DESY Summer Students Session 2006

# Current status of the CKM matrix and CP-violating phases

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# Plan

# Introduction

The experiments on $\beta$- and muon decay proved that the effective coupling to the W boson is different in quark and lepton sectors. The theory, based on the flavour symmetry, required the quark-lepton universality. The mechanism of the flavour symmetry breaking in the quark sector was proposed by Cabibbo.

Cabibbo-Kobayashi-Maskawa Matrix

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} = \hat{\mathbf{V}}_{\mathbf{CKM}} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

weak eigenstates mass eigenstates

# Parametrizations

## Standard Parametrization

$$\hat{V}_{CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

## Wolfenstein parametrization

$$\hat{V}_{CKM} \approx \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda(1 + iA^2\lambda^4\eta) & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2(1 + i\lambda^2\eta) & 1 \end{pmatrix}$$

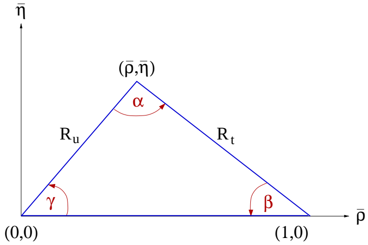
# Unitarity conditions

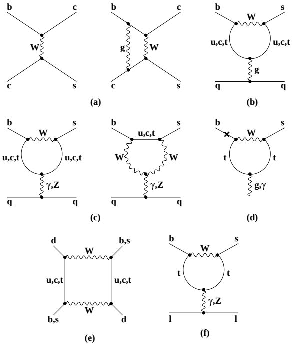
Unitarity imposes:

$$\sum_{j=1}^{3} V_{ij} V_{jk} = \delta_{ik}$$

In particular:

$$V_{ud} V_{ub}^{*} + V_{cd} V_{cb}^{*} + V_{td} V_{tb}^{*} = 0$$






Feynman diagrams typical for weak transitions

# $|V_{ud}|$

- Measures the coupling between the $u$ and $d$ quark
- Experimental determination:
  - $0^+ \rightarrow 0^+$ nuclear Fermi transitions
    The average value yields: $|V_{ud}| = 0.97377 \pm 0.00027$.
  - neutron $\beta$-decay
    The measured value is: $|V_{ud}| = 0.9717 \pm 0.0013 \pm 0.0004$
  - pion $\beta$-decay
    The obtained value yields: $|V_{ud}| = 0.9728 \pm 0.030$.

# $|V_{us}|$

Experimental determination:

- from kaon semileptonic decays $K^+ \to \pi^0 e^+ \nu(\gamma))$
  The present value yields: $|V_{us}| = 0.2257 \pm 0.0021$
- from kaon, pion and hyperon leptonic decays
  The current value is: $|V_{us}| = 0.2250 \pm 0.0026$

# $|V_{cd}|$ and $|V_{cs}|$

- $|V_{cd}|$ obtained from $\nu_\mu + d \to \mu^- c;\ c \to s\mu^+\nu_\mu$
  $|V_{cd}| = 0.230 \pm 0.011$
- Other methods: semileptonic charm decays
- $|V_{cs}|$ obtained indirectly from W decays
  - $$\frac{\Gamma(W^+ \to c\bar{q})}{\Gamma(W^+ \to hadrons)} = \frac{\sum_{i=d,s,b} |V_{ci}|^2}{\sum_{i=u,c\ j=d,s,b} |V_{ij}|^2}$$
    This ratio is expected to be 1/2 (lepton uniwersality, three-generations CKM)
    $|V_{cs}| = 0.94^{+0.32}_{-0.13}$.
  - From the ratio: $\Gamma(W^+ \to hadrons)/\Gamma(W^+ \to l^+\nu_l)$.

# $|V_{cb}|$

Experimental determination of $|V_{cb}|$ is based on the semileptonic decays of the $b$ -quark, $b \to c l \nu_l$.

- exclusive $B \to (D, D^*) l \nu_l$ decay
  From the formula
  $\frac{d\Gamma}{d\omega} = \frac{G_F^2}{4\pi^3}\sqrt{\omega^2 - 1}\ m_{D(*)}^3 (m_B - m_{D(*)})^2 \mathcal{G}(\omega)|V_{cb}|^2|\mathcal{F}(\omega)|^2$,
  $|V_{cb}| = (40.9 \pm 1.8) \times 10^{-3}$
- inclusive $B \to X_c l \nu_l$ reactions
  Inclusive decays provide more precise metod of determining the value of $|V_{cb}|$, namely: $|V_{cb}| = (41.7 \pm 0.7) \times 10^{-3}$.
- The average value: $|V_{cb}| = (41.7 \pm 0.6) \times 10^{-3}$.

# $|V_{ub}|$

The value of $|V_{ub}|$ is obtained by measuring rare $b \to u$ events.

- Inclusive $B \to X_u l \nu_l$ decays.
  The branching ratio is about factor 60 lower than for $B \to X_c l \nu_l$ processes.
  Experimental cut-offs in the phase space
  - analysis of regions of phase space where $B \to X_c l^- \bar{\nu}_l$ are kinematically excluded
  - cutting on the hadronic invariant mass $m_X$, namely $m_X < m_D$.

  Problems
  - shape functions
  - detector resolution
- The value obtained in the exclusive measurment $B \to \pi l \nu_l$ yields $|V_{ub}| = (3.3^{+0.6}_{-0.4}) \times 10^{-3}$.

# Determination of $|V_{tb}|$

$|V_{tb}|$ can be measured directly in the $t \to W^+ b$ decay, described by the tree diagram. The branching ratio for this process:

$$R \equiv \frac{\mathcal{B}(t \to W + b)}{\mathcal{B}(t \to W + q)} = \frac{|V_{tb}|^2}{|V_{td}|^2 + |V_{ts}|^2 + |V_{tb}|^2}$$

has been measured during RUN II of the Tevatron by CDF and DΦ, giving: $R = 1.12^{+0.27}_{-0.23}$ and $R = 1.03^{+0.19}_{-0.17}$, respectively. This results in $|V_{tb}| = 1.058^{+0.520}_{-0.480}$, $|V_{tb}| = 1.015^{+0.436}_{-0.413}$, respectively.
Both are consistent with the SM, within large errors.

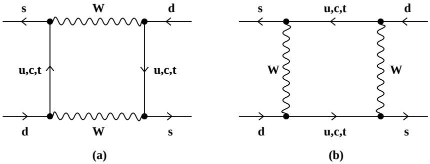
# Determination of $|V_{td}|$ and $|V_{ts}|$

Both are determined from top quark contributions in the loop induced $b$-quark transitions. Very precise method provided by the oscillations of $B_q^0 \bar{B}_q^0$ mesons ($q = c, d$).

$$\Delta m_q = \frac{G_F^2}{6\pi^2} \eta_B m_{B_q} (f_{B_q}^2 B_q) m_W^2 S(x_t) |V_{tq} V_{tb}^*|^2 \quad q = (d, s)$$

Assuming $|V_{tb}| = 1 \Rightarrow |V_{td}| = (7.4 \pm 0.8) \times 10^{-3}$.
There are theoretical uncertainties connected with extrapolating the light-quark mass (typically $\approx$ 100 MeV)to the physical value of $m_d$ ($\approx$ 8 MeV) in the Lattice calculations. To reduce uncertainties, the lattice calculations determine the ratio $\xi = (f_{B_s}\sqrt{B_s})/(f_{B_d}\sqrt{B_d})$.
$|V_{td}/V_{ts}| = 0.208^{+0.001}_{-0.002}$ (exp) $^{+0.008}_{-0.006}$ (theo).

# CPV in mixing

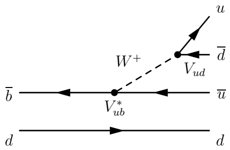


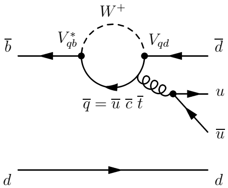

The CP violation was first observed in the neutral Kaons. It is caused by the presence of two box diagrams, providing $K^0 \bar{K}^0$ mixing.

# CPV in decay amplitudes

The asymmetry in the decays of opposite charged particles appear if the dominant contribution comes from at least two different diagrams with different strong and weak phases. The strong phase is the same for $A(M^+ \to f^+)$ and $\bar{A}(M^- \to f^-)$ meson decays amplitudes, as strong interactions are CP-conserving. The corresponding weak phases are of opposite sign.

# CPV in interference of mixing and decay



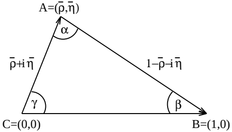




$B^0$ and $\bar{B}^0$ decays to $\pi^+\pi^-$. This type of CP violation is possible only in neutral mesons decays. Interference between the processes: $B^0 \to \pi^+\pi^-$ and $B^0 \to \bar{B}^0 \to \pi^+\pi^-$ causes the time-dependent asymmetry.

$$\mathcal{A}_{\pi\pi} = \frac{\Gamma(\bar{B}^0(t) \to \pi^+\pi^-) - \Gamma(B^0(t) \to \pi^+\pi^-)}{\Gamma(\bar{B}^0(t) \to \pi^+\pi^-) + \Gamma(B^0(t) \to \pi^+\pi^-)}$$

# Charmonium modes

The reaction $B^0 \to J/\Psi K^0_{S,L}$ is mediated by box diagrams with $b$ and $\bar{d}$ on the external lines. The dominating loop contribution stems from $t$.




The phase appearing in the mixing yields $2\beta$.
The world average (BABAR and BELLE measurments) is
$\sin 2\beta = 0.687 \pm 0.032$.
The average values: $\sin 2\beta_{J/\Psi\pi^0} = 0.69 \pm 0.25$, and $\sin 2\beta_{D^+D^-} = 0.29 \pm 0.63$ and $\sin 2\beta_{D^{*+}D^{*-}} = 0.75 \pm 0.23$ are consistent within (sizeable) uncertainties.

# Measurments of $\sin 2\alpha$

The direct measurements of $\sin(2\alpha)$ are obtained through $b \to u\bar{u}d$ transitions.

$B^0$ charmless decays are theoretically less clean than used to extract $sin2\beta$, because they are mediated by $b \to d$ penguin and $b \to u\bar{u}d$ tree diagrams of different phases and the same order in $\lambda$. The additional relative phase $\Delta\alpha$ is extracted from the isospin analysis.

Constraints:

- From $B^0 \to \pi^+\pi^-$ $0^\circ < \alpha < 17^\circ$ or $73^\circ < \alpha < 180^\circ$.
- From $\rho^+\pi^-$, $\rho^-\pi^+$ and $\rho^0\pi^0$ $\alpha = (99^{+13}_{-8})^\circ$
- From $B \to \rho\pi$ decays $\alpha = (99^{+13}_{-8})^\circ$

# Present status of $\gamma$

Techniques used for extraction of $\gamma$

- $B^{\pm} \to DK^{\pm}$ decays
  The golden method to measure the angle $\gamma$ is from the interference of $B^{+} \to \bar{D}^{0}K^{+}$ and $B^{+} \to D^{0}K^{+}$ transitions. The extracted constraints on $\gamma$ yield: $\gamma = (63^{+15}_{-12})^{\circ}$
- $B \to D^{(*)\pm}\pi^{\mp}$ decays
  The analysis of interference between CKM-favoured amplitude of $B^{0} \to D^{(*)-}\pi^{+}$ and doubly CKM-supressed amplitude of $B^{0} \to D^{(*)+}\pi^{-}$. The obtained value yields $\sin(2\beta + \gamma) = 0.8^{+0.18}_{-0.24}$.

# The unitarity tests

Constraints on CKM elements:

- The first row

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9992 \pm 0.0011. \qquad (1)$$

- The second row

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 0.968 \pm 0.181.$$

More precise determination is obtained from subtracting (1) from the LEP-2 measurment:

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1.003 \pm 0.027. \qquad (2)$$

- The first column

$$|V_{ud}|^2 + |V_{cd}|^2 + |V_{td}|^2 = 1.001 \pm 0.005. \qquad (3)$$

# The unitarity tests

Constraints on The UT angles:

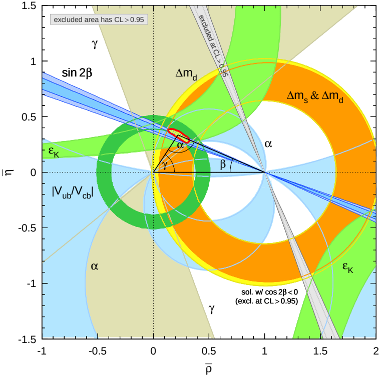
- The sum

$$\alpha + \beta + \gamma = (184^{+20}_{-15})^\circ. \quad (4)$$

The experimentally obtained values of the CKM parameters and phases are consistent with the three generations SM.

All combined results are used for the global fit of the Wolfenstein parameters

$$\lambda = 0.2272 \pm 0.0010,\ A = 0818^{+0.007}_{-0.017},\ \bar{\rho} = 0.221^{+0.064}_{-0.028},\ \bar{\eta} = 0.340^{+0.017}_{-0.045}$$




Constrains obtained from the global fit confirm the SM!