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FROM UNINTEGRATED GLUON DISTRIBUTIONS TO PARTICLE PRODUCTION IN HIGH ENERGY SCATTERING

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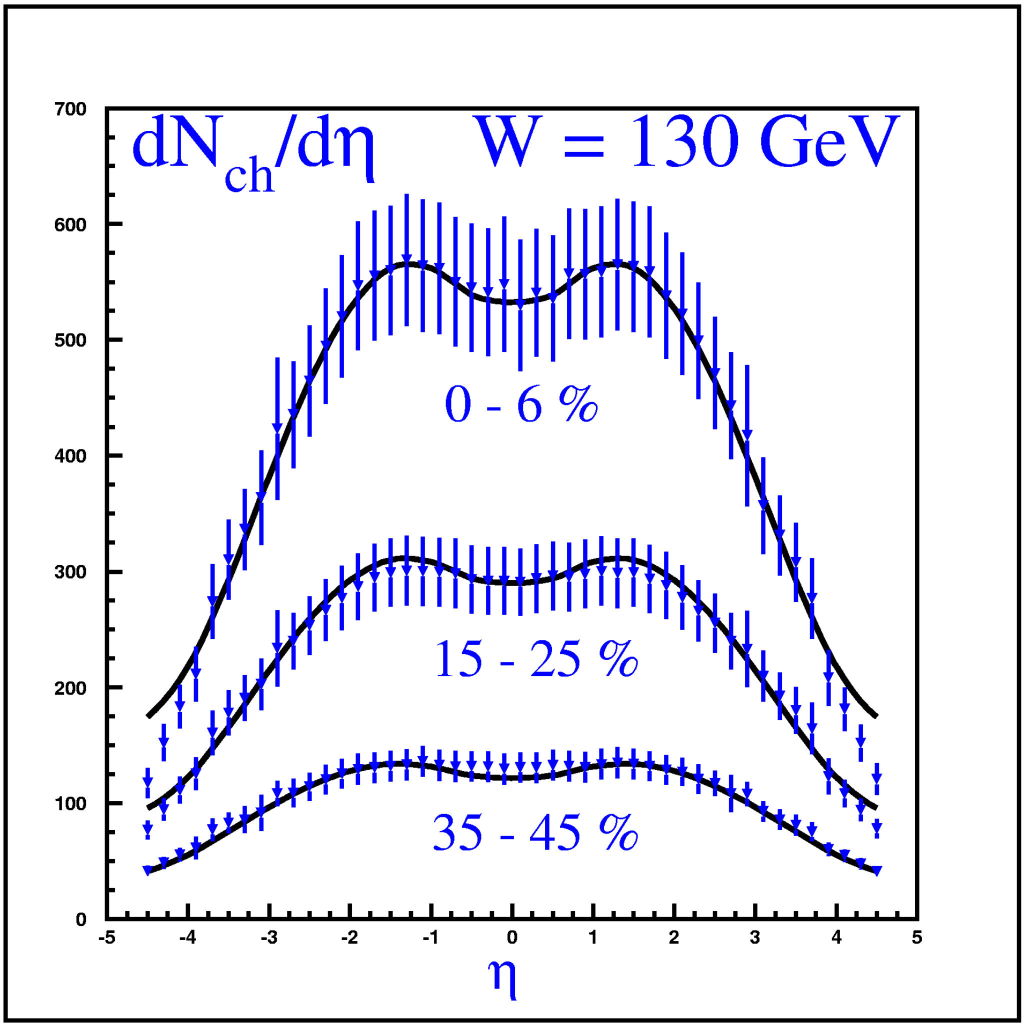
Motivation

Charged particle pseudorapitity spectra in relativistic heavy ion collisions at RHIC (Karzeev-Levin)

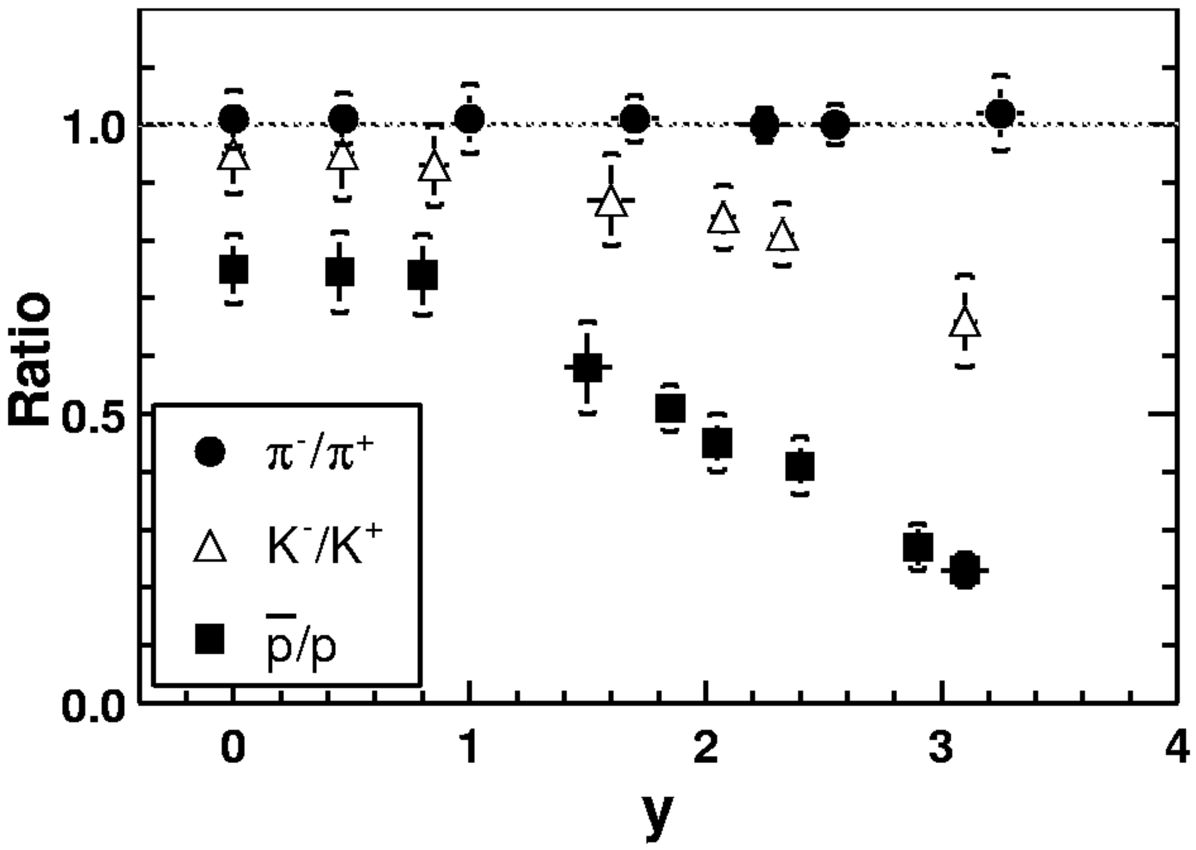


Gluon-gluon fusion ($\rightarrow$ unintegrated gluon distribution)

Motivation

Particle ratios at RHIC

(BRAHMS collaboration)



$K^-/K^+ \neq 1$!!!

and (pseudo)rapidity dependent

THESE RESULTS ARE INCONSISTENT !

Unint.gluon distr. from HERA

Go from A-A to N-N scattering first and check the KL approach there

IDEA:

(a) fix $\mathfrak{F}$ to the F_2 HERA data

(b) use $\mathfrak{F}$ for particle production in N-N collisions

In the dipole representation:

$$\begin{aligned} \sigma_{tot}^{\gamma^* N} &= \sum_q \int dz \int d^2\rho \, |\Psi_{\gamma^* \to q\bar{q}}(Q, z, \rho)|^2 \\ &\times \sigma_{(q\bar{q})N}(x, \rho) \, . \end{aligned} \tag{1}$$

$$\begin{aligned} &\sigma_{(q\bar{q})N}(x_g, \rho) \\ &= \frac{4\pi}{3} \int \frac{d^2\kappa_t}{\kappa_t^2} \left[1 - \exp(i\vec{\kappa}_t \vec{\rho})\right] \alpha_s \mathfrak{F}(x_g, \kappa_t^2) \\ &= \frac{4\pi^2}{3} \int \frac{d\kappa_t^2}{\kappa_t^2} \left[1 - J_0(\kappa_t \rho)\right] \alpha_s \mathfrak{F}(x_g, \kappa_t^2) \, . \end{aligned} \tag{2}$$

Unint. gluon distr. - BFKL

At very low x – BFKL equation?
After some simplifications:

$$-x \frac{\partial f(x, q_t^2)}{\partial x} = \frac{\alpha_s N_c}{\pi} q_t^2$$

$$\int_0^\infty \frac{dq_{1t}^2}{q_{1t}^2} \left[\frac{f(x, q_{1t}^2) - f(x, q_t^2)}{|q_t^2 - q_{1t}^2|} + \frac{f(x, q_t^2)}{\sqrt{q_t^4 + 4q_{1t}^4}} \right]$$

The homogeneous BFKL equation can be solved numerically
(Askew-Kwieciński-Martin-Sutton)

Here a simple parametrization (Eskola et al.)

$$f(x, \kappa_t^2) = \frac{C}{x^\lambda} \left(\frac{\kappa_t^2}{q_0^2} \right)^{1/2} \frac{\tilde{\phi}_0}{\sqrt{2\pi\lambda'' \ln(1/x)}}$$
$$\exp \left[- \frac{\ln^2(\kappa_t^2/\bar{q}^2)}{r 2\lambda'' \ln(1/x)} \right] \qquad (3)$$

In the above expression $\lambda = 4\bar{\alpha}_s \ln 2$, $\lambda'' = 28\,\bar{\alpha}_s \zeta(3)$, $\bar{\alpha}_s = 3\alpha_s/\pi, \zeta(3) = 1.202$. $\bar{q} = q_0 = 1$, $C\bar{\phi}_0 = 1.19$ and $r = 0.15$.

Unint. gluon distr. - GBW

$\mathcal{F}$ from the Golec-Biernat-Wüsthoff parametrization of σ_{dN} (parameters fitted to the HERA data)

$$\alpha_s \mathcal{F}(x,\kappa_t^2) = \frac{3\sigma_0}{4\pi^2} R_0^2(x) \kappa_t^2 \exp(-R_0^2(x)\kappa_t^2) \, , \tag{4}$$

where

$$R_0(x) = \frac{1}{GeV} \left(\frac{x}{x_0} \right)^{\lambda/2} . \tag{5}$$

From their fit to the data:

$\sigma_0 = 29.12$ mb,

$x_0 = 0.41 \cdot 10^{-4}$,

$\lambda = 0.277$.

Other parameters:

$m_0 = 0.14$ GeV (effective quark mass),

$\alpha_s = 0.2$ (to extract gluon distribution)

Karzeev-Levin gluon distr.

Let's take the most simplified functional form:

$$\mathcal{F}(x,\kappa^2) = \begin{cases} f_0 & \text{if } \kappa^2 < Q_s^2, \\ f_0 \cdot \frac{Q_s^2}{\kappa^2} & \text{if } \kappa^2 > Q_s^2. \end{cases} \tag{6}$$

Saturation momentum from Golec-Biernat-Wüsthoff model $Q_s^2(x) = 1\ \mathrm{GeV}^2 \cdot \left(\frac{x_0}{x}\right)^\lambda$.

Let's fit f_0 to the HERA data

($\rightarrow$ see next page)

Kimber-Martin-Ryskin approach

At very small x

$$\mathcal{F}(x,\kappa^2) = \frac{\partial}{\partial Q^2}\left[x g(x,Q^2)\right]|_{Q^2=\kappa^2} \; . \qquad (7)$$

Breaks at larger x when derivative negative
$\rightarrow$ Sudakov form factor

$$\mathcal{F}(x,\kappa^2,\mu^2) = \frac{\partial}{\partial Q^2}\left[T_g(Q^2,\mu^2) x g(x,Q^2)\right]|_{Q^2=\kappa^2} \; . \qquad (8)$$

Resumming virtual contributions to
DGLAP equation

$$f_a(x,\kappa^2,\mu^2) = T_a(\kappa^2,\mu^2)\cdot\frac{\alpha_s(\kappa^2)}{2\pi}$$
$$\sum_{a'}\int_x^{1-\delta} P_{aa'}(z)\left(\frac{x}{z}\right) a'\left(\frac{x}{z},\kappa^2\right) dz \; . \qquad (9)$$

Specializing to the gluon distribution

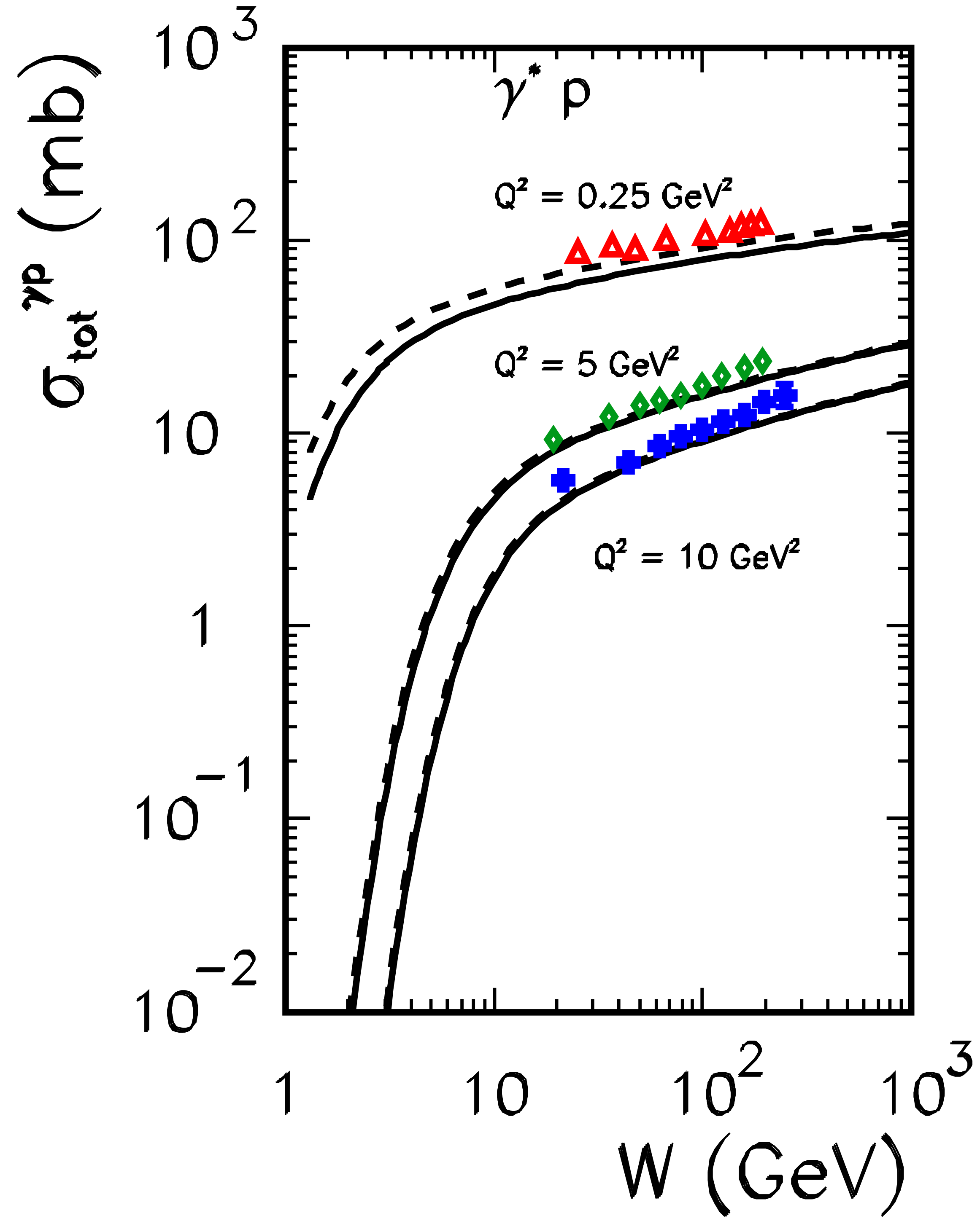
$$T_g(\kappa^2,\mu^2) = \exp\left(-\int_{\kappa^2}^{\mu^2} \frac{dp^2}{p^2}\frac{\alpha_s(p^2)}{2\pi} \int_0^{1-\delta} dz z \left[P_{gg}(z) + \sum_q P_{qg}(z) \right] \right. . \tag{10}$$

A dependence on a second scale μ^2 !
Production of particles sensitive to rather low κ $\rightarrow$ take GRV g(x)
At lower κ_t^2 an extrapolation is needed.
if $\kappa^2 < \kappa_{t0}^2 = 0.5\ \mathrm{GeV}^2$

$$\mathcal{F}(x,\kappa^2) = f(x,\kappa^2)/\kappa^2 = f(x,\kappa_0^2)/\kappa_0^2 \ . \tag{11}$$

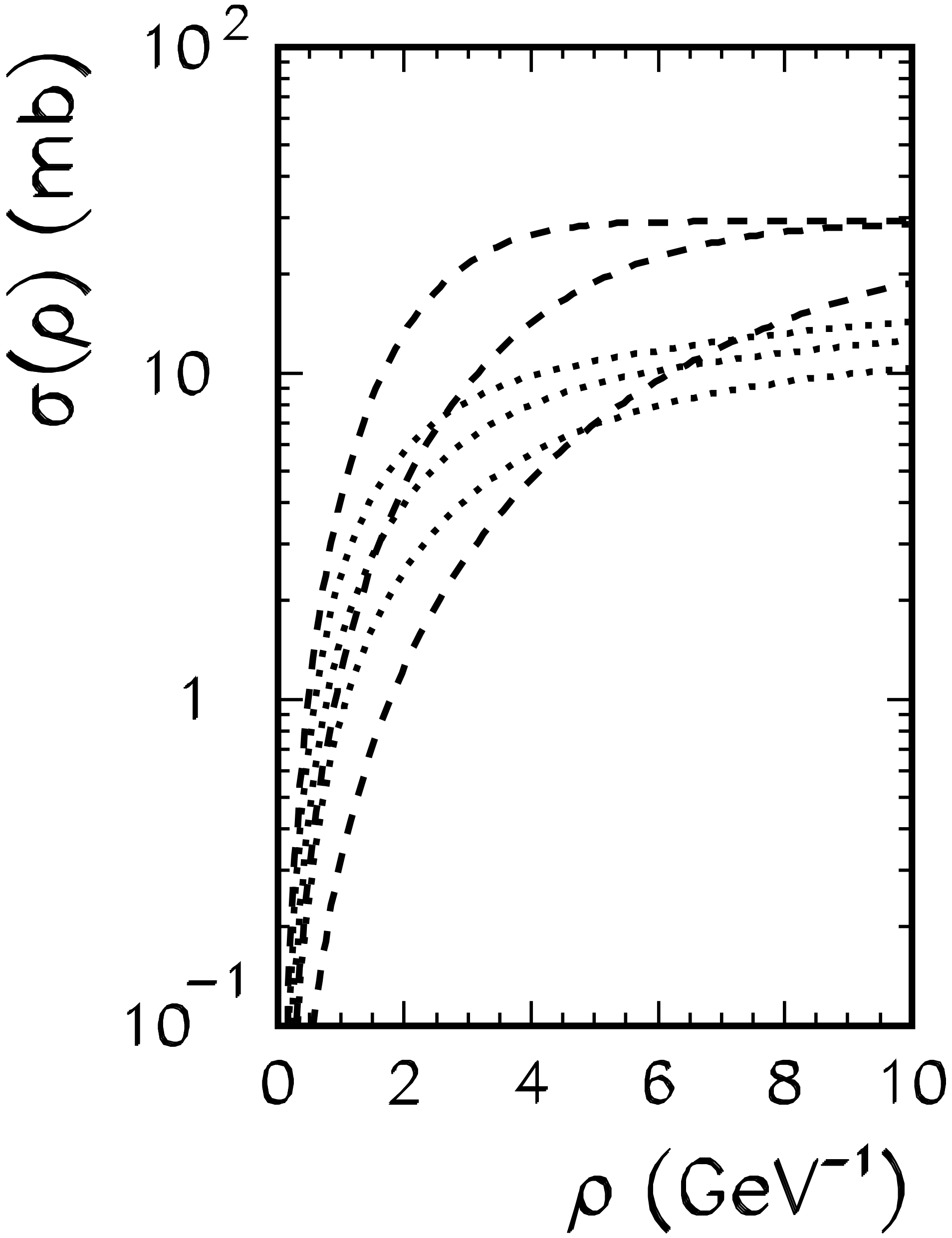
In the present analysis $\mu^2 = p_t^2$. In accord with the interpretation of T_g as a survival probability: $T_g = 1$ if $p_t < \kappa_1$ or $p_t < \kappa_2$)

$\sigma_{tot}^{\gamma^* p}$



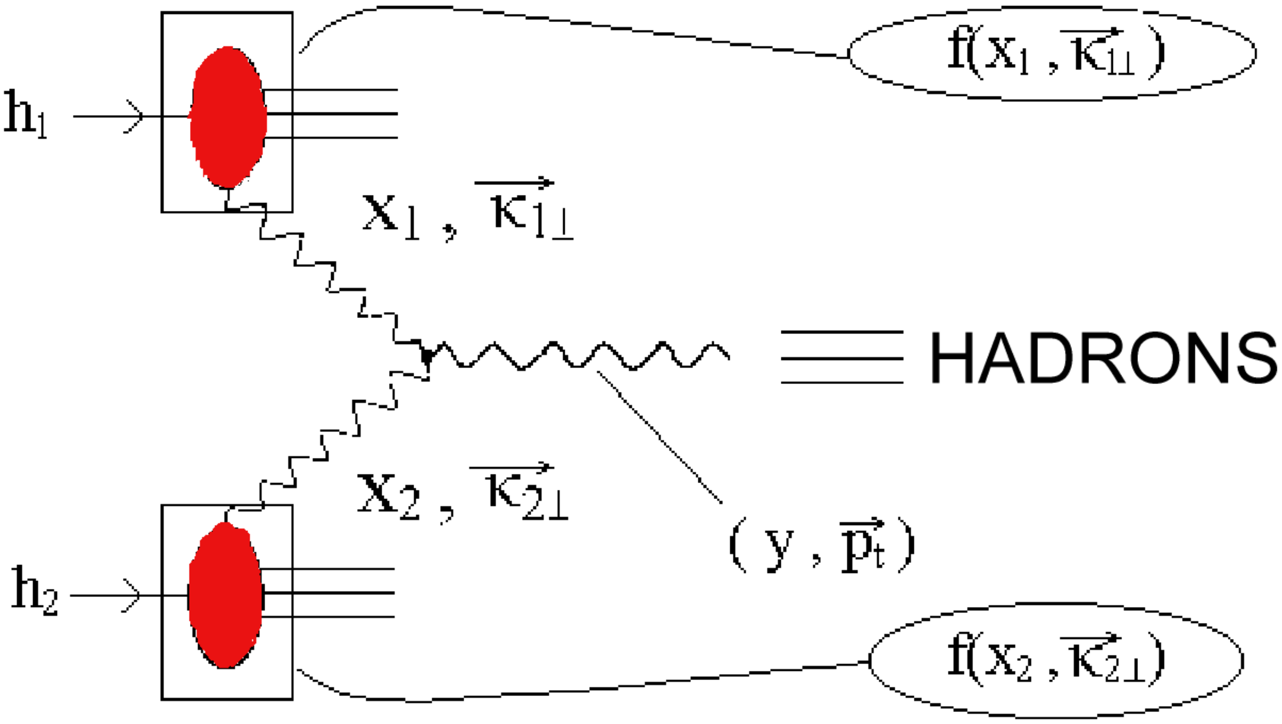
dipole-nucleon interaction

KL versus GBW; $x = 10^{-2}$, 10^{-3}, 10^{-4}



Inclusive gluon production

consider reaction: $h_1 + h_2 \to g$



$$\frac{d\sigma}{dy d^2 p_t} = \frac{16 N_c}{N_c^2 - 1} \frac{1}{p_t^2}$$
$$\int \alpha_s(\Omega^2) \mathcal{F}_1(x_1, \kappa_1^2) \mathcal{F}_2(x_2, \kappa_2^2)$$
$$\delta(\vec{\kappa}_1 + \vec{\kappa}_2 - \vec{p}_t)\ d^2\kappa_1 d^2\kappa_2\ . \qquad (12)$$

(Gribov-Levin-Ryskin, Gyulassy-McLerran)

$\mathcal{F}_1$, $\mathcal{F}_2$ - unintegr. gluon distr. in h_1, h_2,

$x_{1/2} = \frac{p_t}{\sqrt{s}} \cdot \exp(\pm y)$.

$\Omega^2 = \max(\kappa_1^2, \kappa_2^2, p_t^2)$.

Let introduce: $\vec{q}_t = \vec{\kappa}_1 - \vec{\kappa}_2$

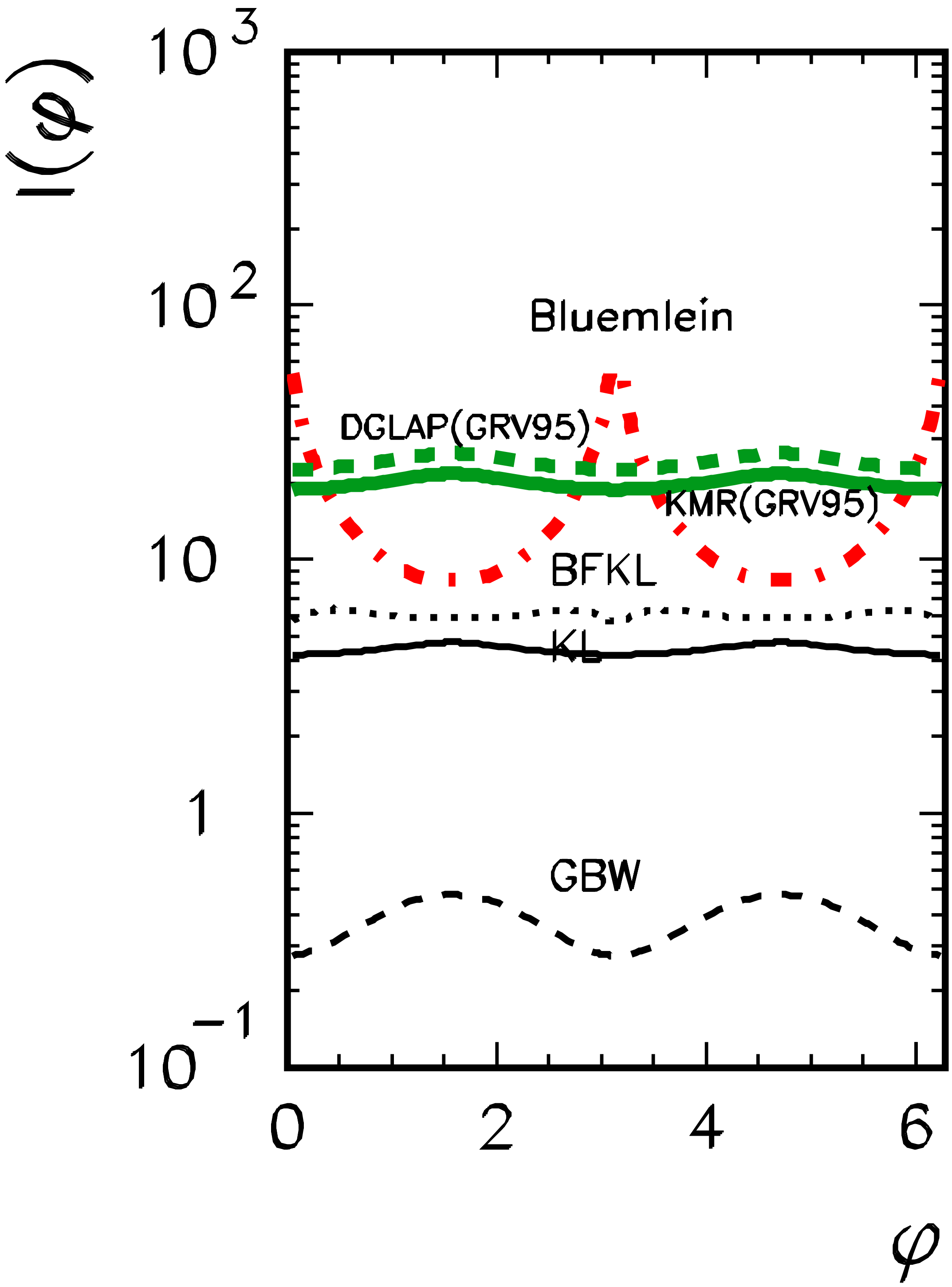
$$\frac{d\sigma}{dy d^2 p_t} = \frac{16 N_c}{N_c^2 - 1} \frac{1}{p_t^2} \frac{1}{4} \int \alpha_s(\Omega^2) \mathcal{F}_1\left(x_1, \left(\frac{\vec{p}_t + \vec{q}_t}{2}\right)\right) \mathcal{F}_2\left(x_2, \left(\frac{\vec{p}_t - \vec{q}_t}{2}\right)\right) d^2 q_t$$

$$\frac{d\sigma}{dy d^2 p_t} = \int I(\phi)\, d\phi\, , \qquad (13)$$

where

$$I(\phi) = \frac{4 N_c}{N_c^2 - 1} \frac{1}{p_t^2} \int \alpha_s(\Omega^2) \mathcal{F}_1\left(x_1, \kappa_1^2\right) \mathcal{F}_2\left(x_2, \kappa_2^2\right)\; q_t dq_t\ . \qquad (14)$$

azimuthal correlation



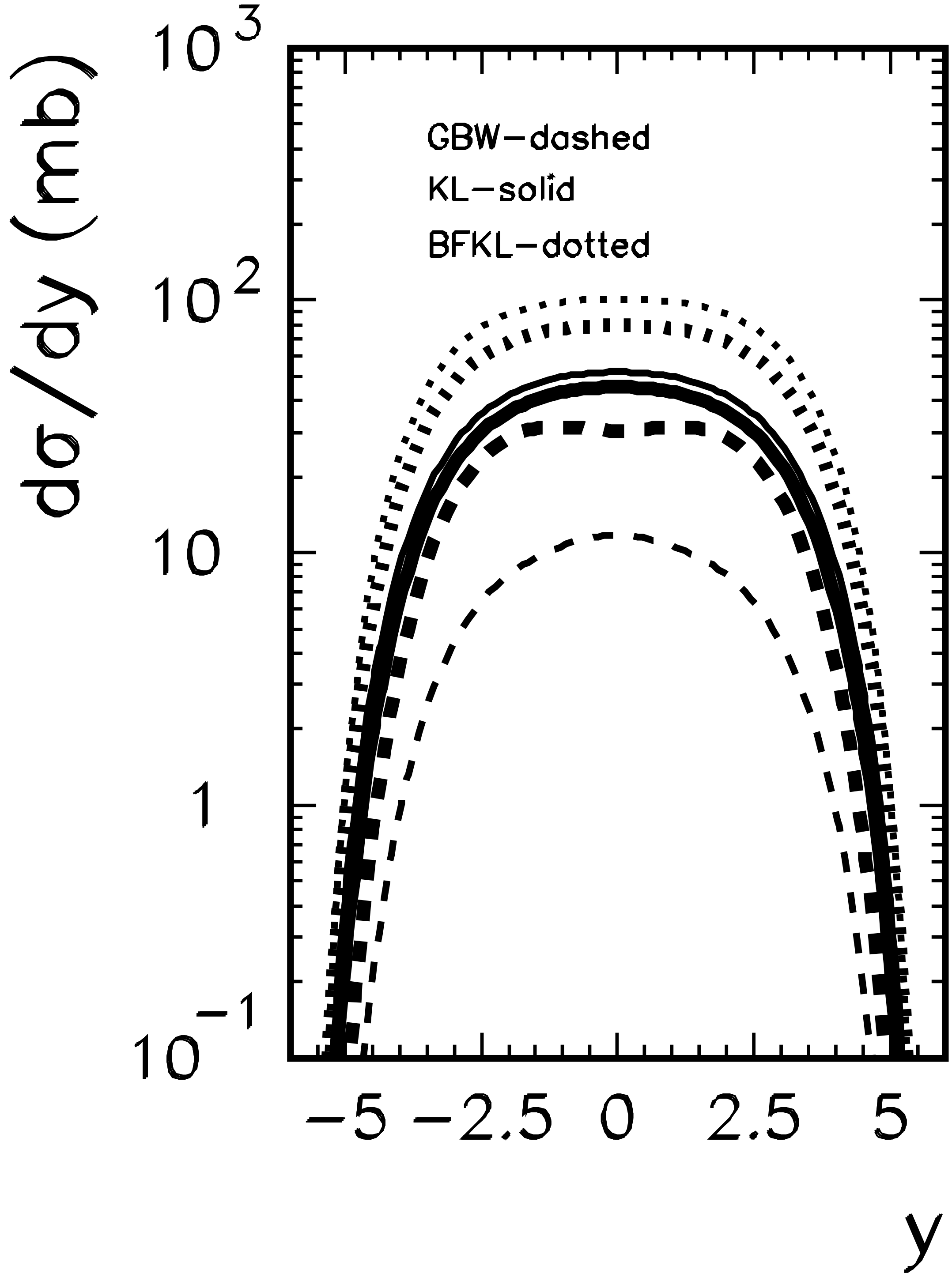
Approximate formula

It was suggested by Karzeev-Levin to use approximate formula

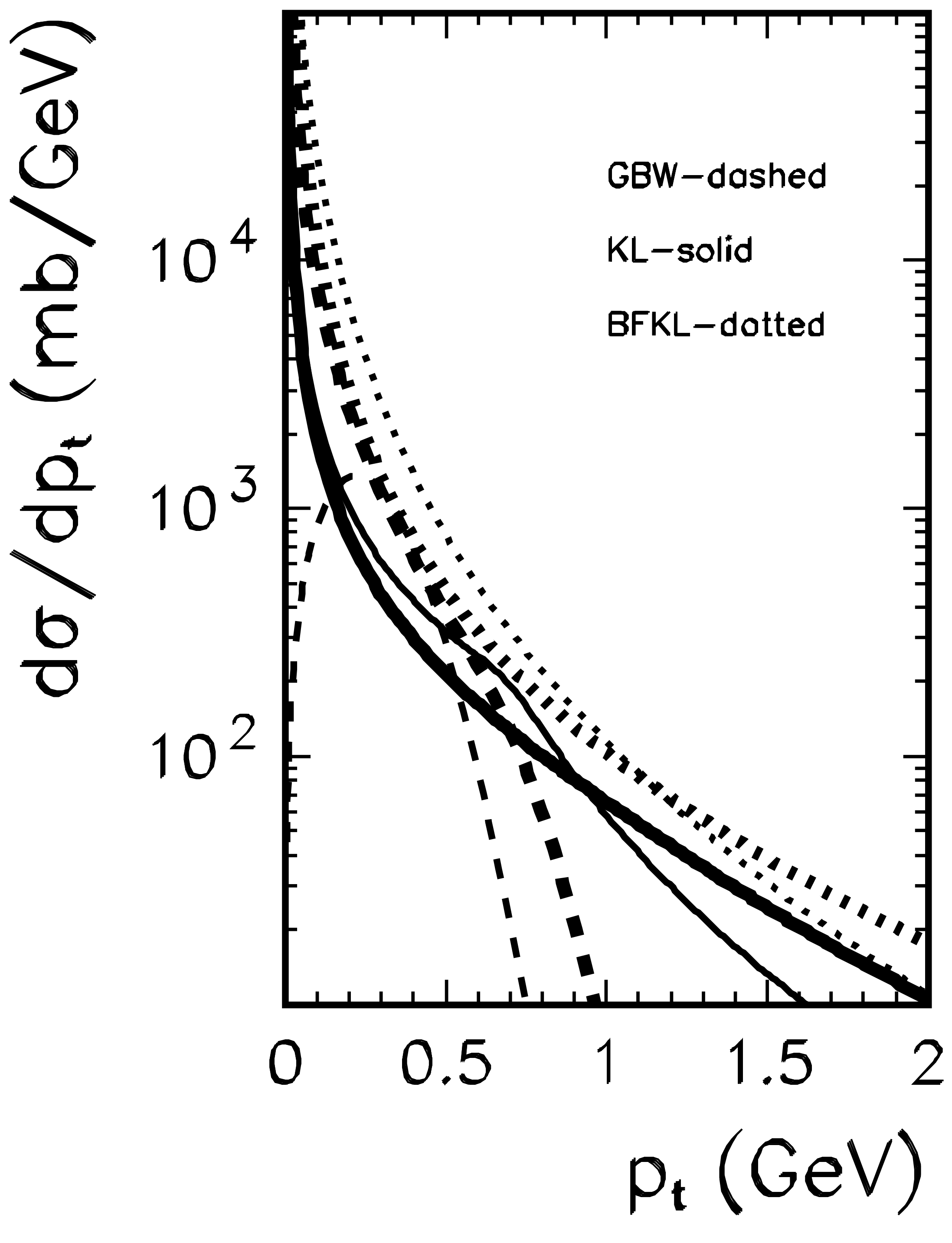
$$\frac{d\sigma}{dyd^2p_t} = \frac{4N_c\alpha_s}{N_c^2-1}\frac{1}{p_t^2} \int \left[\mathcal{F}_1(x_1,p_t^2)\mathcal{F}_2(x_2,q_t^2) + \mathcal{F}_1(x_1,q_t^2)\mathcal{F}_2(x_2,p_t^2)\right] dq_t^2 \qquad (15)$$

Let's compare this approximate formula with the original one.

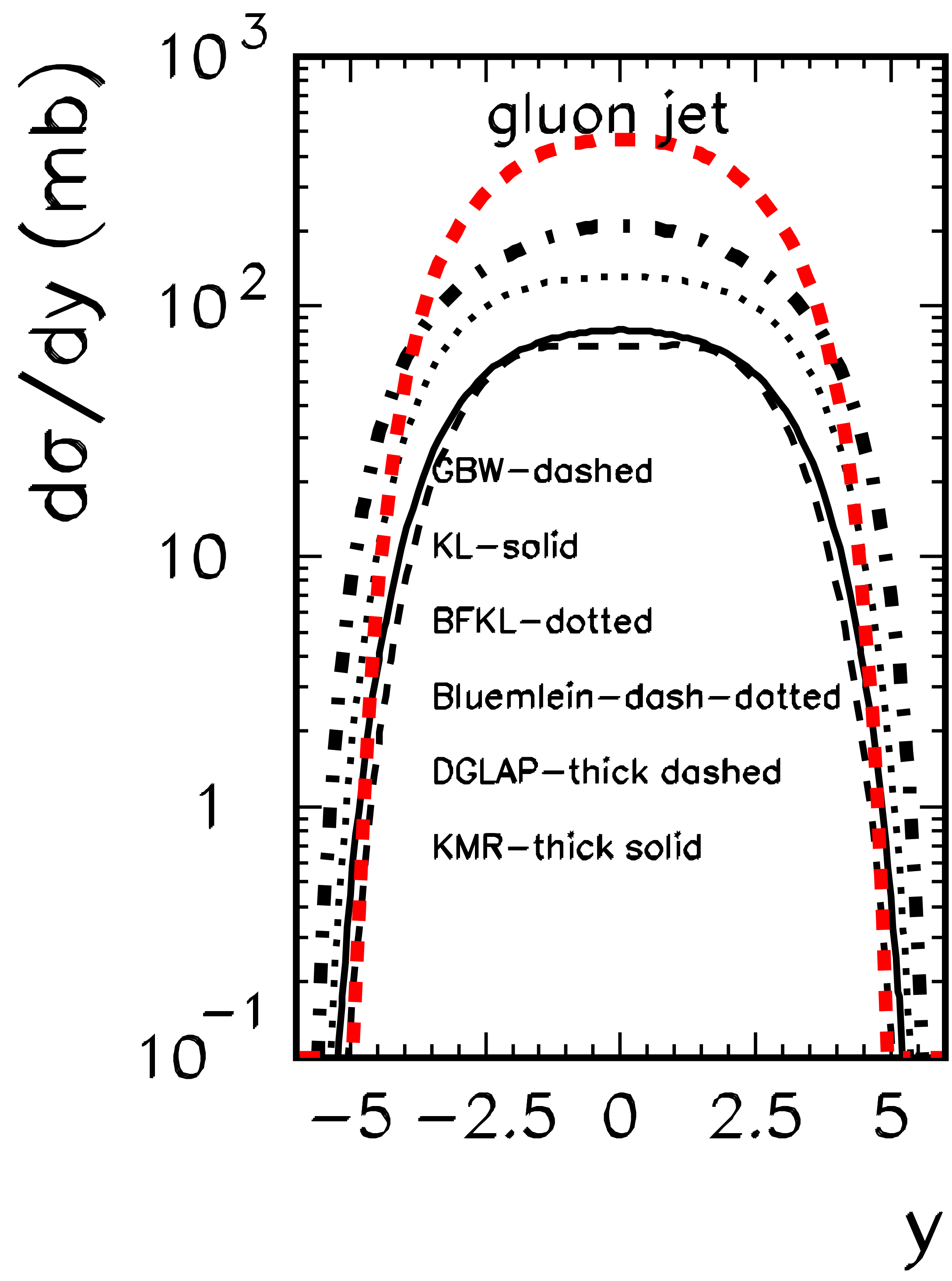
exact versus approximate



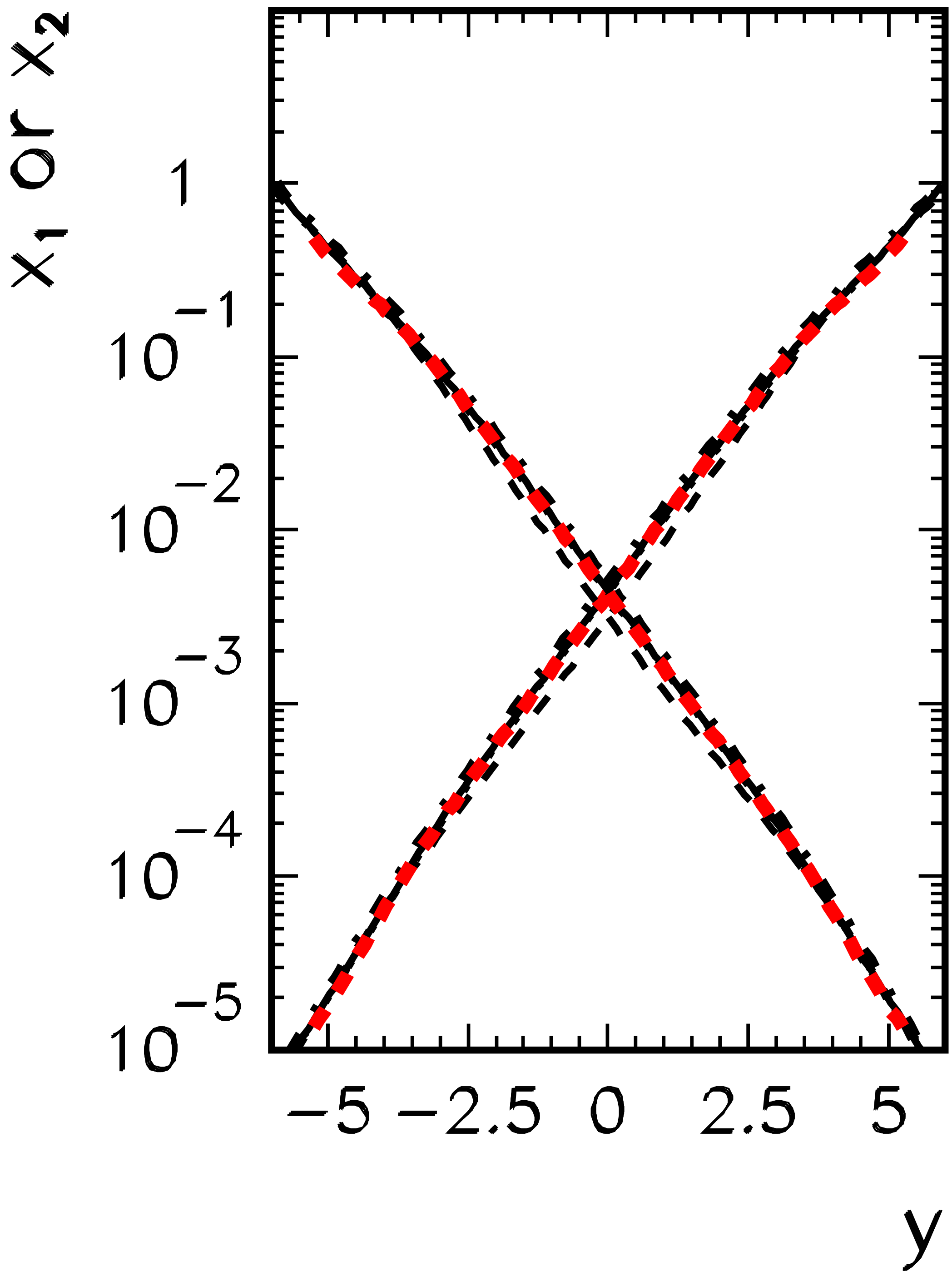
exact versus approximate



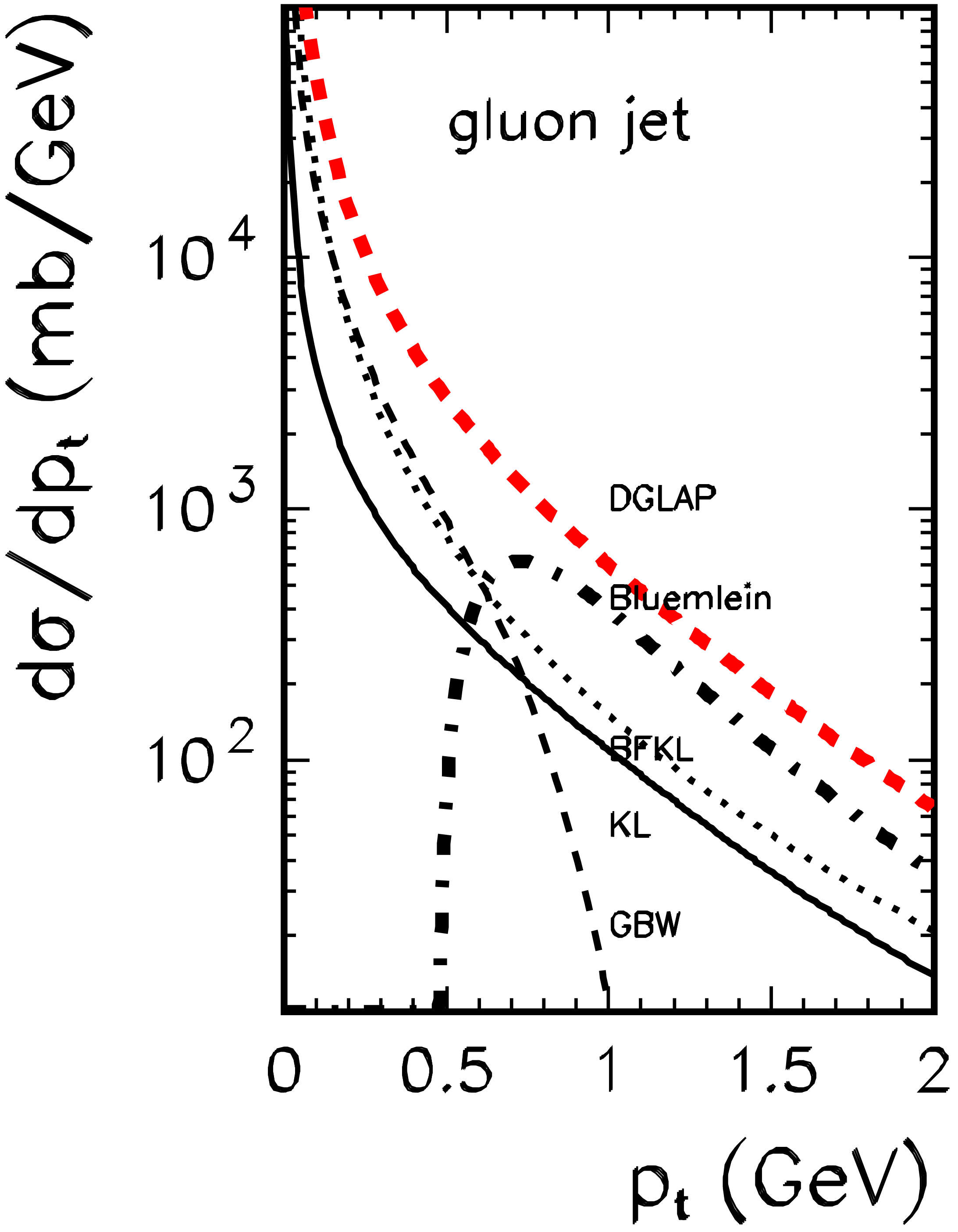
gluon rapidity distribution



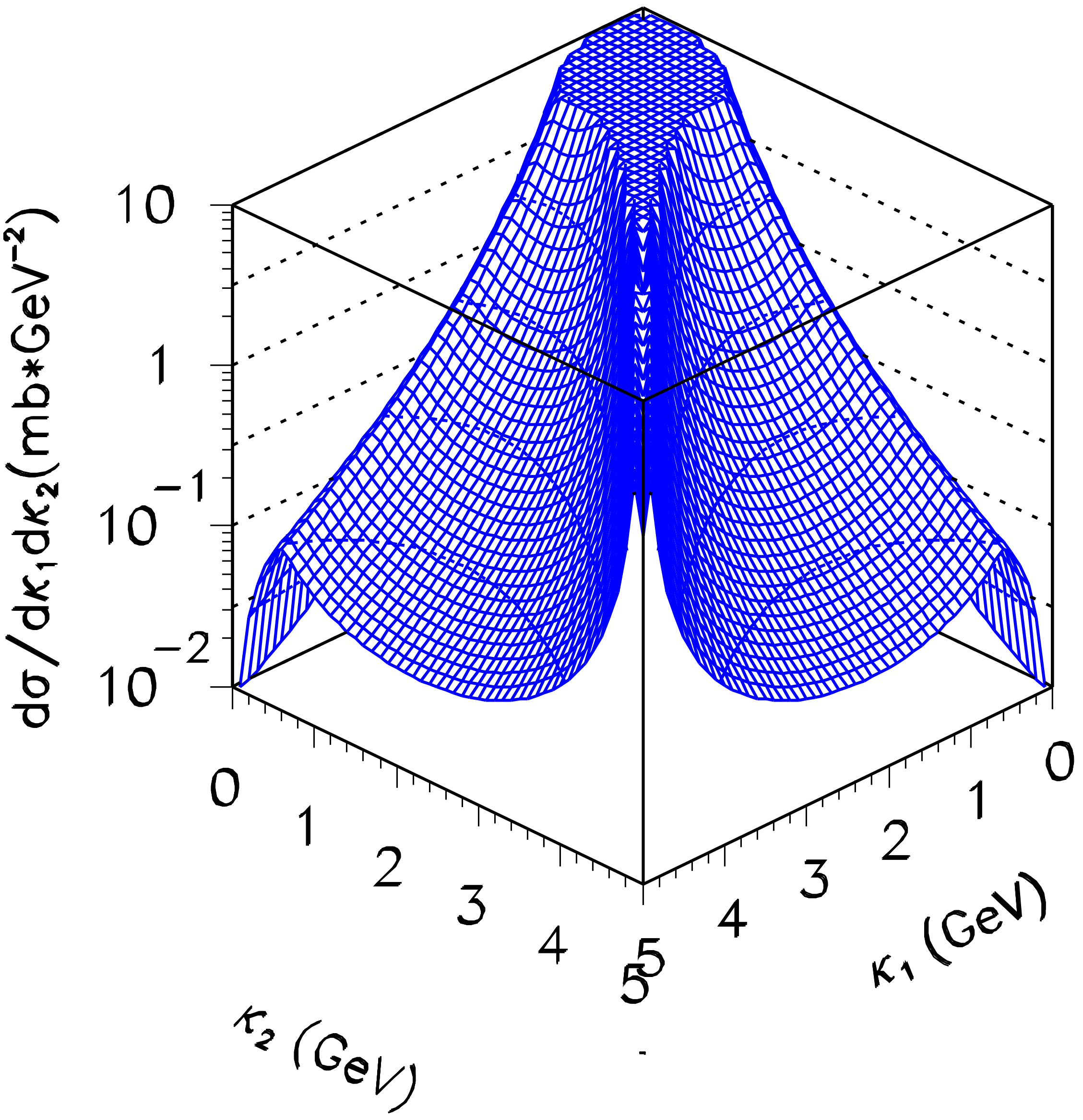
$< x_1 >$ and $< x_2 >$



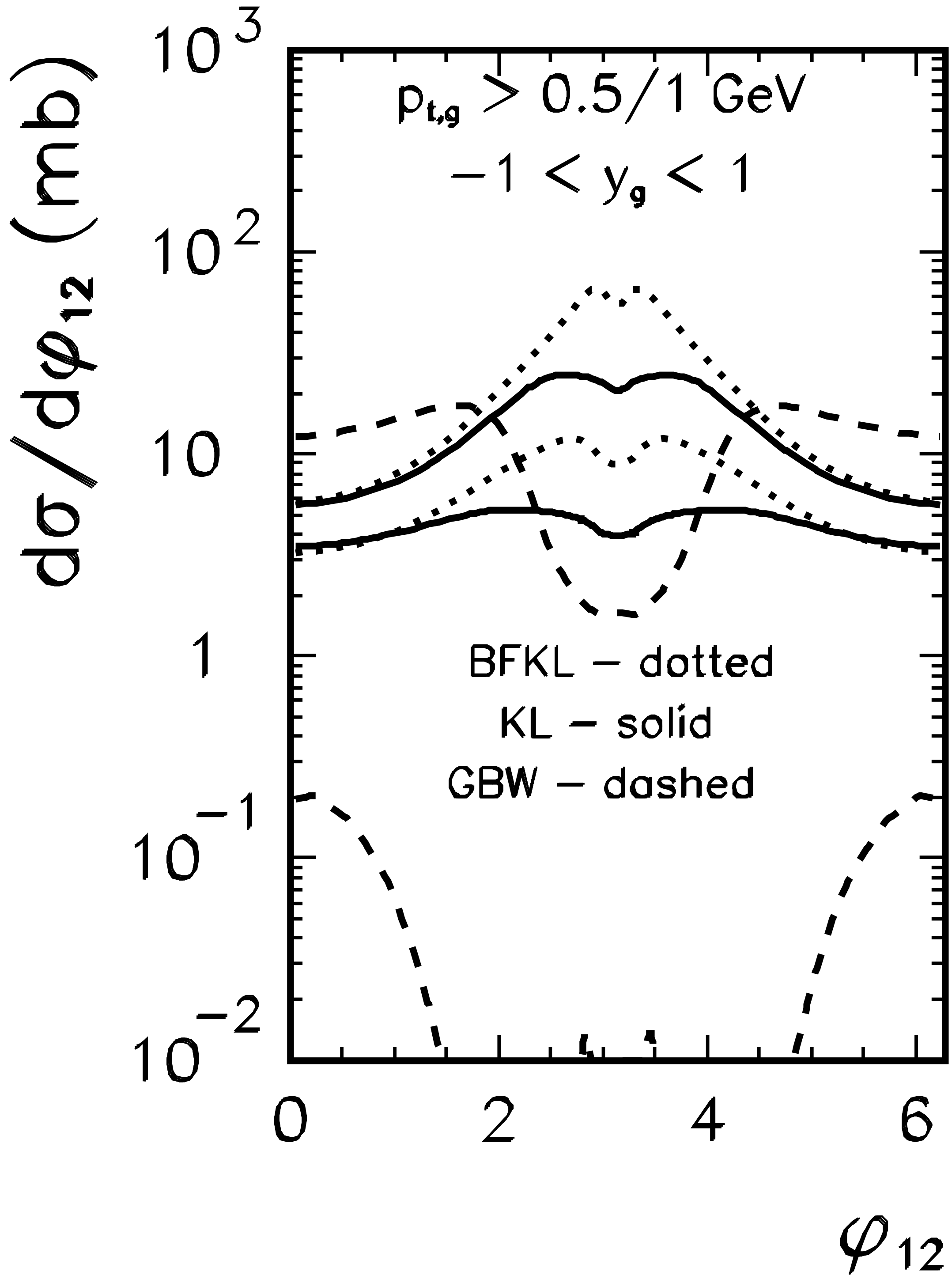
gluon trans. mom. distribution



transverse momenta of uPDF



Φ_{12} correlations



Kinematics of fragmentation

$$y_h = \frac{1}{2}\left[\frac{\sqrt{\frac{m_h^2+p_{t,h}^2}{p_{t,h}^2}+\sinh^2\eta_h}+\sinh\eta_h}{\sqrt{\frac{m_h^2+p_{t,h}^2}{p_{t,h}^2}+\sinh^2\eta_h}-\sinh\eta_h}\right] . \tag{16}$$

Collinearity of gluons and hadrons

$$y_g = \operatorname{arsinh}\left(\frac{m_{t,h}}{p_{t,h}}\sinh y_h\right) . \tag{17}$$

Let's define z: $E_h = zE_g$, then

$$p_{t,g} = \frac{p_{t,h}}{z} J(m_{t,h}, y_h) , \tag{18}$$

$$J(m_{t,h}, y_h) = \left(1 - \frac{m_h^2}{m_{t,h}^2\cosh^2 y_h}\right)^{-1/2} . \tag{19}$$

From gluons to hadrons

$$\frac{d\sigma(\eta_h, p_{t,h})}{d\eta_h d^2 p_{t,h}} = \int dy_g d^2 p_{t,g} \int dz\, D_{g\to h}(z, \mu_D^2) \qquad (20)$$

$$\delta(y_g - \eta_h)\, \delta^2\left(\vec{p}_{t,h} - \frac{z\vec{p}_{t,g}}{J}\right) \cdot \frac{d\sigma(y_g, p_{t,g})}{dy_g d^2 p_{t,g}} \, .$$

$$\frac{d\sigma(\eta_h, p_{t,h})}{d\eta_h d^2 p_{t,h}} = \int_{z_{min}}^{z_{max}} dz \frac{J^2 D_{g\to h}(z, \mu_D^2)}{z^2} \qquad (21)$$

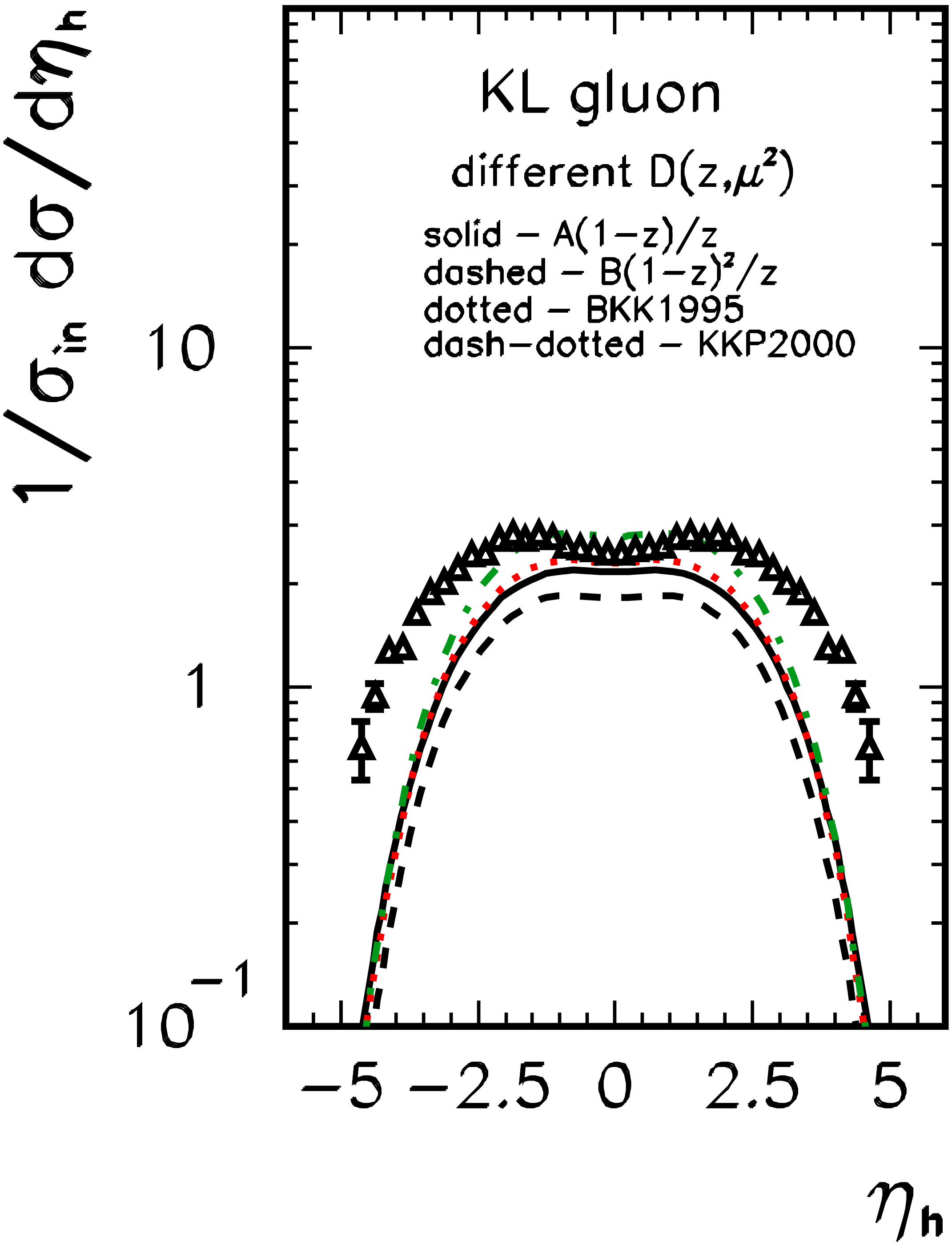
$$\left. \frac{d\sigma(y_g, p_{t,g})}{dy_g d^2 p_{t,g}} \right|_{\substack{y_g = \eta_h \\ p_{t,g} = J p_{t,h}/z}} \, .$$

Fragmentation functions from e^+e^- collisions:

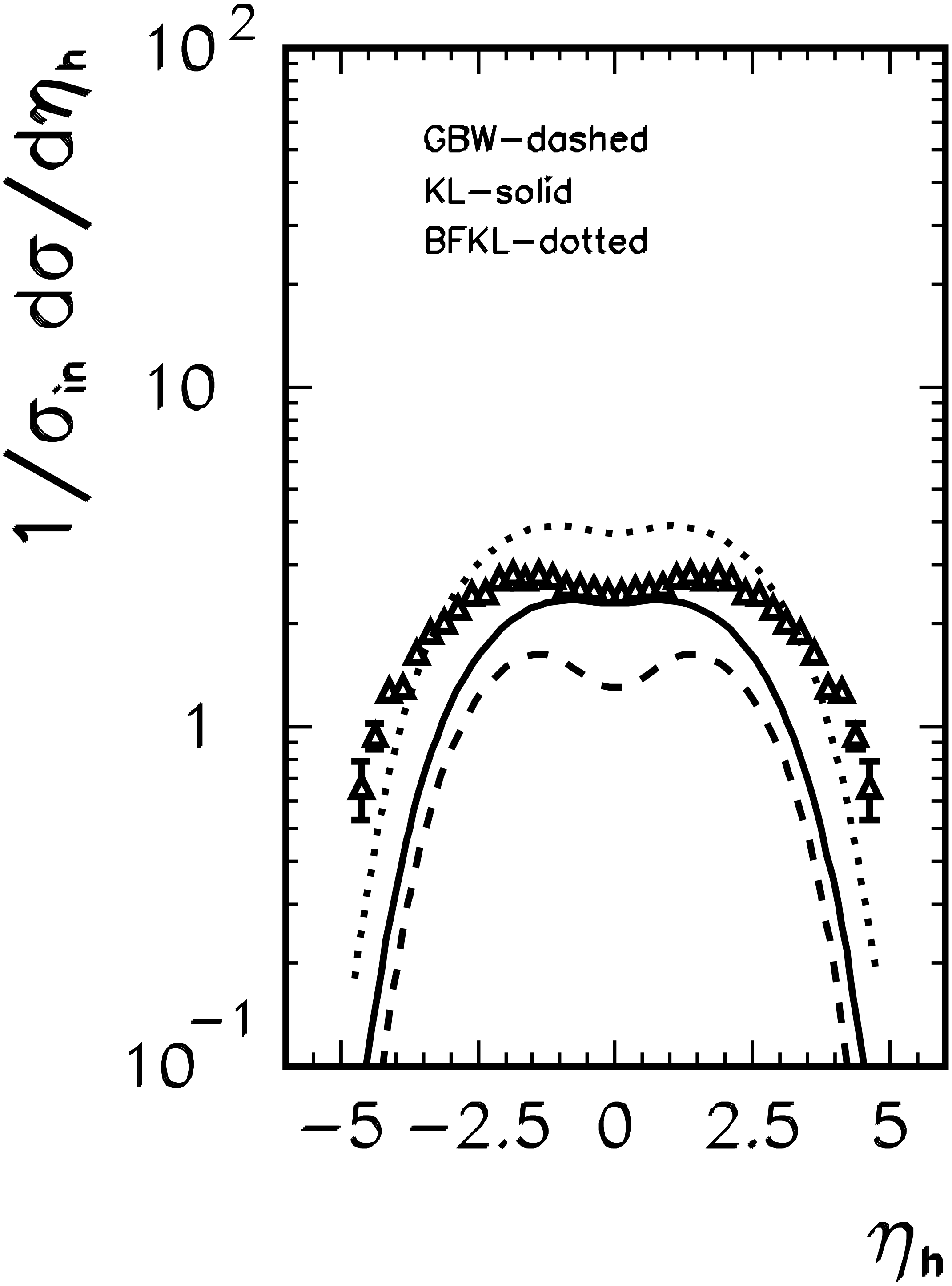
(a) Binnewies-Kniehl-Kramer (BKK95)

(b) Kniehl-Kramer-Pötter (KKP00)

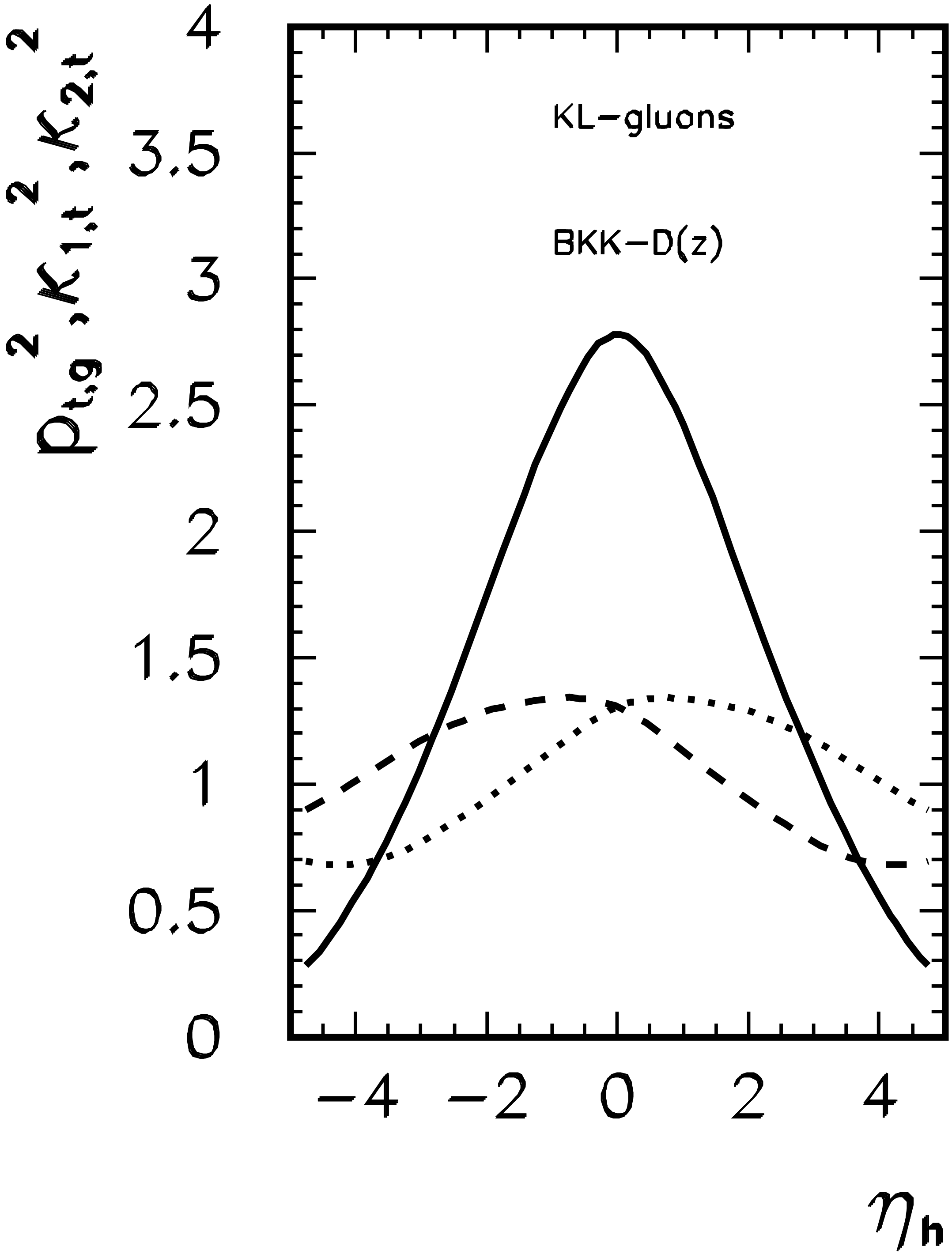
pseudorapidity dist. of particles



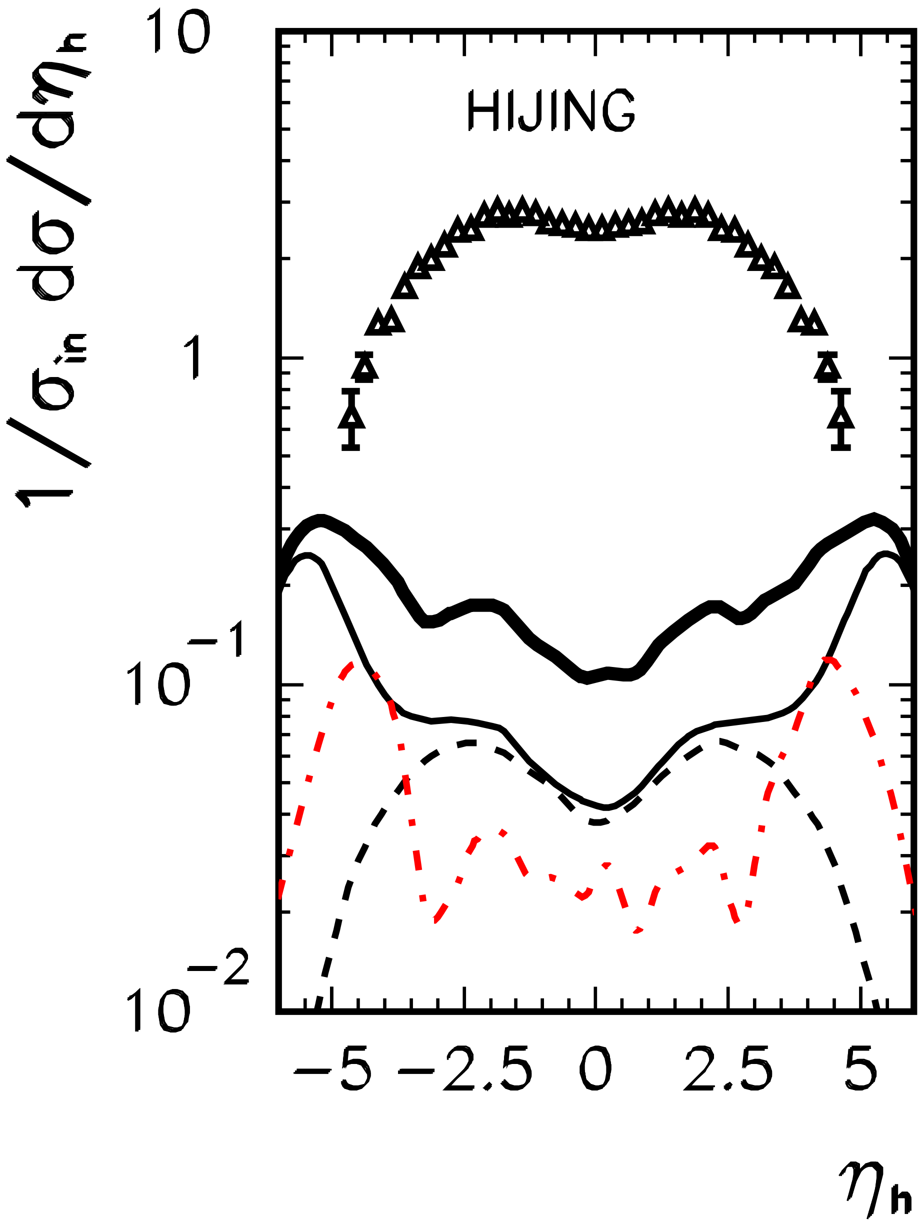
pseudorapidity dist. of particles



average trans. mom.

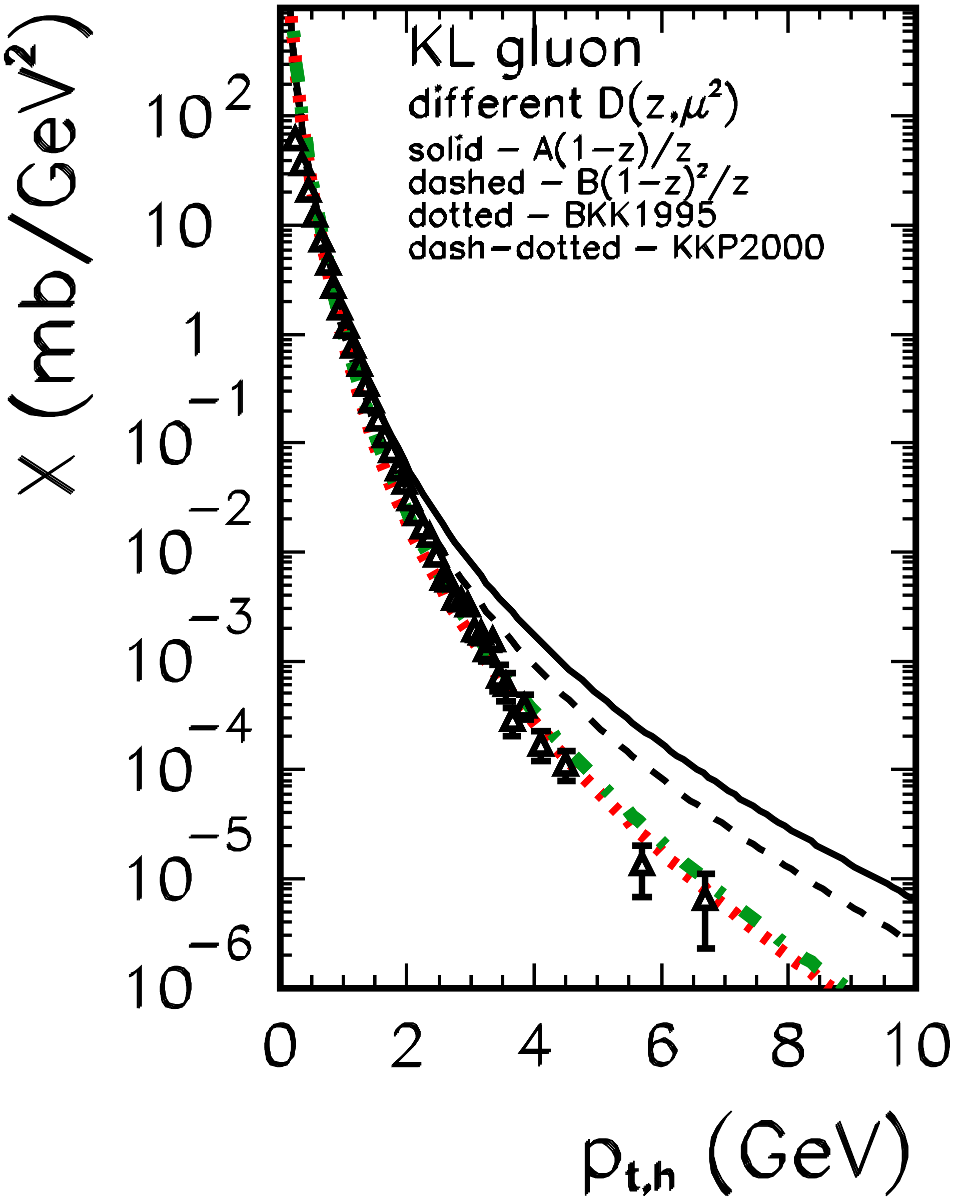


Other processes



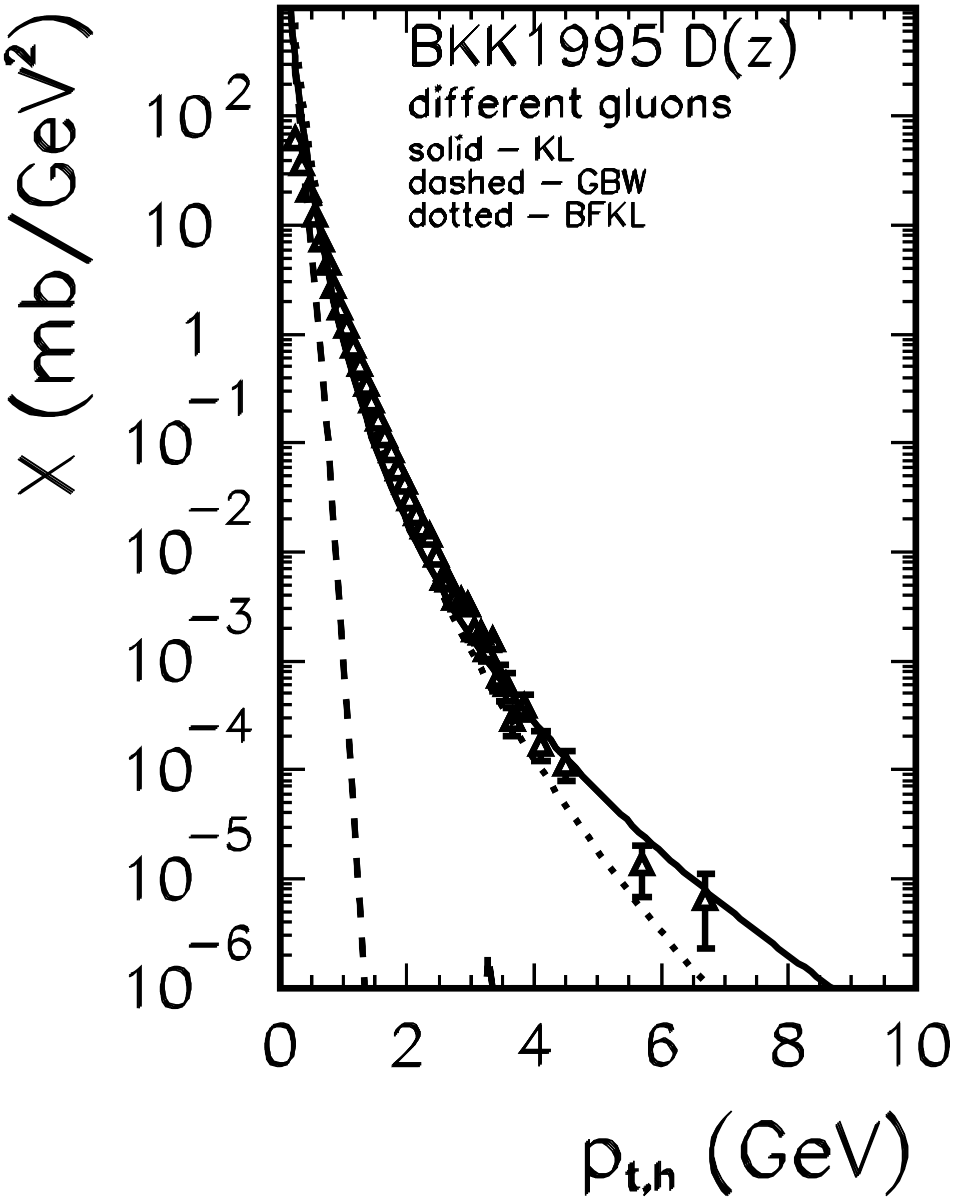
W = 200 GeV

Trans. mom. distribution



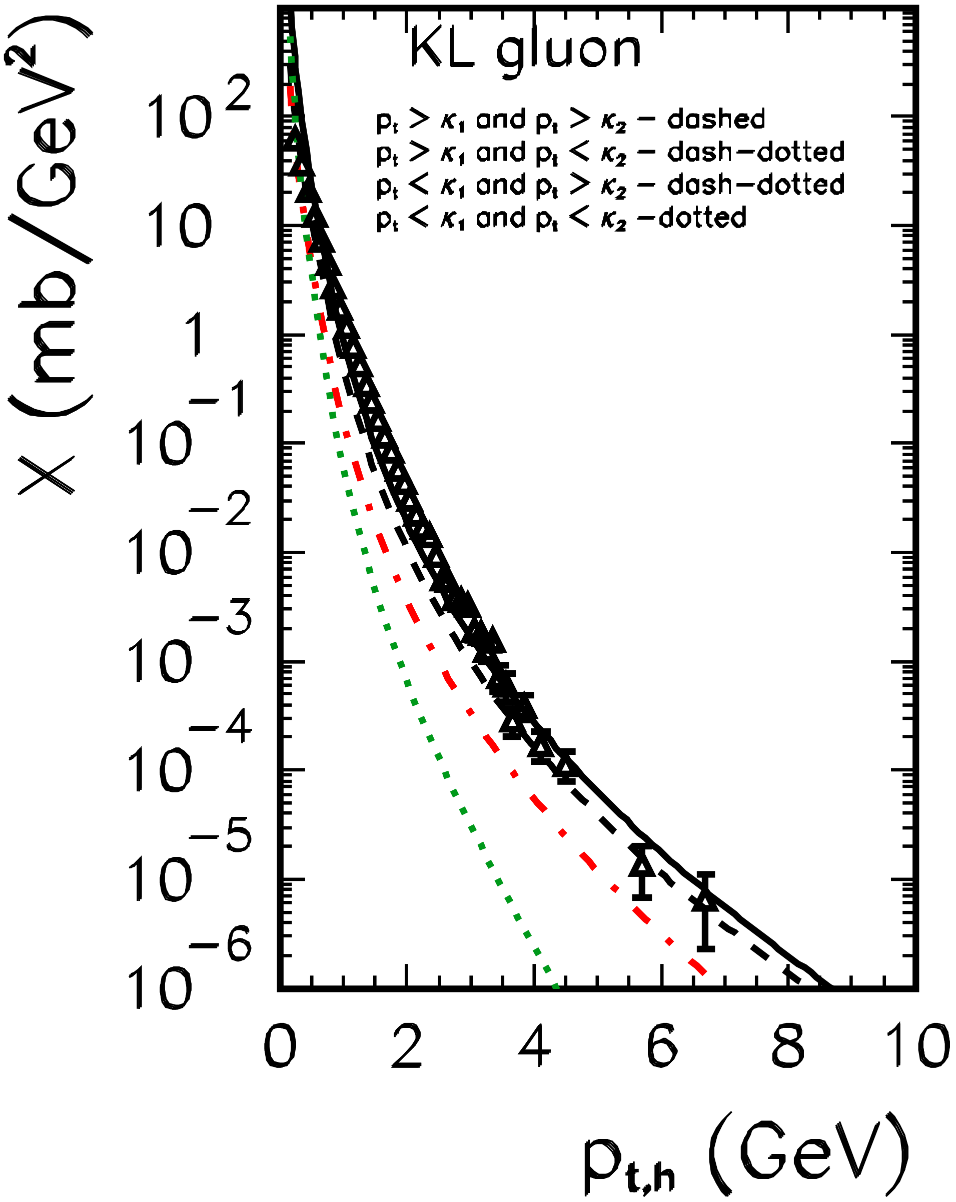
W = 200 GeV, $\eta_h \in$ (-2.5,2.5)

Trans. mom. distribution



W = 200 GeV, $\eta_h \in$ (-2.5,2.5)

interrelations between momenta



W = 200 GeV, $\eta_h \in$ (-2.5,2.5)

Summary/Conclusions

- Distribution of gluons and charged pions in nucleon-nucleon collisions calculated from different $\mathcal{F}(x_g, \kappa^2)$: GBW, BFKL, KL, KMR, etc.
- KL distribution tested against ep DIS data from HERA (GBW and BFKL also describe HERA data)
- GBW, KL, BFKL almost exhaust $\frac{d\sigma}{d\eta_h}(0)$
- KL, BFKL well describe transverse momentum distribution of $\pi^{\pm}$ if $D_{g\to\pi}$ taken from e^+e^-.
- missing strength in the fragmentation region due to missing mechanisms.