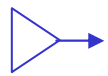
Heavy Quarks in PQCD and Global Analysis

- Heavy Quarks and Renormalization Schemes in PQCD:
  - Textbook, Variable-flavor # (zero-mass partons)
  - Fixed-flavor #



Generalized ($m_Q \neq 0$) Variable flavor Number Scheme
(adopted, in principle, by all)

- Implementations of the GVFNS–when do they matter and how?
  - Perturbative expansion is not the important thing in the threshold region; proper treatment of the physics and kinematics is.
  - Robust and simple implementations of the GVFNS: SACOT, ACOTχ
- Applications

  Charm production in DIS and precision global analysis: CTEQ6HQ

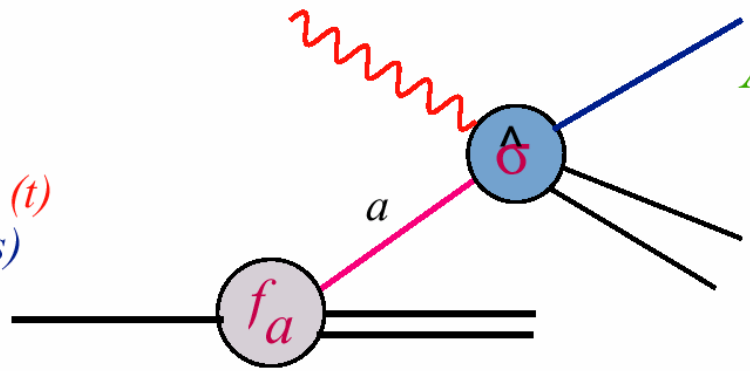
Conventional QCD Parton Model

Zero-mass Variable Flavor Number Scheme (ZVFNS)

Zero-mass Factorization Theorem

$$\sum_{a = g,\ u,\ d,\ s,\ c,\ b,\ (t)}$$

(all active flavors)



All parton masses $m_a = 0$

MSbar Subtr.

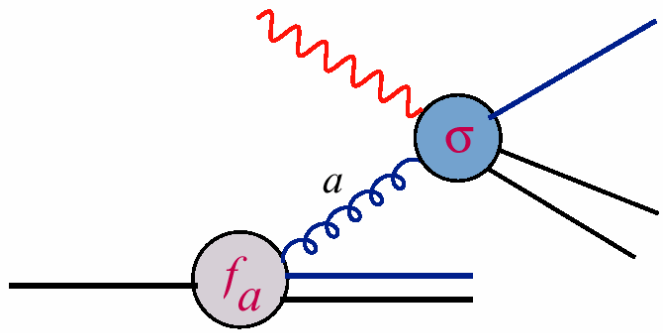
active flavor : all quarks with $m_H < Q$: $n_{fl}(Q)$

Textbooks, most PDFs, most NLO, NNLO hard-cross-section (Wilson coefficient) calculations, practically all MC programs, …

For Charm/bottom Production only: Fixed 3/4-flavor Scheme

Fixed-flavor-number (FFN) Scheme

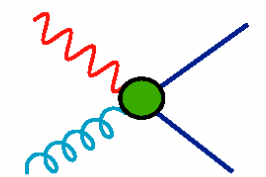
Σ
$a = g, u, d, s$
(light fl. only)

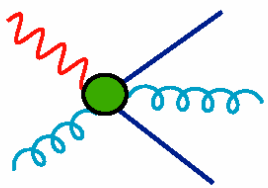


heavy quark:
m_H *"large"*
No col. Subtr.

number of quark flavor n_{fl} *=3 (for c)* fixed, indep. of Q

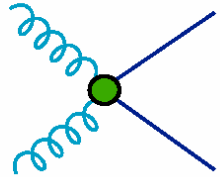
Lepto-production | **hadro-production**

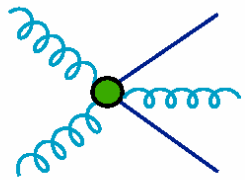




"Heavy flavor creation" (HC)

"Gluon-fusion process"





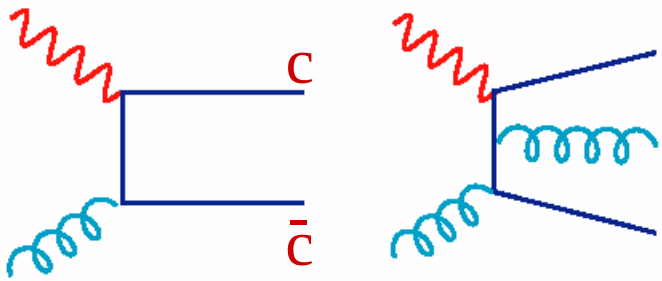
NLO calculation:

Laenen, Smith,
van Neerven ...et.al

Nason, Dawson, Ellis
Mangano, Ridolfi, Fixione .

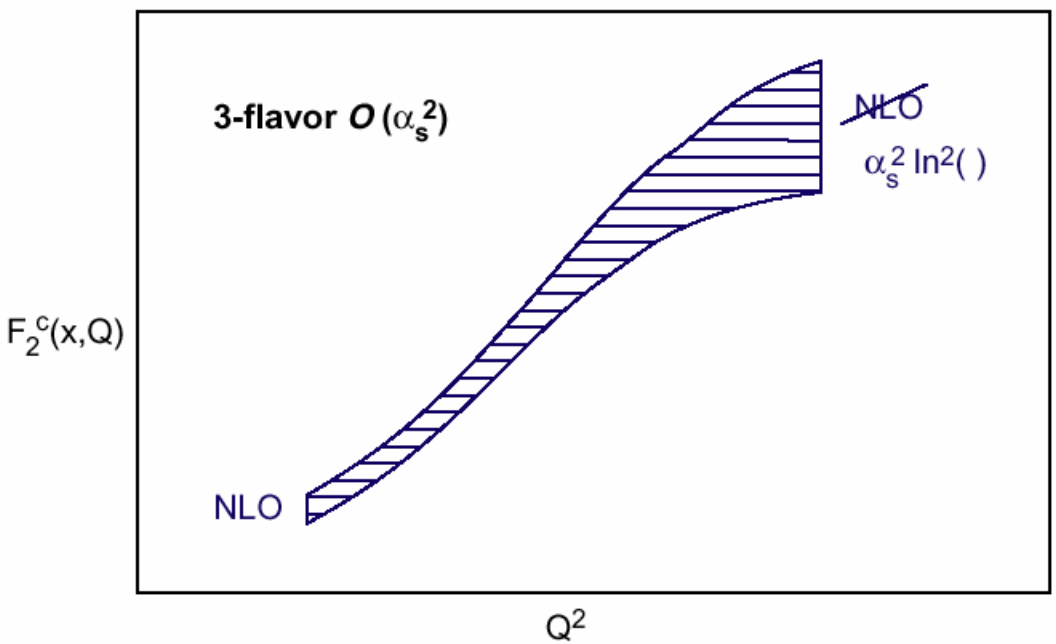
3-flavor order α_S^2 calculation

(no charm parton at any energy scale)



+ ....

- Implicitly assumes $m_c^2 \geq Q^2$;

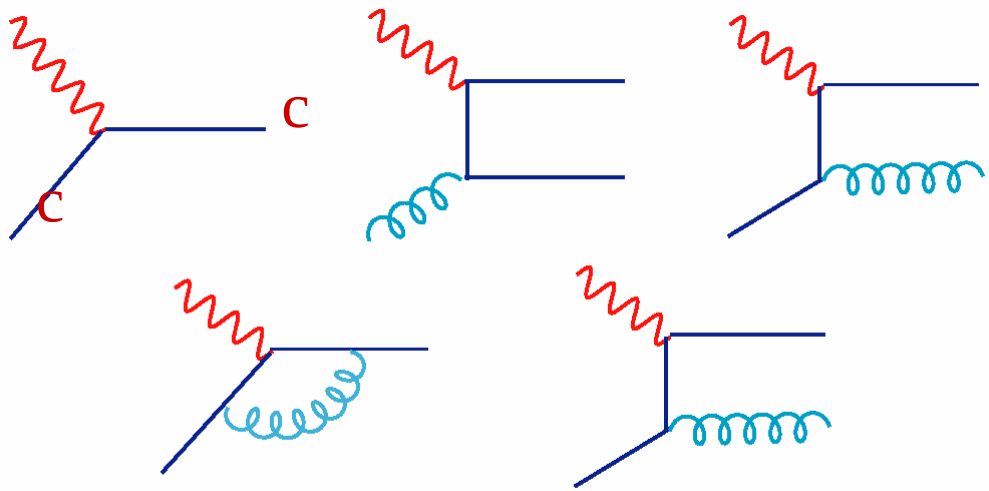


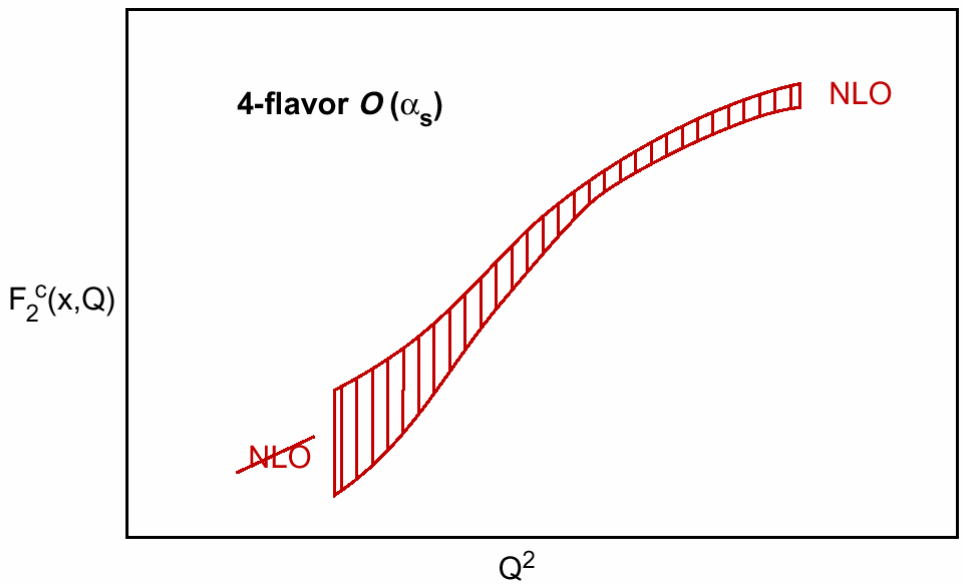
Expansion is in powers of

$$\alpha_s ln(Q^2/m_c^2)$$

Becomes questionable when $Q^2 \gg m_c^2$; and "NLO" loses its meaning!

Return to VFNS: Charm Production in the 4-flavor Scheme





Implicitly assumes that $M_C << Q$

This is a good approx. at high energies: the leading logs in $log^n(Q/M_C)$ are resummed to all orders.

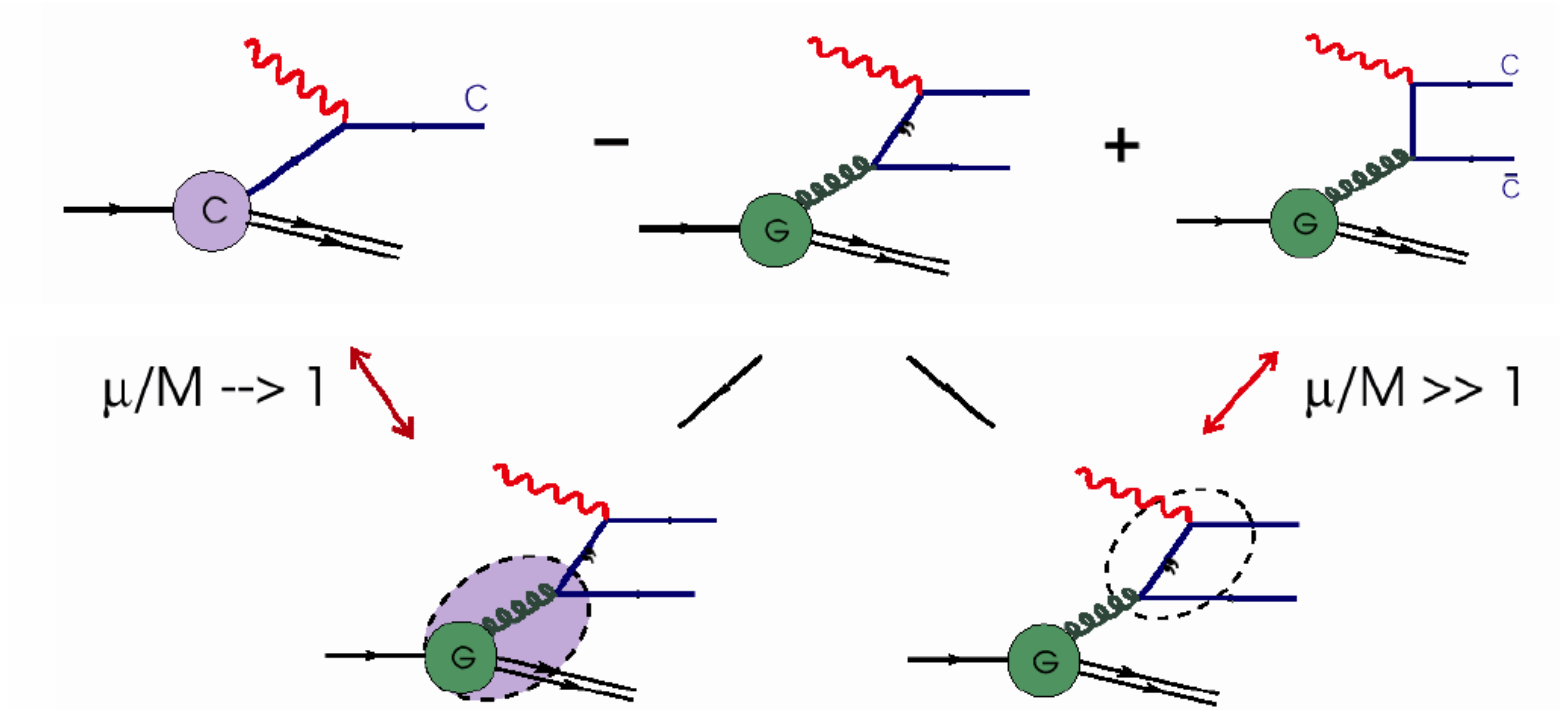
Near threshold, $M_C \sim Q$, this is not expected to be a natural picture.

Can it be made to work better than the expection?

Yes! for good (simple) physical reasons....

Good Physical Reason #1 (profound):
Collins '97: m=0 is not necessary for the parton picture and the factorization thm–The VFNS is fully compatible with $M_{c,b} \neq 0$

Basic Ideas (ACOT94)



This basic idea has been adopted by all recent works on heavy quark prod. (Cacciari etal, Robert-Thorne, Frixione etal, Smith, Van Neerven etal., Kniehl, Kramer, Spiesberger, …)

Good Physical Reason #2 ("trivial" but important)

Perturbative Expansion is not Everything–especially near the threshold region where *kinematic effects can be dominant*.

- Failure to enforce kinematic requirements in a natural way leads to instabilities in the calculation and artificial behaviors of physical predictions in the threshold region.

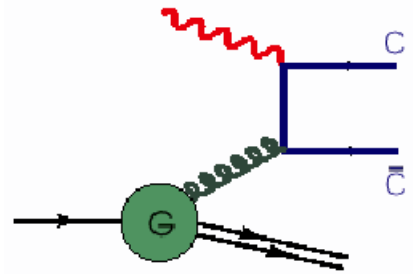
  (almost all previous applications, including ACOT)

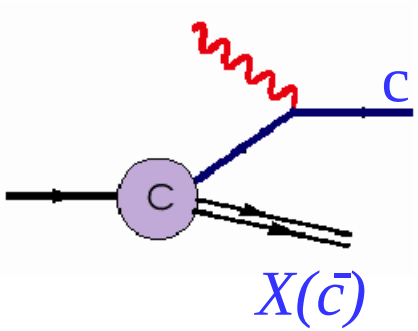
- Proper attention to kinematic constraints in the implementation of the PQCD expansion leads to radiative stability (small higher-order corrections, insensitivity to scheme and scale choices, hence robust physical predictions from the threshold to high energy scales.

- Can potentially restore order to the variety of existing (confusing, and overly elaborate) "schemes"

What Threshold Kinematic Effect?

Whereas the (4 flv. NLO) gluon-fusion process obey charm production kinematics,

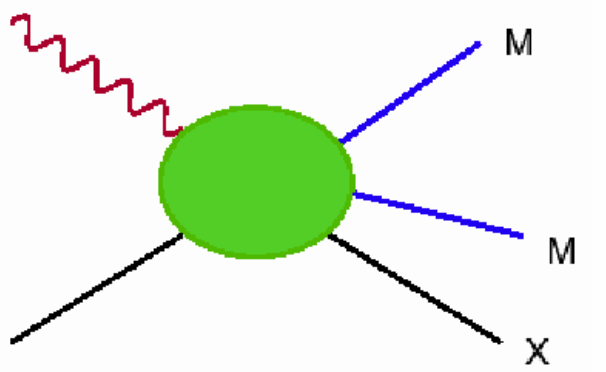




The (LO 4flv) charm-excitation process does not have the proper threshold behavior for producing charm, unless the missing anti-charm in the target fragment is taken into account.

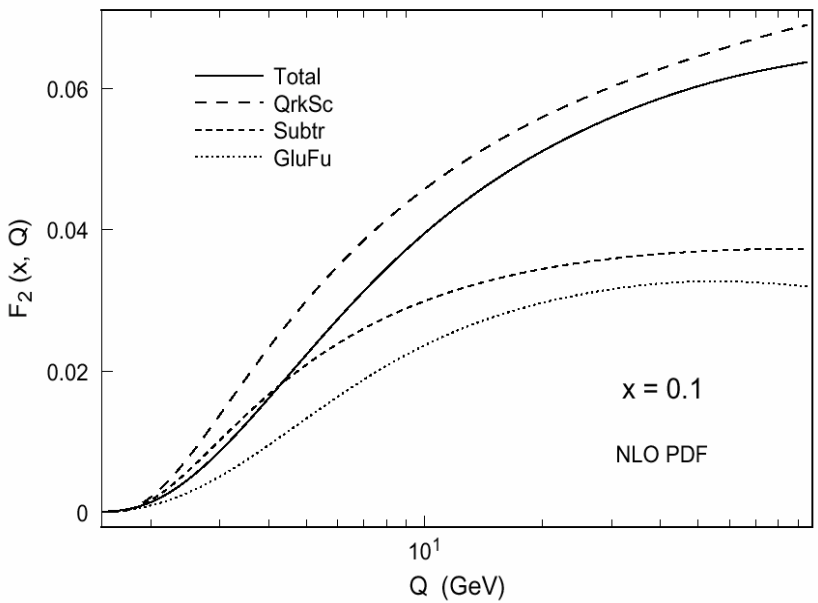
(This has not been done until recently)

- Well known: Near the threshold for producing at least 2 mass-M particles in the final state, the phase space is $\propto \Delta = 1 - 4M^2/W^2$.

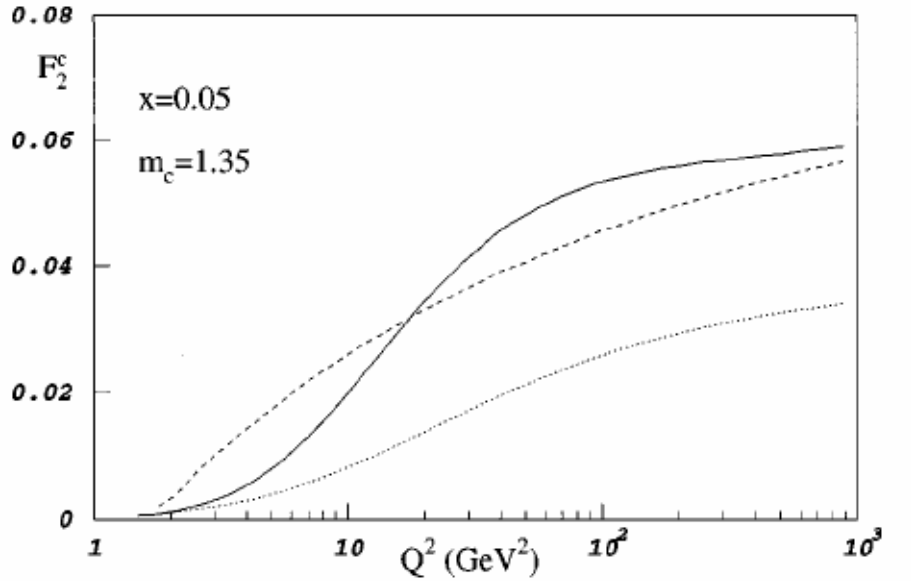


(General phase-space consider-ations)

What happens when the kinematic requirement is not naturally respected?



ACOT: PR ’94
(result quite sensitive to scale choice)



Thorne-Roberts: PL, ’97
(approach to asymptote seems artificial)

Although the two procedures are formally “equivalent” in the perturbative QCD sense, the threshold behavior of physical predictions *are highly dependent on the prescription* adopted in each implementation. Other groups use arbitrary functions to effect the transition from threshold to high energies.

Can one get robust predictions in the threshold region in the (generalized) VFNS?

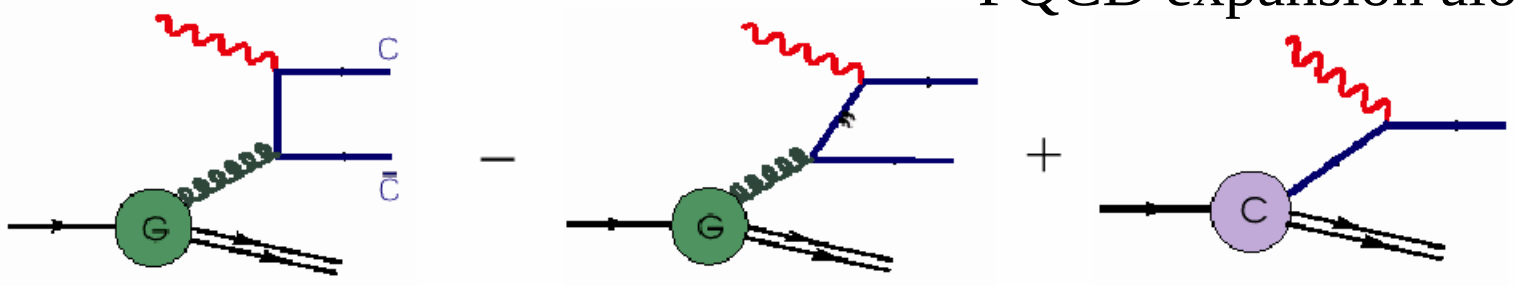
Yes, if the dominant kinematic threshold behavior is naturally built into the perturbative expansion:

Key: use the proper ***rescaling variable***.

Kretzer, Schmidt, T.

A kinematic problem requires a kinematic solution

PQCD expansion alone:



$$\alpha_s(\mu)\int_\chi^1 \frac{dz}{z} g(z,\mu)\omega^1(\frac{\chi}{z},\frac{M}{Q}) \quad \text{(unambiguous)}$$

$$\left[-\alpha_s(\mu)\ln(\frac{\mu}{M})\int_\zeta^1 \frac{dz}{z} g(z,\mu)P_{g\to c}(\frac{\zeta}{z}) + c(\zeta,\mu)\right] \otimes \omega^0(\frac{M}{Q})$$

(flexible)

- $\chi = x\,(1 + 4M^2/Q^2) = 1 - \Delta\,(1-x)$ is dictated by kinematics of gluon fusion – confirming the general phase-space consideration. Thus, the first term is OK as is

The inherent ***flexibility*** for the last two term makes it easy to satisfy the *all-important kinematic requirements*, as follows …

The rescaling variable solves the kinematic problem naturally for the "flavor excitation" and subtraction term:

(perturbatively)

- $\zeta(x, M/Q)$, being associated with the subtraction and resumed terms, is only subject to the requirements:

  (i) $\zeta \to x$ at high energies (just like χ);

  (ii) ζ is the same in both terms in [ ], ie. they are *matched*.

- The *choice of* ζ is dependent upon how one treats the kinematics of the last two terms, which is *not dictated by the factorization theorem*.

But the general phase-space argument suggests that *the most natural choice* is $\zeta = \chi = x\,(1 + 4M^2/Q^2)$!!

Then all terms have the expected threshold behavior:

ie. they $\to 0$ as $W \to 2M$ – as *dictated by physics*.

Conclusion: The M≠0 VFNS perturbative expansion can satisfies both PQCD and kinematic requirements with:

$$\alpha_s(\mu)\int_\chi^1 \frac{dz}{z} g(z,\mu)\omega^1(\frac{\chi}{z},\frac{M}{Q}) \quad +$$

(Kretzer, Schmidt, Tung)

$$\left[-\alpha_s(\mu)\ln(\frac{\mu}{M})\int_\zeta^1 \frac{dz}{z} g(z,\mu)P_{g\to c}(\frac{\zeta}{z}) + c(\zeta,\mu)\right] \otimes \omega^0(\frac{M}{Q}) \quad ;$$

provided we choose: $\zeta = \chi = x\,(1 + 4M^2/Q^2)$ (ACOTχ)

One further (independent) simplification:

$\omega^0(\frac{M}{Q})$ Can be replaced by $\omega^0(M=0)$ without loss of any accuracy and generality.

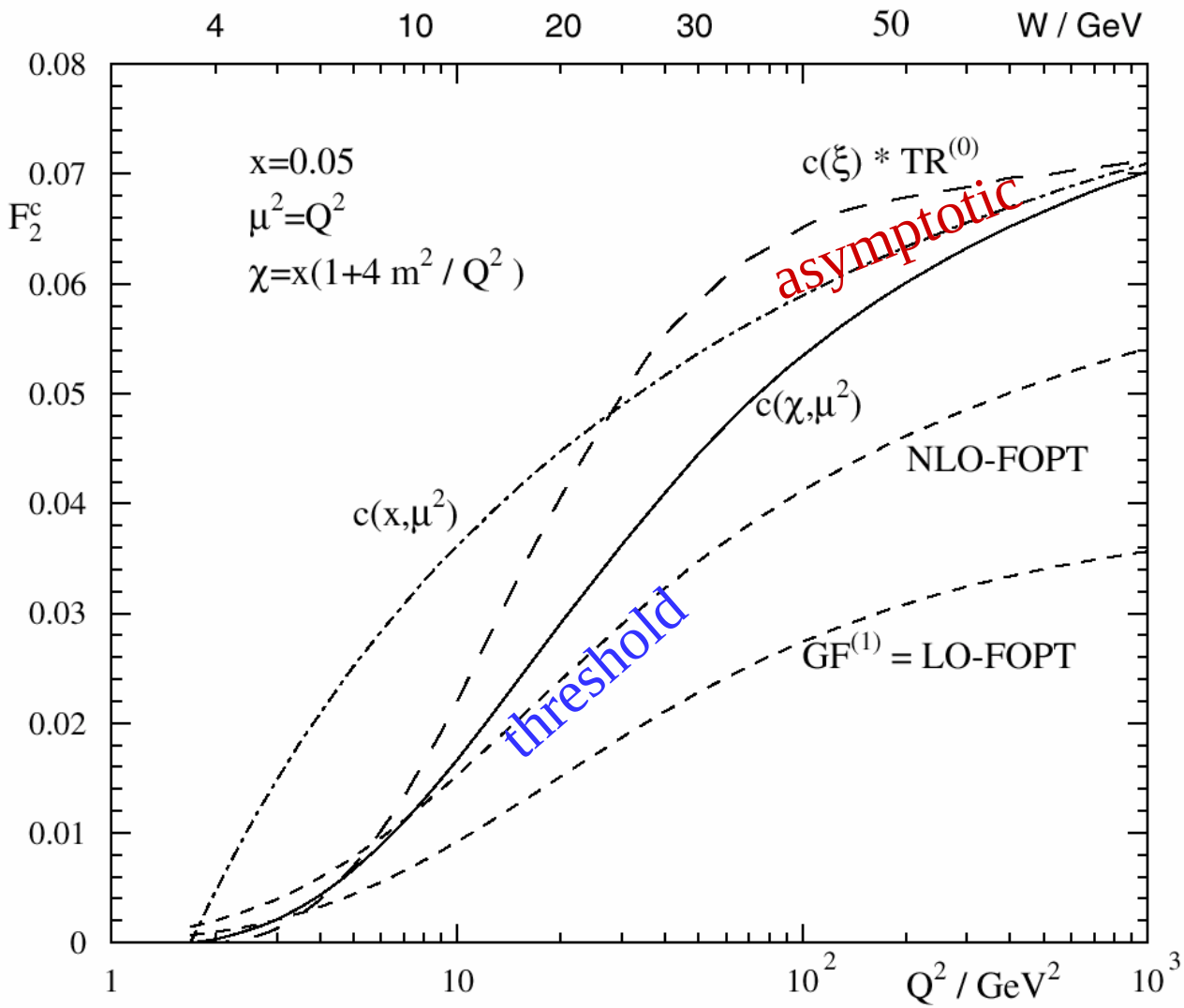
Collins '97; Kramer, Olness, Soper: SACOT

Adding to the above the small contribution from the *(M=0)* order α_s $\gamma^* c \to c\, g$ term, one gets a full NLO 4-flv formula that is very simple to compute, in addition to being numerically robust.

Results:

Smooth interpolation from correct threshold to asymptotic behavior even at LO.

Comparison of Q-dependence of RT and ACOT order α_s^0 results with Fixed-3-flavor order α_s^1 and α_s^2 results



Compare: Leading order ansatz for VFNS ($m \neq 0$) F_2^c

ACOTχ:

$c(\chi)$;

$\chi = x\,(1 + 4M^2/Q^2)$

Thorne-Roberts: (lepto-production)

Choose coefficient function to ensure continuity of the evolution of the physical F2 across the threshold:

$$\left.\frac{dF_2^c(x,Q^2,m_c^2)}{d\ln Q^2}\right|_{\substack{n_f flv \\ Q<M}} = \left.\frac{dF_2^c(x,Q^2,m_c^2)}{d\ln Q^2}\right|_{\substack{(n_f+1) flv \\ Q>M}}$$

which leads to the (non-local) prescription:

$$C_{2,HH}^{VFNS,0}(Q^2/m_H^2) \otimes p_{qg}^0 = \frac{\partial C_{2,g}^{FFNS,1}(x,Q^2/m_H^2)}{\partial \ln Q^2}$$

which results in the Leading-Order formula:

$$\begin{aligned} C_c^{(0)\,VF}(\epsilon) \otimes c_+(Q^2) &= -\int_x^{x_0} dz\, \frac{dC_g^{(1)\,FF}(z,\epsilon)}{d\ln Q^2} \left(\frac{x}{z}\right)^2 \frac{dc_+(x/z,Q^2)}{d(x/z)} \\ &+ 3\int_x^{x_0} dz\, \frac{dC_g^{(1)\,FF}(z,\epsilon)}{d\ln Q^2}\, \frac{x}{z}\, c_+(x/z,Q^2) \\ &- 2\int_x^{x_0} dz\, \frac{dC_g^{(1)\,FF}(z,\epsilon)}{d\ln Q^2} \int_{x/z}^1 dz'\, r(z')\, \frac{x}{zz'}\, c_+(x/zz',Q^2) \end{aligned}$$

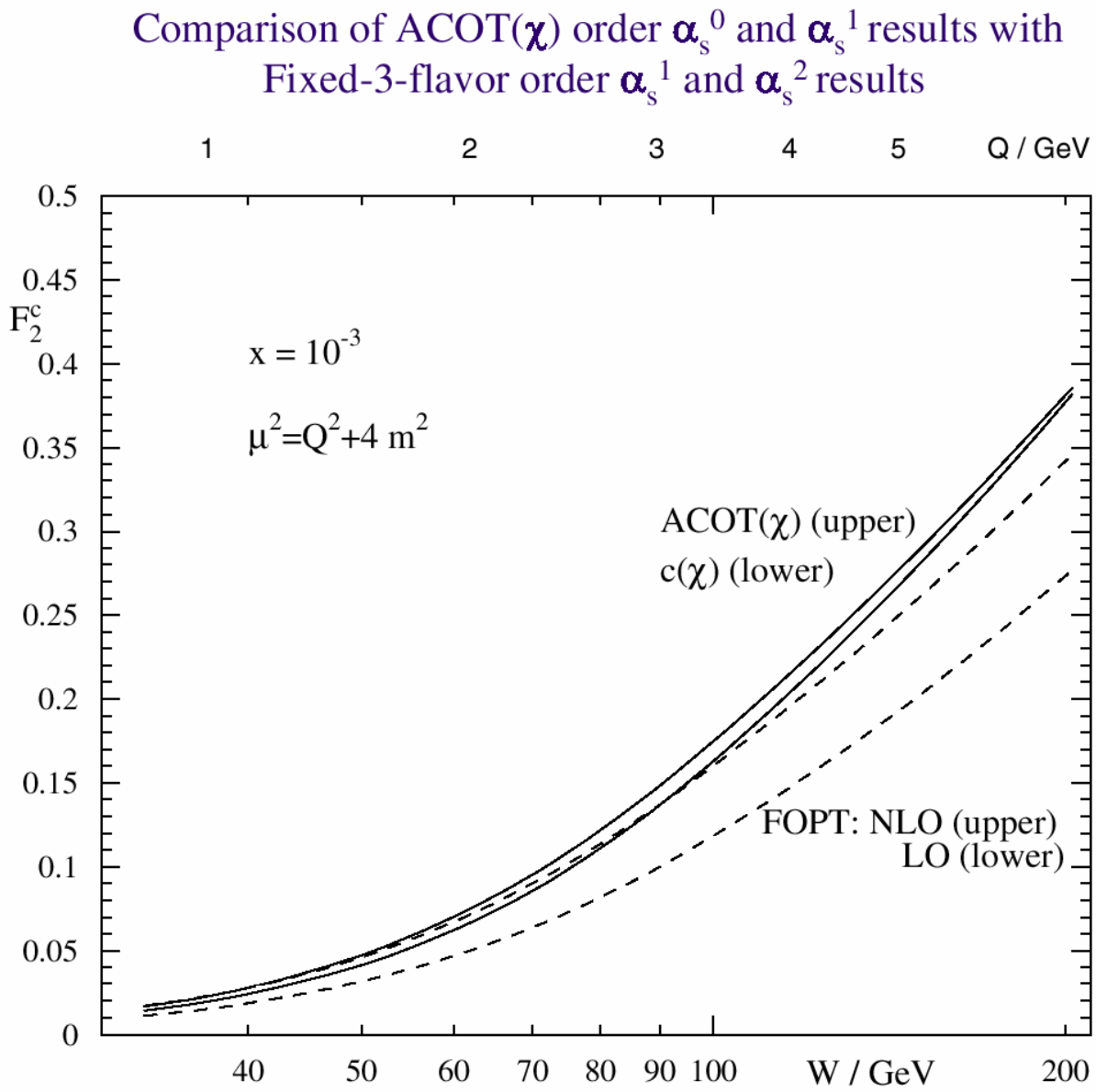
with $x_0 = (1+4\epsilon)^{-1}$ and $r(z)$ is given by

$$r(z) = z^{\frac{1}{2}}\left[\cos\left(\frac{\sqrt{7}}{2}\ln\frac{1}{z}\right) + \frac{3}{\sqrt{7}}\sin\left(\frac{\sqrt{7}}{2}\ln\frac{1}{z}\right)\right].$$

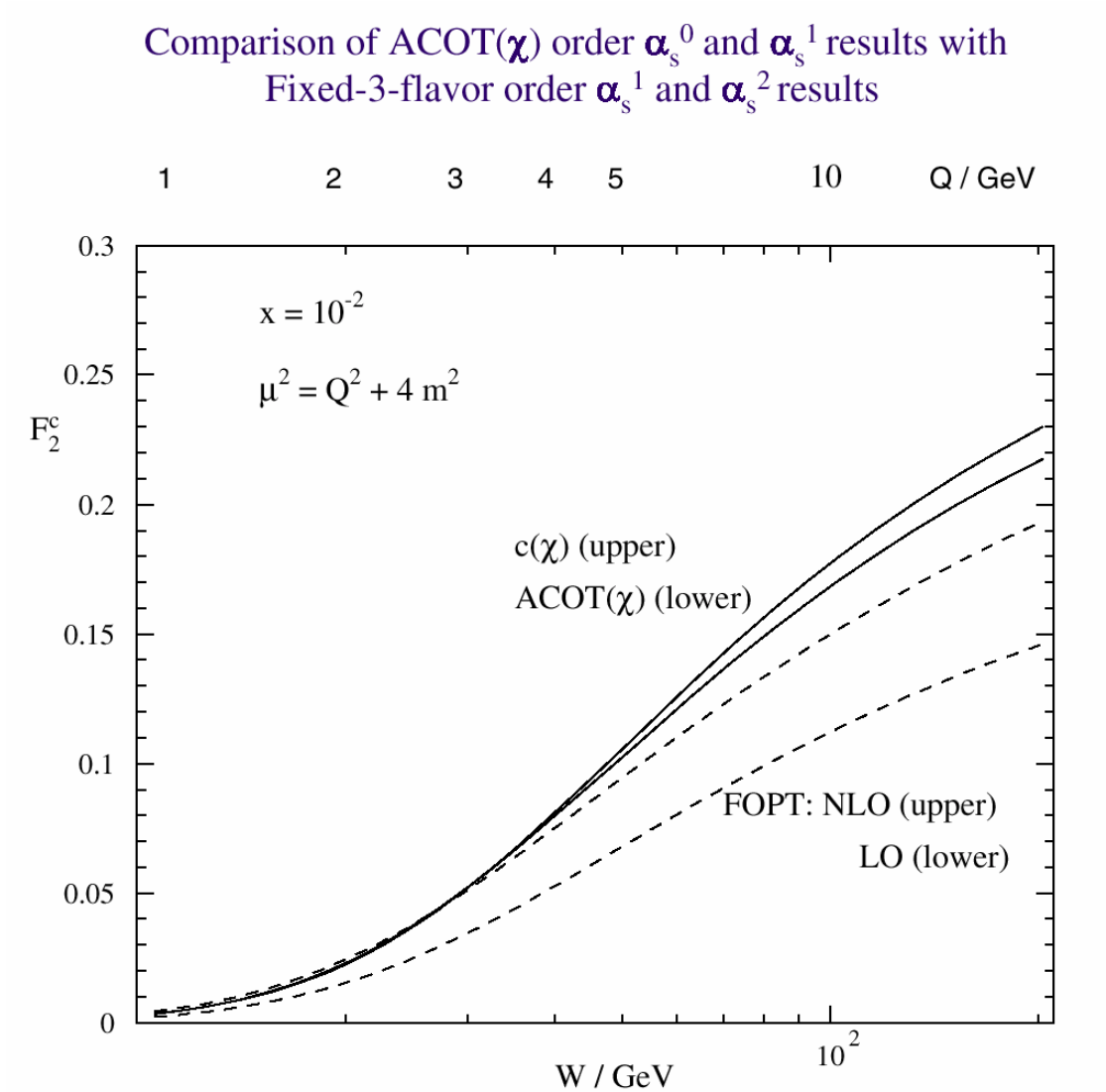
Radiative Stability: very small NLO correction:

(contrast with fixed-order expansion.)

The results are hence, not surprisingly, completely stable wrt the choice of scale μ.



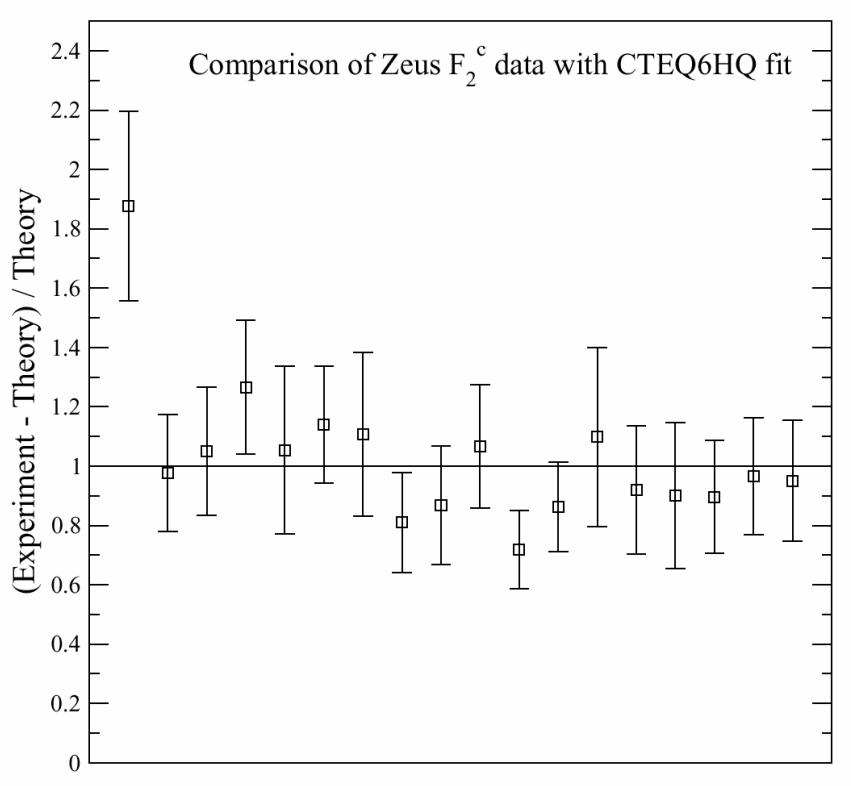
Another x

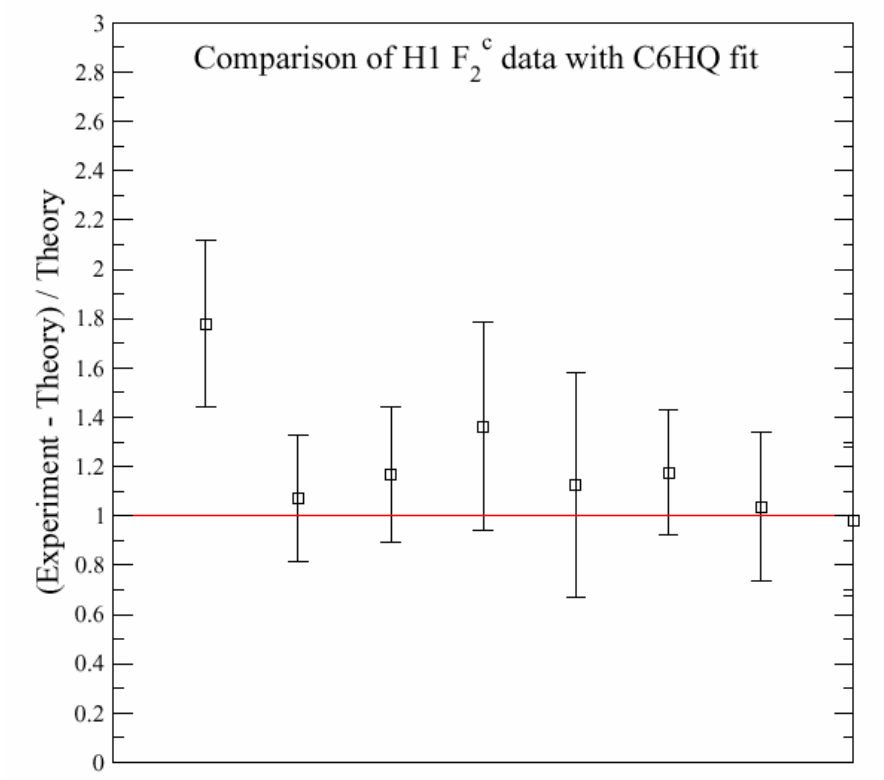

Comparison of ACOT(χ) order α_s^0 and α_s^1 results with
Fixed-3-flavor order α_s^1 and α_s^2 results
1
2
3
4
5
10
Q / GeV
0.3
0.25
0.2
0.15
0.1
0.05
0
F_2^c
$x = 10^{-2}$
$\mu^2 = Q^2 + 4\ m^2$
c(χ) (upper)
ACOT(χ) (lower)
FOPT: NLO (upper)
LO (lower)
10^2
W / GeV

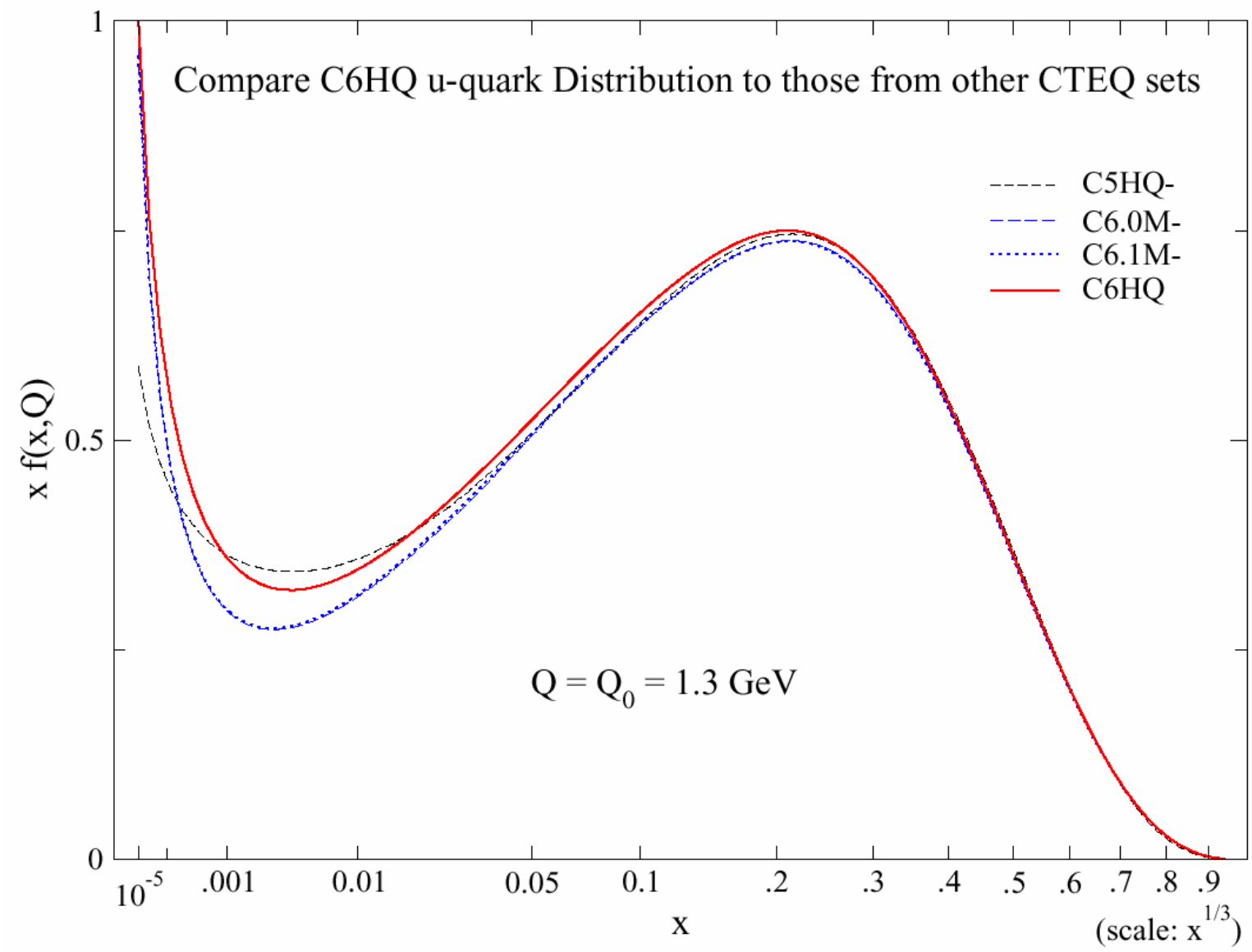
Application: CTEQ6HQ
Global Analysis using the generalized VFNS (m≠0)

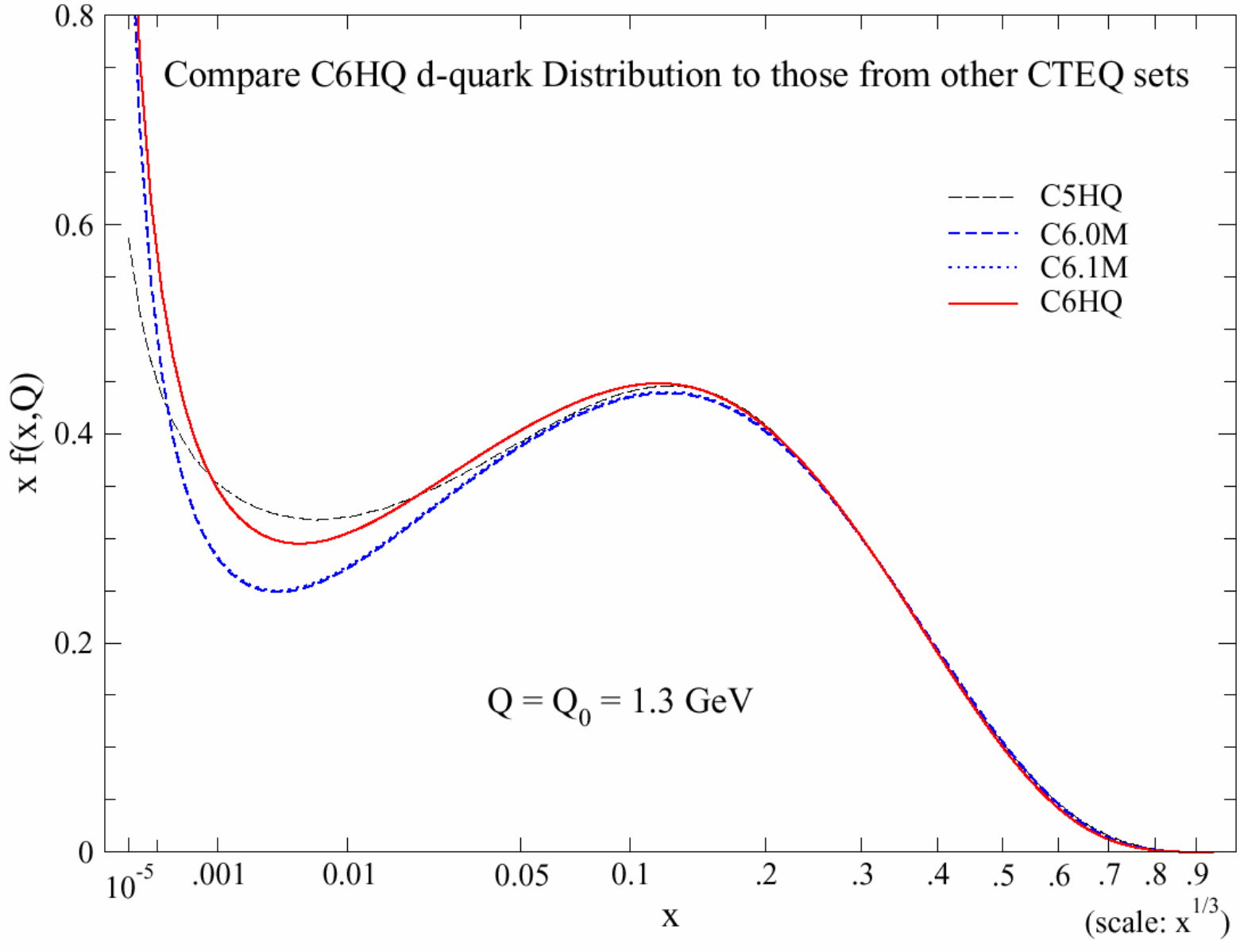
- Use the same inputs as the standard CTEQ6 analysis, with the exception of the GVFNS (vs. ZVFNS);
- Obtain slightly better fit than CTEQ6M, but the difference is not significant;
- These new parton distributions are useful for precision calculations which are sensitive to the charm quarks;
- For general applications with zero-mass hard matrix elements, CTEQ6M is still the more appropriate PDFs to use.

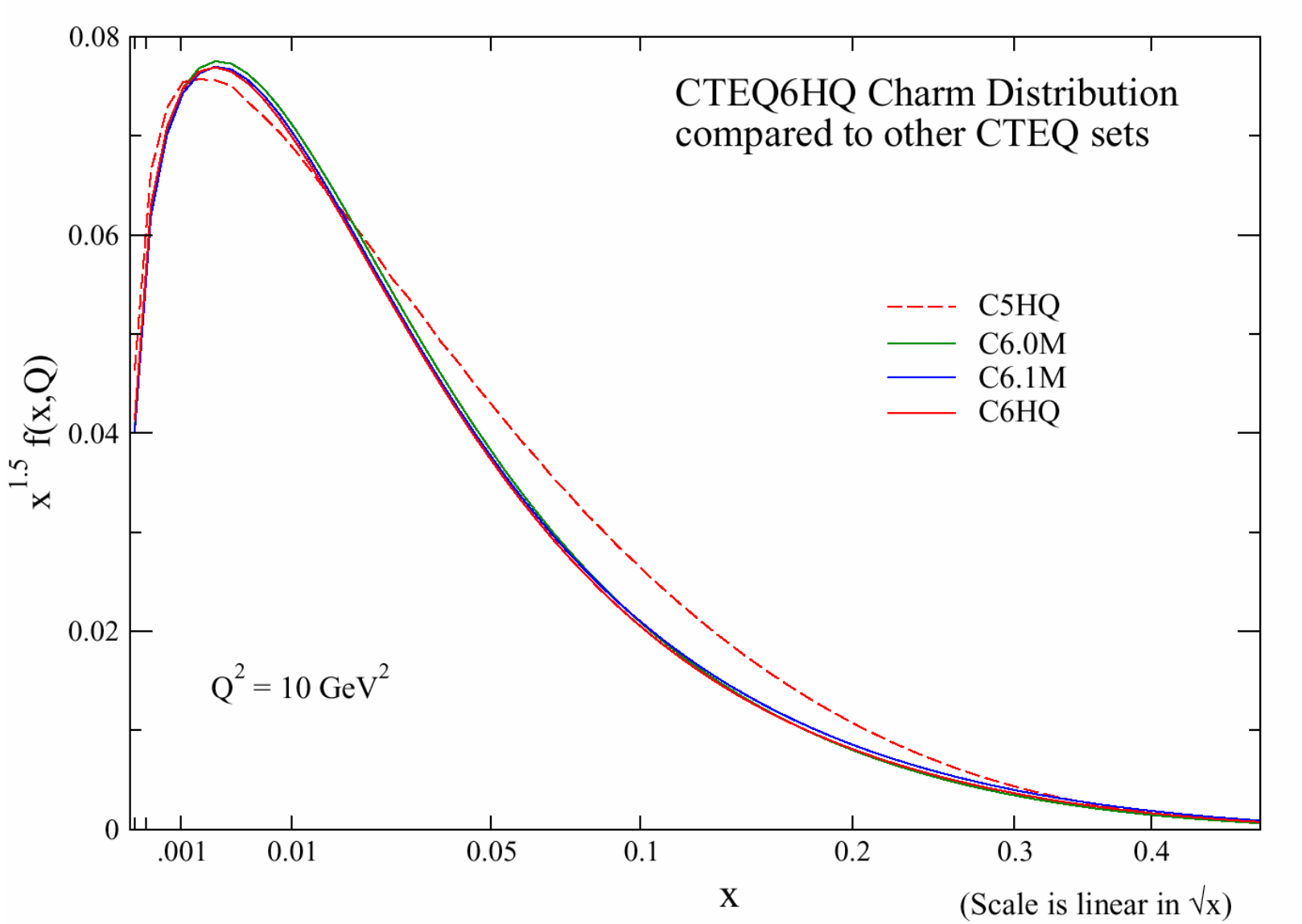
Comparison of CTEQ6HQ fit with charm production data

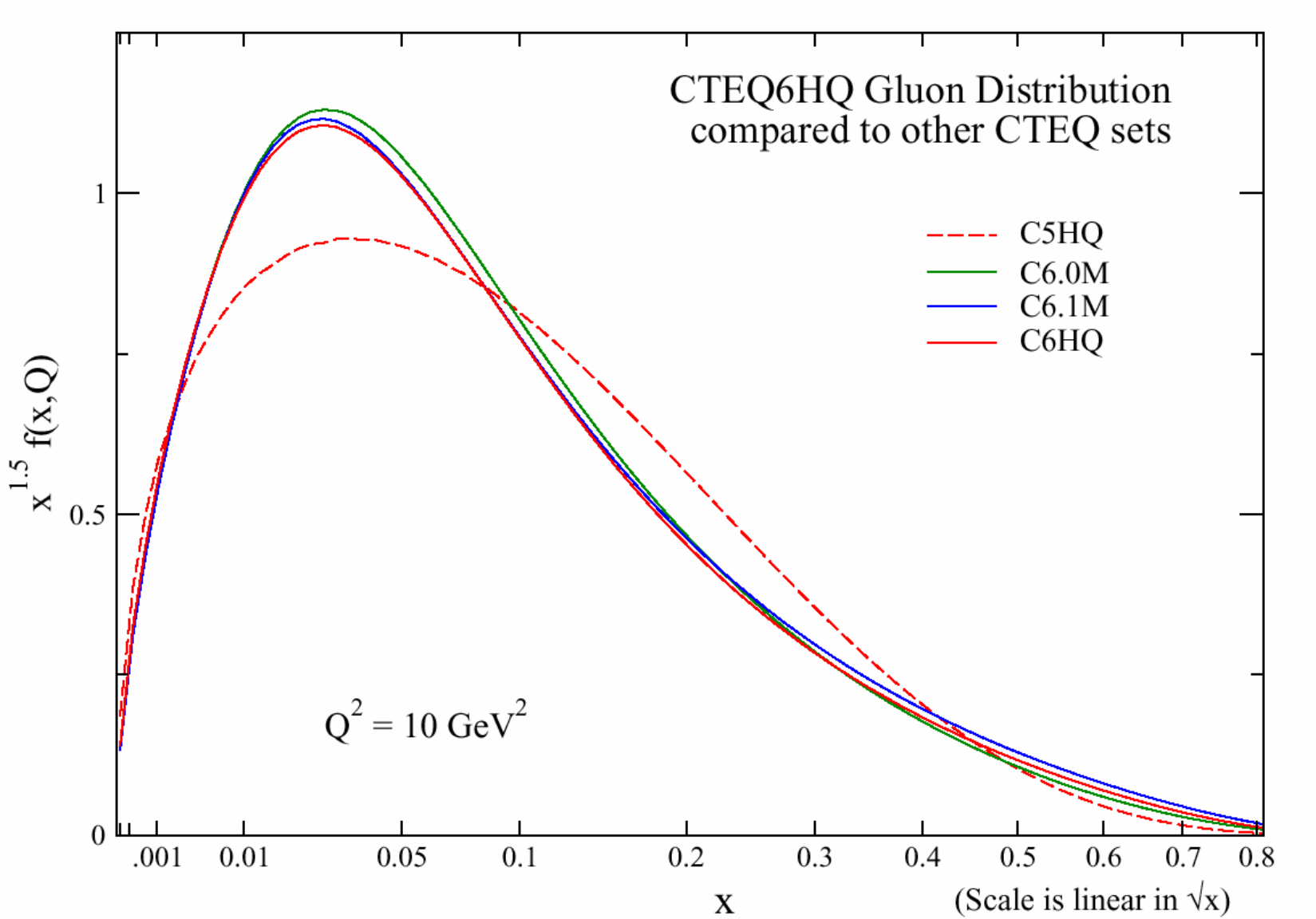




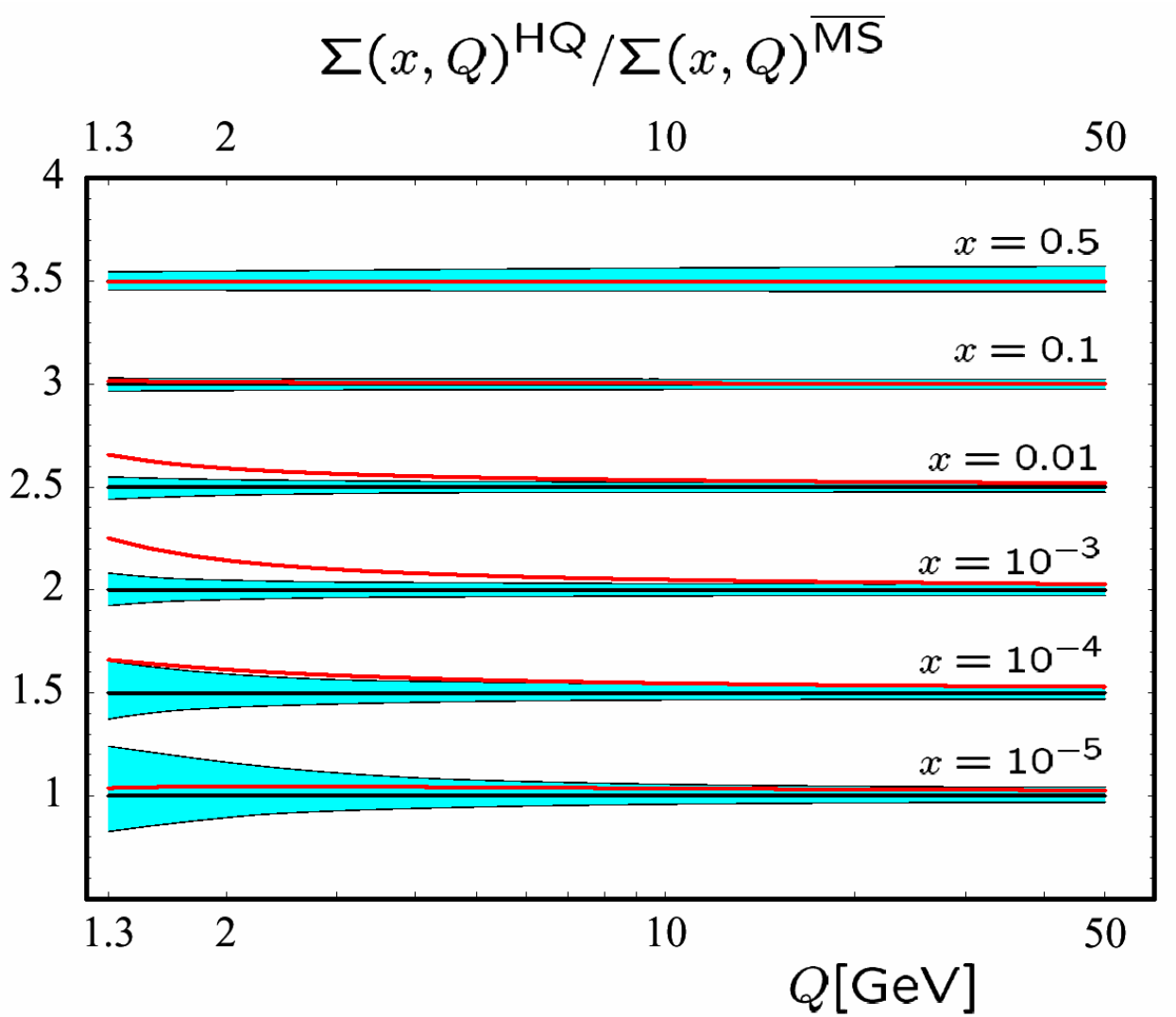

Compare C6HQ u-quark Distribution to those from other CTEQ sets
C5HQ-
C6.0M-
C6.1M-
C6HQ
x f(x,Q)
1
0.5
0
Q = Q0 = 1.3 GeV
10-5
.001
0.01
0.05
0.1
.2
.3
.4
.5
.6
.7
.8
.9
x
(scale: x1/3)


Compare C6HQ d-quark Distribution to those from other CTEQ sets
C5HQ
C6.0M
C6.1M
C6HQ
x f(x,Q)
Q = Q0 = 1.3 GeV
x
(scale: x1/3)
0.8
0.6
0.4
0.2
0
10-5
.001
0.01
0.05
0.1
.2
.3
.4
.5
.6
.7
.8
.9

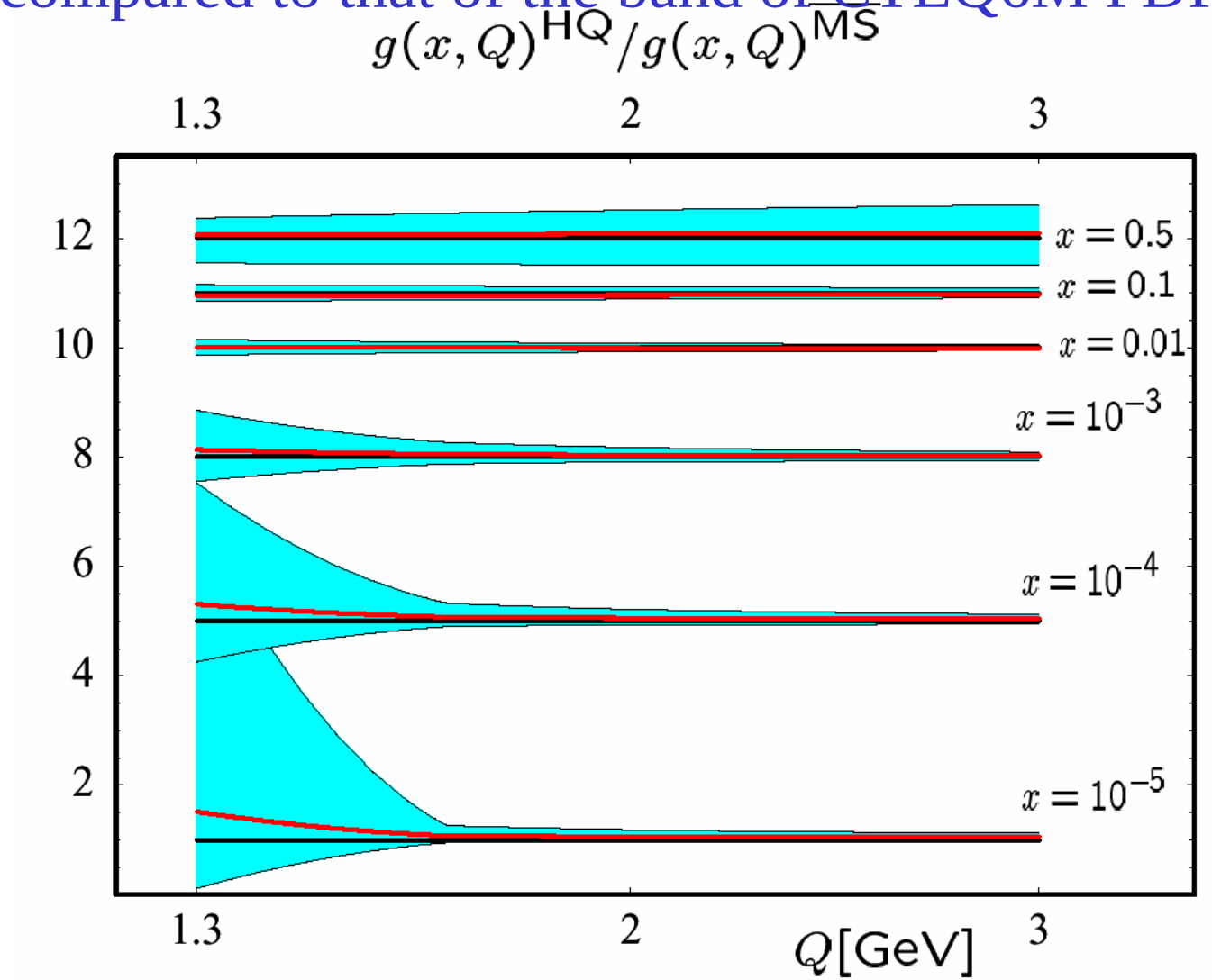

CTEQ6HQ Charm Distribution
compared to other CTEQ sets
C5HQ
C6.0M
C6.1M
C6HQ
$Q^2 = 10\ GeV^2$
$x^{1.5}\ f(x,Q)$
0.08
0.06
0.04
0.02
0
.001
0.01
0.05
0.1
0.2
0.3
0.4
x
(Scale is linear in √x)


CTEQ6HQ Gluon Distribution
compared to other CTEQ sets
C5HQ
C6.0M
C6.1M
C6HQ
$x^{1.5}$ f(x,Q)
1
0.5
0
Q^2 = 10 GeV^2
.001
0.01
0.05
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
x
(Scale is linear in √x)

Scale dependence of CTEQ6HQ singlet quark compared to that of the band of CTEQ6M PDFs



Scale dependence of CTEQ6HQ gluon distribution compared to that of the band of CTEQ6M PDFs



Wish Item

With a stable, robust, and simple procedure to incorporate heavy quark mass effects, hopefully a uniform “scheme” is adopted by all groups in future global analysis involving both lepto- and hadro-production processes. Then both experimentalists and theorists can concentrate on the real physics issues facing heavy quark physics, rather than be bewildered by the variety of different, and relatively arbitrary choices of ad hoc prescriptions in the many existing calculations and phenomenological analyses.