get

$$-\ddot{\phi} - 3H\dot{\phi} = \kappa_4^2 \frac{\partial V(\phi)}{\partial \phi} - \kappa_4^2 \alpha_\phi T \tag{25}$$

If we consider matter as an isotropic fluid, we can write the trace of the energy momentum tensor as

$$T = -(1 - 3\omega)\rho$$

then

$$\begin{aligned} -\ddot{\phi} - 3H\dot{\phi} &= \kappa_4^2 \frac{\partial V(\phi)}{\partial \phi} + \kappa_4^2 \alpha_\phi (1 - 3\omega)\rho \\ &= \kappa_4^2 \frac{\partial V(\phi)}{\partial \phi} + \kappa_4^2 \frac{1}{A} \frac{\partial A}{\partial \phi}(1 - 3\omega)\rho \end{aligned}$$

remembering (16) we have

$$-\ddot{\phi} - 3H\dot{\phi} = \kappa_4^2 \frac{\partial V(\phi)}{\partial \phi} + \kappa_4^2 \frac{\partial A}{\partial \phi}(1 - 3\omega)\rho_0 \tag{26}$$

We can define an effective potential, such as

$$\ddot{\phi} + 3H\dot{\phi} = -\kappa_4^2 \frac{\partial V_{\text{eff}}(\phi)}{\partial \phi} \tag{27}$$

where

$$V_{\text{eff}}(\phi) = V(\phi) + \rho_0(1 - 3\omega)A(\phi) \tag{28}$$

It is important to note that the action used to calculate the equations of motion is *not* the only way in which the scalar field ϕ can couple to matter. If one considers varying constant theories, the matter coupling usually arises from quantum loop effects [7]. However, despite the fact that many Lagrangians are possible, the equation of motion for ϕ usually takes the form of the one given above.

3. The chameleon potential

The key idea of chameleon field theory, apart form the chameleon to matter coupling term, is the self-interaction potential. There are many choices one could make for V. One could take, as it is often seen in the literature, a Ratra-Peebles potential: $V(\phi) = M^4(M/\phi)^n$ [8], where M is some mass scale and n is positive.

With the effective potential described in (28), the value ϕ takes at its minimum will generally depend on the local density of matter. For our choice of conformal transformation, in which $A(\phi)$ increases with ϕ, the potential has a minimum at

$$\frac{V'(\phi_{\text{min}})}{A'(\phi_{\text{min}})} = -\rho_0(1 - 3\omega) \tag{29}$$

When the field is at the minimum of the potential, it becomes a mass given by

$$m^2|_{\phi_{\text{min}}} = \kappa_4^2 V''_{\text{eff}} = -\kappa_4^2 \alpha_\phi (1 - 3\omega) A \frac{d\rho_0}{d\phi(\rho_0)}|_{\phi_{\text{min}}} \tag{30}$$

here we have used the fact that the minimum of ϕ is a function of ρ_0. As is easily seen, the value of the mass at the minimum will depend on $\phi_{\rm min}$ and $V(\phi)$. If $V(\phi)$ is neither constant, linear nor quadratic in ϕ, then the mass will depend on $\phi_{\rm min}$. Since $\phi_{\rm min}$ depends on the background density of matter, the effective mass will also be density-dependent.

For a scalar-tensor theory to be a chameleon theory, the effective mass must increase as the background density increases. It is important to note that the fact that $V_{\rm eff}$ has a minimum does not imply that $V(\phi)$ or $A(\phi)$ have to have one. For example, runaway potentials without a minimum can give rise to effective potentials that *do* have one, as can be seen in figure 2. In figure 3 the potential is taken to be of the form ϕ^4 and has a minimum at $\phi = 0$. The minimum of the effective potential, however, does not coincide with that of V. On both cases, the minimum of $V_{\rm eff}$ is seen to be density dependent.

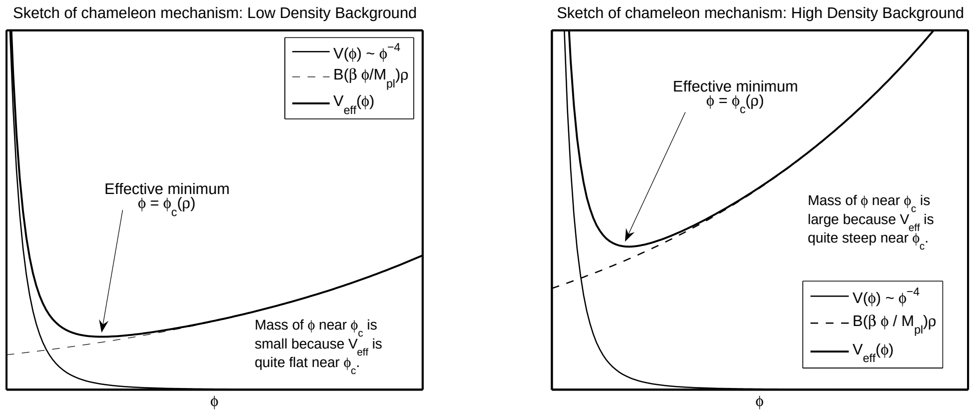

FIG. 2: Chameleon mechanism for a runaway potential: $V \sim \phi^{-4}$. The sketch on the left is for a low background density of matter, whereas the drawing of the right shows what occurs when there is a high density in the surroundings. We can see that the position of the effective minimum, $\phi_{\rm min}$, and the slope of the effective potential near that minimum, depends on the density. A shallow minimum corresponds to a low chameleon mass. The mass of the chameleon can be clearly seen to grow with the background density of matter.

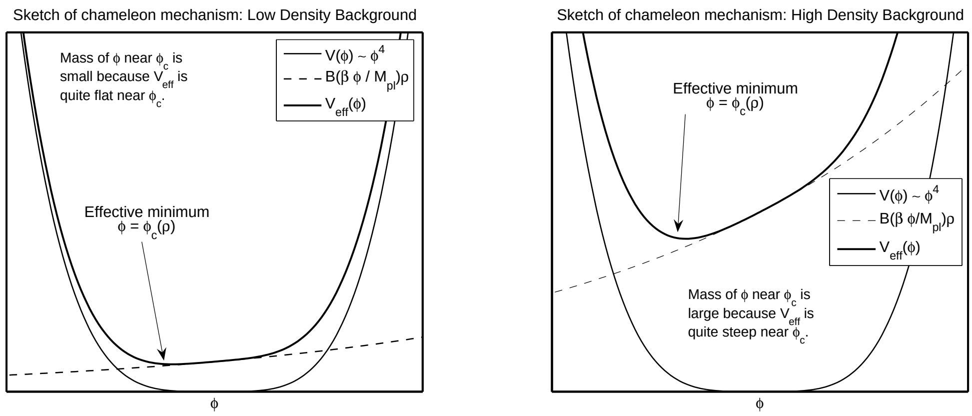

FIG. 3: Chameleon mechanism for a potential with a minimum at $\phi = 0$: $V \sim \phi^4$. The sketch on the left is for a low background density of matter, whereas the drawing of the right shows what occurs when there is a high density in the surroundings. We can see that the position of the effective minimum, $\phi_{\rm min}$, and the slope of the effective potential near that minimum, depends on the density. A shallow minimum corresponds to a low chameleon mass. The mass of the chameleon near $\phi_{\rm min}$ can be clearly seen to grow with the background density of matter.

D. The thin-shell mechanism

In chameleon theories, macroscopic bodies are said to develop a *thin-shell*. This means that the scalar field ϕ is approximately constant everywhere inside the body except for a small region near the surface, where large ($\mathcal{O}(1)$) changes in the value of ϕ occur. Hence, inside a body with such a mechanism $\vec{\nabla}\phi$ vanishes everywhere apart from in the thin-shell. Since the force exerted by such a field is proportional to $\vec{\nabla}\phi$, it is only the surface layer that feels and contributes to the fifth force mediated by ϕ.

The existence of this thin-shell effect allows chameleon theories to evade experimental constraints on the strength of the field's coupling to matter [3, 6]. For example, in the solar system the chameleon is very light and thus mediates a long-range force, however, this force has not been detected because the field only couples to a small fraction of the matter in the large bodies (the fraction in the thin-shell). As a result, the chameleon field has no great effect in planetary orbits and therefore, the constraints on such a long-range force are evaded.

To illustrate the idea of the thin-shell mechanism, let us solve the equation of motion for a spherical body of radius R and density ρ_b embedded in a medium with density ρ_m. We distinguish two different masses and minima: the mass and minimum inside the body (m_b and ϕ_b) and those outside it (m_∞ and ϕ_∞). The Klein–Gordon equation in spherical coordinates reads

$$\frac{\mathrm{d}^2\phi}{\mathrm{dr}^2} + \frac{2}{r}\frac{\mathrm{d}\phi}{\mathrm{dr}} = \frac{\partial V_{\mathrm{eff}}}{\partial\phi} \tag{31}$$

There exist three classes of solution to this equation:

1. Solution I: Outside the sphere

Outside the sphere the effective potential can be approximated by

$$V_{\mathrm{eff}} \approx \frac{1}{2}m_\infty^2(\phi - \phi_\infty)^2 \tag{32}$$

The equation we then have to solve is

$$\frac{\mathrm{d}^2\phi}{\mathrm{dr}^2} + \frac{2}{r}\frac{\mathrm{d}\phi}{\mathrm{dr}} = m_\infty^2(\phi - \phi_\infty) \tag{33}$$

The general solution to this differential equation is

$$\phi(r) = A\frac{e^{-m_\infty(r-R)}}{r} + B\frac{e^{m_\infty(r-R)}}{r} + \phi_\infty, \tag{34}$$

for dimensionless constants A and B. Imposing the condition that $\phi \to \phi_\infty$ as $r \to \infty$ gives $B = 0$, so we have

$$\phi(r) = A\frac{e^{-m_\infty(r-R)}}{r} + \phi_\infty. \tag{35}$$

2. Solution II: Inside the sphere

For $r < R$ there are two classes of solution depending on two different approximations. First of all, we need to define a value R_b which divides the interval $[0, R]$ into two: $[0, R_b]$ in which $\phi \sim \phi_b$ and $[R_b, R]$ where $\phi \gg \phi_b$.

Approximation 1: $\phi \gg \phi_b$. In this case the harmonic approximation for the effective potential (32) is not valid. But taking a look at figure 2, we notice that for $\phi > \phi_{\rm min}$, the bare potential V decays quickly and the term $\rho e^{\beta\phi/M_P}$ dominates. In particular, we have

$$\frac{\partial V_{\rm eff}}{\partial \phi}(\phi) \approx \frac{\beta}{M_P}\rho_b e^{4\beta\phi/M_P} \approx \frac{\beta}{M_P}\rho_b, \tag{36}$$

since usually $\phi \ll M_P$. Taking this into account, our differential equation takes the form

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}r^2} + \frac{2}{r}\frac{\mathrm{d}\phi}{\mathrm{d}r} \approx \frac{\beta}{M_P}\rho_b \tag{37}$$

with the general solution

$$\phi(r) = \frac{\beta}{6M_P}\rho_b r^2 + \frac{C}{r} + D\phi_b, \tag{38}$$

for dimensionless constants C and D.

Approximation 2: $\phi \sim \phi_b$. In this case we can again use the harmonic approximation (32) but this time we will write the solution as

$$\phi(r) = E\frac{e^{-m_b r}}{r} + F\frac{e^{m_b(r-R_b)}}{r} + \phi_b, \tag{39}$$

where E and F are, again, dimensionless constants.

We now have to apply boundary and continuity conditions to obtain the final solution. We distinguish three cases in function of the possible values of R_b

Case 1: $R_b = R$. Here we use solution (35) outside the sphere and (39) inside the sphere. The first condition that we must impose is that $\phi(r)$ cannot diverge when $r \to 0$. In order to achieve this, we must have $E = -Fe^{-m_b R}$ in (39), giving

$$\phi(r) = F\frac{e^{m_b(r-R)} - e^{-m_b(r+R)}}{r} + \phi_b \tag{40}$$

Secondly, we must impose continuity of the solution at the origin, that is $\frac{\mathrm{d}\phi}{\mathrm{d}r} \to 0$ as $r \to 0$ (we can easily check by applying L'Hôpital's rule that our solution for $\phi(r)$ already fulfills this). Therefore, we have

$$\phi(r) = \begin{cases} F\dfrac{e^{m_b(r-R)} - e^{-m_b(r+R)}}{r} + \phi_b & r < R \\ A\dfrac{e^{-m_\infty(r-R)}}{r} + \phi_\infty & r > R. \end{cases}$$

Finally, we must impose continuity conditions at $r = R$

$$\lim_{r\to R^-} \phi(r) = \lim_{r\to R^+} \phi(r) \qquad \text{and} \qquad \lim_{r\to R^-} \frac{\mathrm{d}\phi}{\mathrm{d}r} = \lim_{r\to R^+} \frac{\mathrm{d}\phi}{\mathrm{d}r}$$

These conditions give equations for constants A and F

$$F\frac{1 - e^{-2m_b R}}{R} + \phi_b = A\frac{1}{R} + \phi_\infty,$$
$$F\left[m_b R - 1 + m_b R e^{-2m_b R} + e^{-2m_b R}\right] = A\left[-m_\infty R - 1\right].$$

The solution to these equations is

$$A = \frac{\phi_\infty - \phi_b}{m_b + m_\infty + m_b e^{-2m_b R} - m_\infty e^{-2m_b R}} \left(1 - m_b R - e^{-2m_b R} - m_b R e^{-2m_b R}\right),$$
$$F = \frac{\phi_\infty - \phi_b}{m_b + m_\infty + m_b e^{-2m_b R} - m_\infty e^{-2m_b R}} \left(1 + m_\infty R\right).$$

Case 2: $R_b = 0$ *thick-shell regime***.** In this case we use solutions (34) and (38). The non-divergence condition at the origin requires simply $C = 0$ in equation (38), so we have

$$\phi(r) = \begin{cases} \frac{\beta}{6M_P}\rho_b r^2 + D\phi_b & r < R \\ A\frac{e^{-m_\infty(r-R)}}{r} + \phi_\infty & r > R, \end{cases}$$

The continuity conditions give equations for A and D:

$$\frac{\beta}{6M_P}\rho_b R^2 + D\phi_b = A\frac{1}{R} + \phi_\infty$$
$$\frac{\beta}{3M_P}\rho_b R = A\frac{-m_\infty R - 1}{R^2},$$

with solutions

$$A = -\frac{\beta}{3M_P}\rho_b \frac{R^3}{1 + m_\infty R},$$
$$D = \frac{\phi_\infty}{\phi_b} - \left(\frac{1}{1 + m_\infty R} + \frac{1}{2}\right)\frac{\beta\rho_b R^2}{3\phi_b M_b}.$$

Case 3: $0 < R_b < R$ *thin-shell regime***.** Now the solution is divided between three regions. As in the case $R_c = R$, we have $E = -Fe^{-m_b R_b}$. Then ϕ and $\mathrm{d}\phi/\mathrm{dr}$ are given by:

$$\phi(r) = \begin{cases} F\frac{e^{m_b(r-R_b)} - e^{-m_b(r+R_b)}}{r} + \phi_b & 0 < r < R_b \\ \frac{\beta}{6M_P}\rho_b r^2 + \frac{C}{r} + D\phi_b & R_b < r < R \\ A\frac{e^{-m_\infty(r-R)}}{r} + \phi_\infty & R < r < \infty, \end{cases}$$

In this case there are four continuity equations: two at $r = R_b$,

$$\begin{aligned} F\frac{1 - e^{-2m_b R_b}}{R_b} + \phi_b &= \frac{\beta}{6M_P}\rho_b R_b^2 + \frac{C}{R_b} + D\phi_b \\ F\frac{m_b R_b - 1 + m_b R_b e^{-2m_b R_b} + e^{-2m_b R_b}}{R_b^2} &= \frac{\beta}{3M_P}\rho_b R_b - \frac{C}{R_b^2} \end{aligned} \tag{41}$$

and two at $r = R$,

$$\begin{aligned} \frac{\beta}{6M_P}\rho_b R^2 + \frac{C}{R} + D\phi_b &= A\frac{1}{R} + \phi_\infty, \\ \frac{\beta}{3M_P}\rho_b R - \frac{C}{R^2} &= A\frac{-m_\infty R - 1}{R^2}. \end{aligned} \tag{42}$$

3. The thin-shell suppression factor

We now want to study the behaviour of the chameleon field outside the body. This is of special interest, as it is here where experiments can measure the chameleon force.

In all of the above mentioned cases, the exterior solution can be approximated to

$$\phi\left(r\right) = A\frac{e^{-m_\infty(r-R)}}{r} + \phi_\infty .$$

The constant A gives us the magnitude of ϕ and, thus, the chameleon force.

In the thick-shell case, assuming $m_\infty R \ll 1$, we have

$$\begin{aligned} A &= -\frac{\beta}{3M_P}\rho_b\frac{R^3}{1+m_\infty R} \\ &\approx -\frac{\beta}{4\pi M_P}\left(\frac{4}{3}\pi R^3\rho_b\right). \end{aligned}$$

In the thin-shell case, we assume as in [9] that $\phi = \phi_b$ for $r < R_b$, which translates for us to $F = 0$. Then the two continuity equations (41) give

$$\begin{aligned} C &= \frac{\beta}{3M_P}\rho_b R_b^3, \\ D &= 1 - \frac{\beta\rho_b R_b^2}{2M_P}\frac{1}{\phi_b}. \end{aligned}$$

When substituting in the continuity equation (42), we get

$$\begin{aligned} &\frac{\beta}{3M_P}\rho_b R - \frac{\beta}{3M_P}\rho_b\frac{R_b^3}{R^2} \approx -\frac{A}{R^2} \\ &\Rightarrow A \approx -\frac{\beta}{3M_P}\rho_b\left(R^3 - R_b^3\right). \end{aligned} \tag{43}$$

Next, substituting into (42) gives

$$\begin{aligned} \frac{\beta}{6M_P}\rho_b R^2 + \frac{\beta}{3M_P}\rho_b\frac{R_b^3}{R} + \phi_b - \frac{\beta\rho_b R_b^2}{2M_P} &= -\frac{\beta}{3M_P}\rho_b R^2 + \frac{\beta}{3M_P}\rho_b\frac{R_b^3}{R} + \phi_\infty \\ \Rightarrow \frac{\beta}{2M_P}\rho_b R^2 + \phi_b - \frac{\beta\rho_b R_b^2}{2M_P} &= \phi_\infty \\ \Rightarrow R^2 - R_b^2 &= \frac{2M_P}{\beta\rho_b}(\phi_\infty - \phi_b). \end{aligned}$$

Finally, if we use a Taylor expansion in R_b about R we get

$$\begin{aligned} A &\approx -\frac{\beta}{3M_P}\rho_b\frac{3}{2}R\left(R^2 - R_b^2\right) \\ &= -\frac{\beta}{3M_P}\rho_b\frac{3}{2}R\frac{2M_P}{\beta\rho_b}\left(\phi_\infty - \phi_b\right) \\ &= -\frac{\beta}{4\pi M_P}\left(\frac{4}{3}\pi R^3\rho_b\right)\frac{3M_P\left(\phi_\infty - \phi_b\right)}{\beta\rho_b R^2} \end{aligned}$$

Putting this all together, the external solutions for the thick- and thin-shell regimes are (see [9])

$$\phi_{\text{thick}}(r) \approx -\frac{\beta}{4\pi M_P}\left(\frac{4}{3}\pi R^3\rho_b\right)\frac{e^{-m_\infty(r-R)}}{r} + \phi_\infty$$
$$\phi_{\text{thin}}(r) \approx -\frac{\beta}{4\pi M_P}\left(\frac{4}{3}\pi R^3\rho_b\right)\left(3\frac{M_P\left(\phi_\infty - \phi_b\right)}{\beta\rho_b R^2}\right)\frac{e^{-m_\infty(r-R)}}{r} + \phi_\infty$$

As explained in [9], the difference between these two solutions is the so-called "thin-shell suppression factor"$3\Delta R/R$, where

$$\frac{\Delta R}{R} \equiv \frac{M_b\left(\phi_\infty - \phi_b\right)}{\beta\rho_b R^2}.$$

In general, a "chameleon suppression factor" can be defined as

$$\begin{aligned} W &\equiv -A\left[\frac{\beta}{4\pi M_P}\left(\frac{4}{3}\pi R^3\rho_b\right)\right]^{-1} \\ &= -A\frac{3M_P}{\beta R^3\rho_b}. \end{aligned}$$

Clearly we have $W \approx 1$ in the thick-shell case and $0 < W < 1$ in the other two cases. This gives us a means of quantifying the "thickness of shell" of an object in a given background density.

E. Detecting chameleons with light propagating through a magnetic field

Until now, the best constraints on corrections to general relativity come from gravitational experiments such as the Eöt-Wash experiment, [10] and Lunar Laser Ranging tests for WEP violations, [11, 12]. At very small distances $d \lesssim 10\mu m$, the best bounds on the strength of any fifth force come from measurements of the Casimir force. However, lately the PVLAS experiment could have also detected the chameleon [13] and there could also exist the possibility to detect the chameleon in an optical experiment such as the ALPS.

How do we do this? The idea is quite simple: we have a laser propagating through a cavity with a transverse magnetic field. At each end of this cavity there are two glass windows. Due to the magnetic field, it is possible that some of the photons from the laser turn into massive chameleons. These massive chameleons cannot go through the glass windows, so they will reflect, creating a standing wave inside the cavity. After some time, we turn off the laser. The chameleons that where in the cavity could then, again due to the magnetic field, reconvert into photons that could go through the glass windows and be detected.

Let us go into this in a bit more detail:

The effective Lagrangian at lowest order for this mixing would be

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 + \frac{1}{4}\beta\phi F^{\mu\nu}F_{\mu\nu} \tag{44}$$

where ϕ is our chameleon field and $F_{\mu\nu}$ the electromagnetic field tensor. β is, as already mentioned, the coupling strength

$$\beta = \frac{M_P}{M} \tag{45}$$

The equations of motion can again be derived from the Euler-Lagrange equations. For the scalar field we have:

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)} &= \frac{1}{2}g^{\mu\nu}(\delta^\alpha_\mu\partial_\nu\phi+\delta^\alpha_\nu\partial_\mu\phi)\\ &= \frac{1}{2}(g^{\alpha\nu}\partial_\nu\phi+g^{\mu\alpha}\partial_\mu\phi)\\ &= \frac{1}{2}(\partial^\alpha\phi+\partial^\alpha\phi)\\ &= \frac{1}{2}\partial^\mu\phi\end{aligned}$$

And thus

$$\partial_\mu\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)} = \partial_\mu\partial^\mu\phi = \partial^2\phi \tag{46}$$

$$\frac{\partial\mathcal{L}}{\partial\phi} = -m^2+\frac{1}{4}\beta F^{\mu\nu}F_{\mu\nu} \tag{47}$$

Subtracting (46) and (47) we get

$$(\partial^2+m^2)\phi = \frac{1}{4}\beta F^{\mu\nu}F_{\mu\nu} \tag{48}$$

And for the electromagnetic field:

$$\frac{\partial\mathcal{L}}{\partial(\partial_\alpha A_\gamma)} = -\frac{1}{4}(1-\beta\phi)[g^{\mu\rho}g^{\nu\sigma}\left[\delta^\alpha_\rho\delta^\gamma_\sigma\partial_\mu A_\nu+\delta^\alpha_\mu\delta^\gamma_\nu\partial_\rho A_\sigma\right]-g^{\mu\rho}g^{\nu\sigma}\left[\delta^\alpha_\rho\delta^\gamma_\sigma\partial_\nu A_\mu+\delta^\alpha_\nu\delta^\gamma_\mu\partial_\rho A_\sigma\right]$$
$$-g^{\nu\rho}g^{\mu\sigma}\left[\delta^\alpha_\rho\delta^\gamma_\sigma\partial_\mu A_\nu+\delta^\alpha_\mu\delta^\gamma_\nu\partial_\rho A_\sigma\right]+g^{\nu\rho}g^{\mu\sigma}\left[\delta^\alpha_\rho\delta^\gamma_\sigma\partial_\nu A_\mu+\delta^\alpha_\nu\delta^\gamma_\mu\partial_\rho A_\sigma\right]] = (1-\beta\phi)\left[\partial^\alpha A^\gamma-\partial^\gamma A^\alpha\right]$$

An thus

$$\partial_\alpha\frac{\mathcal{L}}{\partial(\partial_\alpha A_\gamma)} = \partial_\alpha(1-\beta\phi)F^{\alpha\gamma} \tag{49}$$

$$\frac{\partial\mathcal{L}}{\partial A_\gamma} = 0 \tag{50}$$

Subtracting (49) and (50)

$$\partial_\mu F^{\mu\nu} = \beta\partial_\mu(\phi F^{\mu\nu}) \tag{51}$$

Since the background classical field (F) will be much larger than the quantum ones (f), we can linearise the equations of motion with respect to the latter to obtain

$$\begin{aligned}(\partial^2+m^2)\phi &= \frac{1}{2}\beta f_{\mu\nu}F^{\mu\nu}\\ \partial_\mu f^{\mu\nu} &= \beta F^{\mu\nu}\partial_\mu\phi\end{aligned} \tag{52}$$

We can express these equations as function of the magnetic field $\mathbf{B}$ and the vector potential $\mathbf{a}$ as follows (working in the usual gauge $a^0=\nabla\cdot\mathbf{a}=0$)

$$\begin{aligned}(\partial_t^2-\nabla^2+m^2)\phi &= \beta(\nabla\times\mathbf{a})\cdot\mathbf{B}\\ (\partial_t^2-\nabla^2)\vec{a} &= \beta(\nabla\phi)\times\mathbf{B}\end{aligned} \tag{53}$$

We take the evolution equation for a plane wave laser propagating in the z direction perpendicular to $\mathbf{B}$ from [14]. For constant $\mathbf{B}$, the energies of the laser photon and the chameleon are equal. We can therefore remove the time dependence by substituting $\mathbf{a} \to \mathbf{a}e^{-i\omega t}$, $\phi \to \phi e^{-i\omega t}$, where ω is the angular frequency of the laser light. For propagation in the $+z$ direction, we can substitute on the left-hand side of equations, $(\omega^2 + \partial_z^2) = (\omega - i\partial_z)(\omega + i\partial_z) \to 2\omega(\omega + i\partial_z)$, and on the right, $\partial_z \to +i\omega$. We also choose $\mathbf{B} = B\overrightarrow{e_x}$. The equations of motion in matrix form are then

$$\left(1 + i\omega^{-1}\partial_z + \Omega\right) \begin{pmatrix} a^{\|} \\ a^{\perp} \\ \phi \end{pmatrix} = 0, \quad \Omega = \begin{pmatrix} 0 & 0 & +i\delta \\ 0 & 0 & +i\delta \\ -i\delta & -i\delta & -\delta_0 \end{pmatrix}, \tag{54}$$

where

$$\delta = (2\omega)^{-1}\beta B, \ \delta_0 = \frac{m^2}{2\omega^2} \tag{55}$$

and $a^{\|,\perp}$ are respectively the parallel and perpendicular components of the vector potential to $\mathbf{B}$.

To obtain the eigenmodes of propagation, we diagonalise Ω

$$\Omega_{\text{diag}} = U^{-1}\Omega U = \delta_0 \ \text{diag}\left(0, \epsilon^2, -(\epsilon^2 - 1)\right), \tag{56}$$

$$U = E \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\epsilon & -\sin\epsilon \\ 0 & \sin\epsilon & \cos\epsilon \end{pmatrix} \tag{57}$$

where $\epsilon^2 = \frac{1}{2}(1 + \sqrt{1 - 8\delta^2/\delta_0^2})$ and $E = \text{diag}(i, i, 1)$ removes the $\pm i$ factors in Ω.

A state that enters the $\mathbf{B}$ field at z_0, $\Psi(z_0) = (a_{\|}(z_0), a_{\perp}(z_0), \phi(z_0))^T$ will evolve into the state $\Psi(z) = (a_{\|}(z), a_{\perp}(z), \phi(z))^T$, after having traveled a distance $z - z_0 = L \geq 0$ in the $+z$ direction:

$$\Psi(z_0 + L) = e^{i\omega L} V_+(L)\Psi(z_0), \ V_+(L) = U e^{i\omega L \Omega_{\text{diag}}} U^{-1} \tag{58}$$

If we expand $V_+(L)$ in ϵ (assuming small ϵ) we obtain

$$V_+(L) = V_0(L) + \epsilon V_1(L) + \epsilon^2 V_2(L) + O(\epsilon^3) \tag{59}$$

where

$$\begin{aligned} V_0(L) &= \text{diag}(1, 1, e^{-i\zeta}), \ \zeta = \delta_0 \omega L \\ V_1(L) &= (1 - e^{-i\zeta}) \begin{pmatrix} & & 0 \\ & & i \\ 0 & -i & 0 \end{pmatrix} \\ V_2(L) &= (-1 + i\zeta + e^{-i\zeta}) \begin{pmatrix} 0 & 0 & \\ 0 & 1 & \\ & & 0 \end{pmatrix} \\ &+ (1 - e^{-i\zeta}(1 + i\zeta))\text{diag}(0, 0, 1) \end{aligned}$$

For propagation in the $-z$ direction, we have to do the same as before, but, when linearising the equations of motion, we have to now make the following substitutions: $\partial_z \to -\partial_z$ and $\beta \to -\beta$. The latter replacement amounts to $1 \to -1$ in the matrices U and V for the chosen direction of $\mathbf{B}$.

Now, a state $\Psi(z_0)$, entering the $\mathbf{B}$ field at z_0 will evolve into the following one, after having traveled a distance $z_0 - z = L$ in the $-z$ direction:

$$\Psi(z_0 - L) = e^{i\omega L} V_-(L)\Psi(z_0), \ V_-(L) = V_+(L)|_{1\to -1} \tag{60}$$

Let us now consider the optical effects of the evolution equation. Suppose, as it is usually the case, that the laser light propagates in the $+z$ direction and enters a transverse $\mathbf{B} = B\overrightarrow{e_x}$ field. If the laser is initailly polarised at an angle θ measured anti-clockwise in the (xy) plane with respect to the field, it will evolve into the following state after traveling a distance L inside the magnetic field:

$$\begin{pmatrix} a_{\|} \\ a_{\perp} \\ \phi \end{pmatrix}(L) = e^{i\omega L} V_{+}(L) \begin{pmatrix} \cos\theta \\ \sin\theta \\ 0 \end{pmatrix} \equiv e^{i\omega L} \begin{pmatrix} \eta\cos(\theta+\Delta\theta) \\ \eta\sin(\theta+\Delta\theta)e^{i\varphi_\perp} \\ \rho e^{i\sigma} \end{pmatrix} \tag{61}$$

and, to $\mathcal{O}(\epsilon^2)$,

$$\begin{aligned} \eta\cos(\theta+\Delta\theta) &= \cos\theta \\ \eta\sin(\theta+\Delta\theta) &= \sin\theta - \epsilon^2(1-\cos\zeta)\sin\theta \\ \varphi_\perp &= \epsilon^2 \sin\theta \frac{1}{\sin\theta}(\zeta - \sin\zeta) \\ \rho e^{i\sigma} &= 2\epsilon\sin\theta\sin\frac{\zeta}{2}\exp\left(-\frac{i}{2}\zeta\right) \end{aligned}$$

where $\eta^2 + \rho^2 = 1$ must hold in order to have maximum probability 1. $\Delta\theta$ is the amount by which the polarisation of the initial beam has been rotated

$$\Delta\theta = -\epsilon^2 \sin^2\frac{\zeta}{2}\sin 2\theta \tag{62}$$

The relative phase shift of the parallel and perpendicular components of the light beam results in the ellipticity measured by $\tan\chi$ where $\chi \in [-\frac{\pi}{4}, \frac{\pi}{4}]$ is determined by $\sin 2\chi = \sin 2(\theta+\Delta\theta)\sin\varphi_\perp$. Thus, to $\mathcal{O}(\epsilon^2)$, we can approximate

$$\tan\chi \approx \chi \approx \frac{1}{2}\epsilon^2(\zeta - \sin\zeta)\sin 2\theta \tag{63}$$

A more detailed calculation of rotation and ellipticity can be found in [15].

The production probability of chameleons is

$$P[(\gamma \text{ at } \theta) \to \phi] = |\rho|^2 = 4\epsilon^2\sin^2\theta\sin^2\frac{\zeta}{2} \tag{64}$$

The chameleons will then evolve into the state

$$e^{i\omega L}\left(0, i\epsilon(1-e^{-i\zeta}), e^{-i\zeta} + \epsilon^2[1 - e^{-i\zeta}(1+i\zeta)]\right)^T \tag{65}$$

The probability that a chameleon will then reconvert into a photon is hence

$$P[\phi \to \gamma] = 4\epsilon^2\sin^2\frac{\zeta}{2} \tag{66}$$

III. CONCLUSIONS AND OUTLOOK

In this report I have tried to give an overview of my work at DESY. I started with a brief introduction to hidden sector particles and their detection using light shining through walls experiments, and then moved to chameleons.

In the part of my report devoted to the latter, I explained scalar-tensor theories and how their action is applicable to chameleons. From this action, I derived the equation of motion and discussed the several choices one has for the potential in chameleon theories, showing that these scalar fields possess an effective potential whose minimum depends on the matter density of the surroundings. I then moved on to explain how chameleons evade detection by the means of a thin-shell mechanism and how this problem may be overcome by the use of optical setups.

There still remain, however, many unanswered questions. What is the exact function of the chameleon potential? We have assumed an inverse-power-law potential, but it could also have been an exponential type one or some other sort. What values are allowed for the coupling constant β? Until today, the existing experiments do not agree in giving a limit to this problematic constant. Astronomical bounds and string theory predict a small β, whereas recent data from PVLAS give it an order of magnitude of 10^{13}. Determining the value of the coupling would allow us to know more precisely the origin of the chameleon scalar field and in which energy scale is it likely to be found.

The chameleon model is intriguing because of its promise of explaining cosmic acceleration and dark energy nature using the property of density dependence. However, there is still a long way to go. I have here used the perfect fluid approximation, which is not easily evaded in chameleon gravity. This must be regarded as a flaw in the model, since the real Universe contains inhomogeneities which do not allow the local density of matter to be well-defined. A more general solution to the chameleon equation would allow us to make more realistic predictions involving chameleons.

Another interesting point is the liberty of choice one has in specifying the conformal transformation in (14). This transformation defines the coupling of the field to matter and hence determines very important features in the model, such as the thin-shell condition or the chameleon force.

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