Automated resummation of final state QCD observables

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- event shape variables and jet rates
- NLO versus NLL
- full automated resummation
- conclusions and outlook

QCD and event shapes

✗ QCD *is* a special theory in the SM: strongly coupled domain is in the reach of current experiments (and unavoidable)

➥ wealth of non-trivial phenomena at formally PT scales
(divergent PT series, large NP effects, . . .)

✗ Main aim of QCD is the full understanding of these phenomena, both for their fundamental interest and for studies of new physics

☞ QCD observables: the more exclusive, the more difficult to predict

Interesting compromise between IR sensitivity and PT calculability

☛ event shapes and jet rates

Ideal laboratory to study the interface between PT and NP

Event shape variables

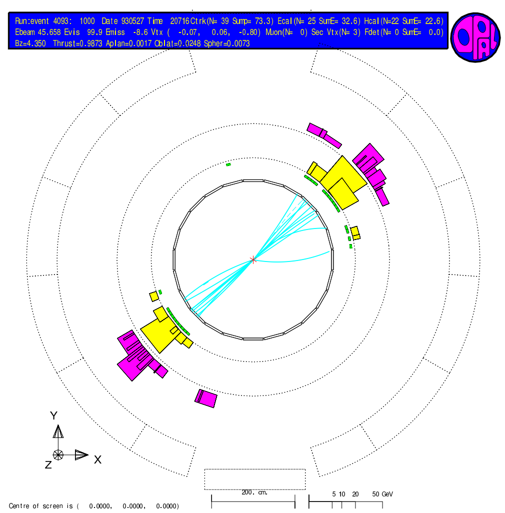
Event shape variables v are *IRC safe observables* which describe the topology of an event's hadronic final state in high energy collisions

They are constructed recombining momenta in a single number whose value gives an insight into geometrical properties of the event

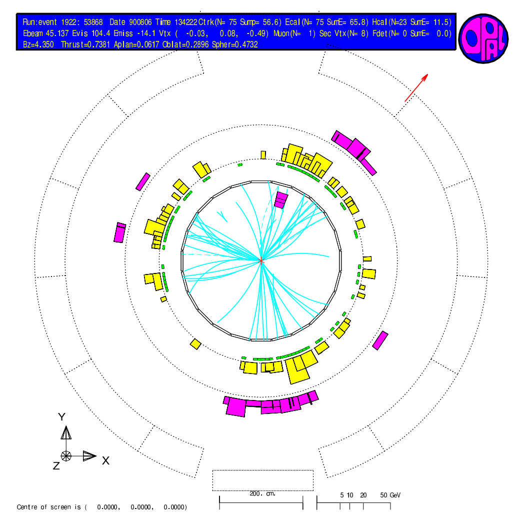
- Thrust: longitudinal particle alignment

$$T \equiv \frac{1}{Q} \max_{\vec{n}_T} \sum_i |\vec{p}_i \cdot \vec{n}_T| = \frac{1}{Q} \sum_i |p_{iz}|$$

Pencil-like event: $\tau \equiv 1 - T \ll 1$



Planar event: $T \simeq 2/3$



Distributions: the apperance of infrared logarithms

✗ Mean values and distributions of event shapes widely used to

- ✓ measure α_s and colour factors (test the $SU(3)$ gauge group)
- ✓ investigate (large) non-perturbative effects

✗ Most discriminatory studies make use of event shape *distributions*

$$\Sigma(v) = \int dV \frac{1}{\sigma}\frac{d\sigma}{dV}\,\theta(v - V(k_1,\dots,k_n))$$

They are characterized by *two different kinematical scales*

- ❖ hard scale of the process Q
  (CM energy in e^+e^-, virtuality of gauge boson in DIS, M_Z in DY, $\hat{s}$ in hh, . . .)
- ❖ observable scale $v\,Q$
  (determined by secondary emissions)

☛ Where $v \ll 1$ physical problem with 2 widely different scales! Theoretical comparisons require the resummation of large infrared logarithms $L \equiv \mathrm{Ln}(1/v)$

$$\alpha_s \ll 1 \quad \text{BUT} \quad L \gg 1 \qquad \Rightarrow \text{need to resum terms } (\alpha_s L)^n \sim 1$$

State-of-the-art of event-shape variables

Perturbative predictions

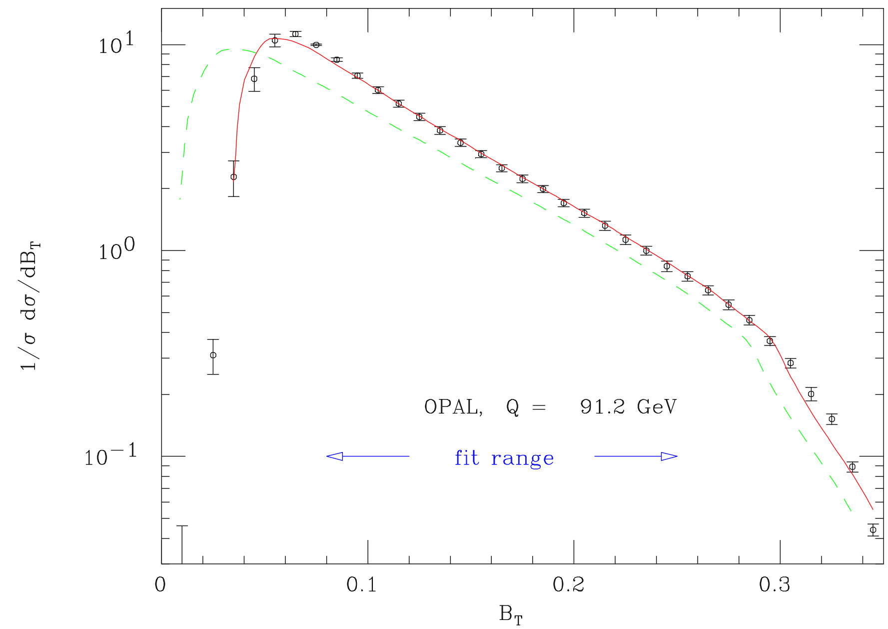
- $v = \mathcal{O}(1)$ NLO prediction

$$\Sigma(v) = 1 + \alpha_s \Sigma_1(v) + \alpha_s^2 \Sigma_2(v)$$

- $v \ll 1$ NLL resummation

$$\Sigma(v) = e^{\alpha_s^n \ln^{n+1} v + \alpha_s^n \ln^n v}$$

➥ Matching of NLO and NLL



Non-perturbative hadronisation effects

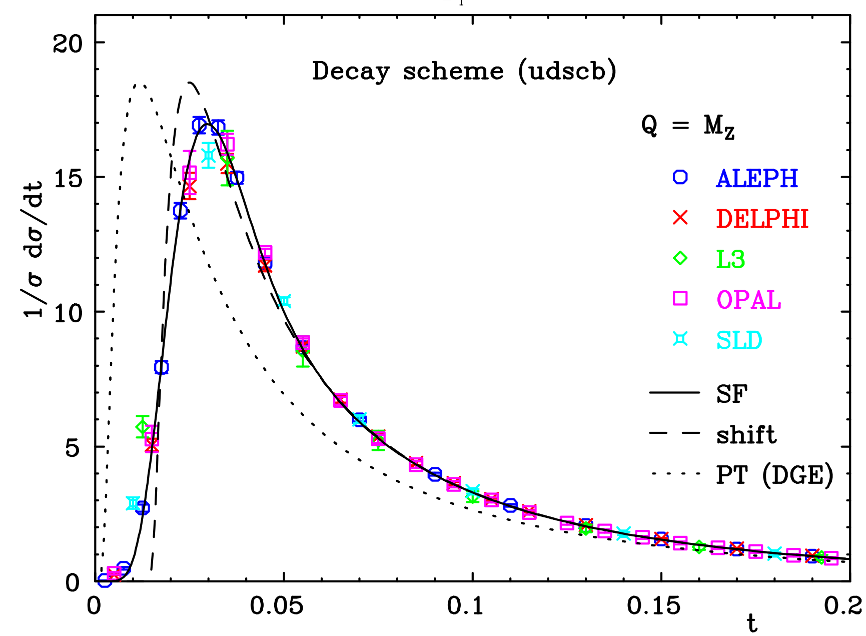
- $\alpha_s \ln v \gtrsim 1$ $1/Q$ corrections $\Rightarrow$ shift

Dokshitzer, Marchesini, Webber

- $\alpha_s \ln v \gg 1$ confinement effects are as large as perturbative contributions
➥ all $(1/Q)^n$ needed $\Rightarrow$ shape function

Korchemsky Sterman Gardi

➥ Merging of PT and NP contributions



☛ *fits of α_s, color factors and study of non-perturbative effects*

NLO versus NLL

Fixed order (NLO) predictions

- ☛ emission probabilities are calculable (in principle) at any given order
- ☛ the number of produced partons is finite
- ☛ for a given PT order the weight is observable independent, usually computed via Fixed Order Monte Carlos, all physics is hidden in them

☺ NLO predictions obtained easily by any user just providing the definition of the observable in the form of a computer routine

Resummed (NLL) predictions

- ☛ emission probabilities are known but mix different logarithmic orders $\Rightarrow$ analytical work needed to reorganise the PT series
- ☛ one needs to know the value of the observable for an arbitrary number of emissions

☹ A separate analytical calculation must be performed for each observable $\Rightarrow$ At least one theoretician is needed!

A selection of analytical NLL predictions

- S. Catani, G. Turnock, B. R. Webber and L. Trentadue, Phys. Lett. B **263**, 491 (1991)
  "Thrust distribution in e^+e^- annihilation"
- S. Catani, G. Turnock and B. R. Webber, Phys. Lett. B **272**, 368 (1991)
  "Heavy jet mass distribution in e^+e^- annihilation"
- S. Catani, G. Turnock and B. R. Webber, Phys. Lett. B **295**, 269 (1992)
  "Jet broadening measures in e^+e^- annihilation"
- Y. L. Dokshitzer, A. Lucenti, G. Marchesini and G. P. Salam, JHEP **9801**, 011 (1998)
  "On the QCD analysis of jet broadening"
- S. Catani and B. R. Webber, Phys. Lett. B **427**, 377 (1998)
  "Resummed C-parameter distribution in e^+e^- annihilation"
- V. Antonelli, M. Dasgupta and G. P. Salam, JHEP **0002** (2000) 001
  "Resummation of thrust distributions in DIS"
- A. Banfi, G. Marchesini, Y. L. Dokshitzer and **GZ**, JHEP **0007**, 002 (2000)
  "QCD analysis of near-to-planar 3-jet events" (thrust minor)
- A. Banfi, Y. L. Dokshitzer, G. Marchesini and **GZ**, JHEP **0105**, 040 (2001)
  "QCD analysis of D-parameter in near-to-planar three-jet events,"
- S. J. Burby and E. W. Glover, JHEP **0104**, 029 (2001)
  "Resumming the light hemisphere mass and narrow jet broadening distributions in e^+e^- annihilation"
- A. Banfi, G. Marchesini, G. Smye and **GZ**, JHEP **0111**, 066 (2001)
  "Out-of-plane QCD radiation in DIS with high p_t jets"
- A. Banfi, G. Marchesini, G. Smye and **GZ**, JHEP **0108**, 047 (2001)
  "Out-of-plane QCD radiation in hadronic Z_0 production"
- M. Dasgupta and G. P. Salam, Eur. Phys. J. C **24**, 213 (2002)
  "Resummation of the jet broadening in DIS"

☛ On average one paper per observable in a single process always!

Automated resummed predictions

Our goal: build a computer code which resums final state observables in an automated way – as for fixed order calculations.

The user just

✗ fixes the Born process and the number of hard legs

✗ provides the definition of the observable in the form of a computer routine

To achieve this we need a general understanding of the origin of all NLL logs in observable distributions. This reduces to

✓ soft, large angle emissions
✓ hard, collinear radiation
✓ running of the coupling effects
} ❶ single emission properties

✓ recoil (Mellin transforms), recombination effects } ❷ multiple emission properties

Notation: leg $\equiv$ hard parton

Observable properties: single emission properties

Fix a hard Born event and study the behaviour of the observable when a single soft emission k is generated collinear to any given hard leg ℓ. We parametrize

$$\boxed{V(k) \simeq d_\ell \left(\frac{k_t}{Q}\right)^{a_\ell} e^{-b_\ell \eta} g_\ell(\phi)}$$

$k_t \quad \Rightarrow \quad$ transverse momentum wrd the leg
$\eta \quad \Rightarrow \quad$ rapidity wrd the leg
$\phi \quad \Rightarrow \quad$ azimuthal angle

The program

✓ checks the assumed parametrisation

✓ determines the coefficients and functions $a_\ell, b_\ell, d_\ell, g_\ell(\phi)$

This knowledge allows the analytical computation all NLL single emission effects and gives the *single (or simple) emission distribution* $\Sigma_s(v)$

The single emission distribution Σ_s

$\Sigma_s(v)$ is the full NLL resummed distribution but for a simpler observable!

$$V(k_1,\dots k_n) \Rightarrow V_s \equiv \max[V(k_1),\dots,V(k_n)]$$

Simple factorization for V_s $\Rightarrow$ analytical NLL resummation for V_s straightforward!

$$\Sigma_s(v) = \prod_{\ell=1}^{n_{inc}} \underbrace{f_\ell(v^{\frac{2}{a+b_\ell}}\mu_F^2)}_{\text{pdfs}} \otimes \prod_{\ell=1}^{N} \underbrace{J_\ell(L)}_{\text{jet function}} \cdot \underbrace{S\left(T(L/a)\right)}_{\text{soft}}$$

$$\ln J_\ell(L) = -C_\ell\left[r_\ell(L) + r'_\ell(L)\left(\ln \bar{d}_\ell - b_\ell \ln\frac{2E_\ell}{Q}\right) + B_\ell T\left(\frac{L}{a+b_\ell}\right)\right]$$

- soft and collinear emission $\Rightarrow$ jet function $J_\ell(L)$
  (all LL contributions in $r_\ell(L)$, while $r'_\ell(L)$ and $T(L)$ are pure NLL functions)
- hard collinear splitting $\Rightarrow$ evolution of the pdf's and rescaling factors B_ℓ
- soft large angle $\Rightarrow$ QCD coherence and geometry dependence in $S(t)$

Botts and Sterman, Nucl. Phys. B **325** (1989) 62

Banfi, Dokshitzer, Marchesini, GZ JHEP 07 (2000) 002

Observable properties: multiple emission properties

The computation of Σ_s is based on 1 emission, the "hardest" emission, i.e. on the approximation

$$V(k_1,\ldots k_n) \Rightarrow V_s \equiv \max[V(k_1),\ldots,V(k_n)]$$

To account for all NLL effects one needs to relate the value of the observable to *all secondary emissions*, i.e. account for the observable specific mismatch between $V(k_1,\ldots k_n)$ and V_s

- Physically: one needs accurate understanding of the kinematics (recoil)
- Mathematically this translates into performing Mellin transforms

But this last NLL effects has been computed numerically in a general way

Banfi, Salam, GZ *JHEP* 0201 (2002) 018

How can these observable-specific multiple emission effects be computed generally?

How to deal generally with multiple emission effects??

AIM: compute the mismatch between $\Sigma_s(v_s)$ and $\Sigma(v)$ $\Rightarrow$ this gives $\mathcal{F}$!

The 2 distributions are related by a simple convolution

$$\frac{D(v)}{v} = \int \frac{dv_s}{v_s} D_s(v_s) P(v|v_s) \qquad D(v) \equiv \frac{d\Sigma}{dv}$$

☞ $P(v|v_s)$ is the probability to have v given v_s

Since
- ☛ $D_s(v_s) = e^{-R(v_s)}$ $\Rightarrow$ known analytically
- ☛ $v \sim v_s$ $\Rightarrow$ same DL strucure

➥ expand and get $D_s(v_s) =_{NLL} D_s(v) e^{-R' \ln \frac{v}{v_s}}$ $\qquad R' \equiv dR/dL$

Just combining the 2 relations gives

$$\boxed{D(v) =_{NLL} D_s(v)\mathcal{F}(R') \qquad \mathcal{F}(R') = \int \frac{dx}{x} e^{-R' \ln x} p(x) \qquad x = v/v_s}$$

How to compute $\mathcal{F}$ ($\Leftrightarrow$ $p(v|v_s)$) generally?

Numerical implementation:
✎ *the procedure to get $\mathcal{F}$*

Fix a Born configuration and generate *decreasing soft-collinear (SC) emissions* according to phase space

I set $v(k_1) = v_s$ [START FROM: $V_s = v_s$]

II generate a SC emission with $v(k_i) < v(k_{i-1})$

III if $v(k_i) < \epsilon \ll 1$ exit (no resolvable emission), otherwise go to back II.

➥ ARRIVE AT: $V(k_1, k_2, \ldots k_n) \equiv v$!

This gives the probability of having $V = v$ given $V_s = v_s$ and allows so the numerical computation of $\mathcal{F}$!

☞ This simple procedure can be applied to any n-jet process!

[http://www.ippp.dur.ac.uk/~zander/numsum.html]

Requirement on the observable

For the observable to be resummed automatically it should

- ✗ be infrared and collinear safe
- ✗ vanish in the Born limit
- ✗ exponentiate (JADE excluded)
- ✗ be continuously global ($a_\ell = a \ \forall$ hard legs ℓ)
- ✗ behave as $V(k) \simeq d_\ell \left(\frac{k_t}{Q}\right)^{a_\ell} e^{-b_\ell \eta} g_\ell(\phi)$ for 1 SC gluon along leg ℓ
- ✗ satisfy minor (technical) requirements

While this might seem a long list

- ☛ practically the limiting condition is the requirement of globalness
  (all other conditions are satisfied by all observables resummed so far)
- ☛ an essential feature of the program is the ability to perform all checks automatically and to resum the observable only if correctness of the result is guaranteed at NLL

➥ Observables beyond the scope of the program are outwith our current understanding!

The full result

The master formula: $$\boxed{\Sigma(v) =_{NLL} \Sigma_s(v)\mathcal{F}(R')}$$

Banfi, Salam, GZ hep-ph/0304148, submitted to Phys. Rev. Lett.

✓ *Analytical* expression for Σ_s: *pure expansion in double and single logarithms*

$$\Sigma_s(v) = \prod_{\ell=1}^{n_{inc}} \underbrace{f_\ell(v^{\frac{2}{a+b_\ell}}\mu_F^2)}_{\text{pdfs}} \otimes \prod_{\ell=1}^{N} \underbrace{J_\ell(L)}_{\text{jet function}} \cdot \underbrace{S\left(T(L/a)\right)}_{\text{soft}}$$

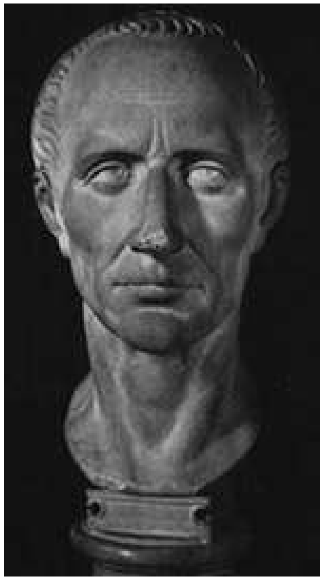
$$\ln J_\ell(L) = -C_\ell\left[r_\ell(L) + r'_\ell(L)\left(\ln \bar{d}_\ell - b_\ell \ln\frac{2E_\ell}{Q}\right) + B_\ell T\left(\frac{L}{a+b_\ell}\right)\right]$$

✓ $\mathcal{F}(R')$ is computed numerically but is *by construction a pure NLL function*

☛ Novel approach to resummation based on a *master formula with applicability conditions* for a *class of observables*. Can such an approach be used in other resummation contexts or seeking for higher precisions?

CAESAR

Computer Automated Expert Semi-Analytical Resummation



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☛ For *NEW predictions* from CAESAR in e^+e^-, DIS and hadronic dijet production listen to ***Andrea Banfi's talk!***

Some remarks concerning the method

☛ Results have the quality of analytic NLL predictions, though no human intervention is required

✓ Any hadronisation model can be applied
(string-, cluster-, dipole fragmentation, power corrections. . .)

✓ Studies of μ_R and μ_F dependence feasible

✓ The answer is free of subleading logs $\Rightarrow$ matching with fixed order feasible

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☛ Comparison with standard Monte Carlo event generators (Herwig, Phythia, . . .)

✗ MCs don't guarantee the correctness of all subleading logs

✗ MC results can be matched mainly at LO, first attempts towards a NLO matching

Frixione & Webber JHEP 06 (2002) 029

✗ MCs can be applied to any QCD observable. Our numerical approach should be seen as a *pseudo-analytical* solution to a class of observables

Physical/mathematical/technical content of CAESAR

- ✓ Born processes currently implemented

e^+e^--collisions:	$e^+e^- \to 2$ jets	$e^+e^- \to 3$ jets
DIS collision:	$p\ e \to 1$ jet	$p\ e \to 2$ jets
Drell Yan collision:	$p_1p_2 \to Z_0 +$ jet	
Hadronic collisions:	$p_1p_2 \to 2$ jets	

$(p_i = q, \bar{q}, g)$

Extention to other processes open and straightforward

- ✓ Implementation of exact analytical formulas whenever possible
- ✓ Recoil in dipole method — Catani & Seymour, Nucl. Phys. B **485** (1997) 291
- ✓ Evolution of colour charge due to soft large angle radiation implemented through matrices in colour space — Kidonakis, Oderda & Sterman, Nucl. Phys. B **531** (1998) 365
- ✓ Parton densities (PDF) evolution code to account for hard collinear initial state radiation — Dasgupta & Salam, Eur. Phys. J. C **24**, 213 (2002)
- ✓ Extended arbitrary precision arithmetic package for tests in the asymptotic region and for the determination of asympthotic properties — Bailey, RNR Technical Report RNR-94-013

Conclusions and outlook

✓ Multiple emission effects computed numerically in an observable independent way

Banfi, Salam, GZ *JHEP* 0201 (2002) 018

✓ We have a numerical program CAESAR which fully automates the resummation of of a class of final state observables in arbitrary hard n-jet processes

Banfi, Salam, GZ hep-ph/0304148, submitted to Phys. Rev. Lett.

Our hope: reproduce and extend the scope of studies now standard in e^+e^- and DIS to hadronic collision (⇒ studies of correlation between colour/energy flow)

For the *near* future: automated matching of NLL result with NLO (⇒ NLOJET++)

Nagy PRL 88 (2002) 122003