

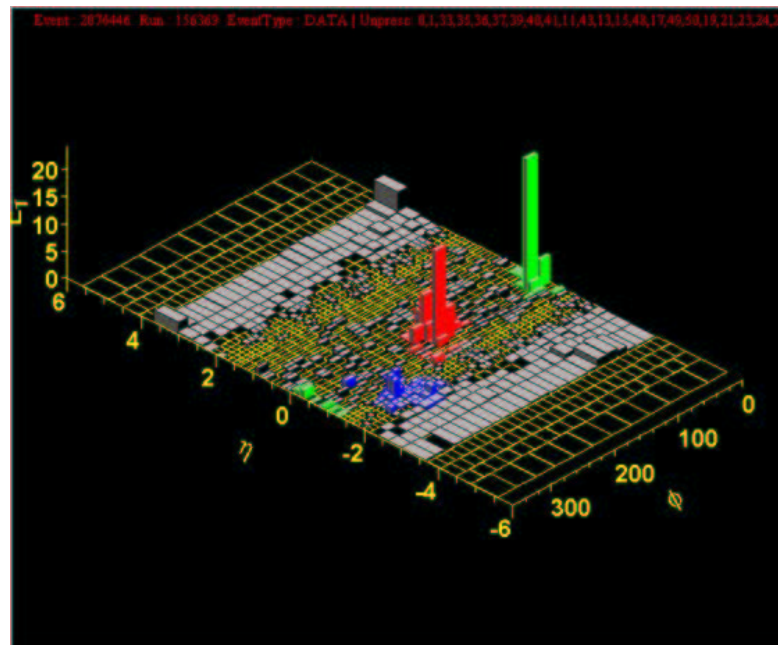
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Work done in collaboration with:

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- ➡ Physical motivations
- ➡ Event shapes in **hh collisions**
- ➡ **Master formula** for resummation
- ➡ First results obtained with **CAESAR**
- ➡ Conclusions and outlook



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Physical motivations

Hadronic collider experiments

😊 constitute the **ideal environment** for the search of **new physics**

😊 are **incredibly rich** from the point of view of **QCD**

In particular **jet observables** at hadronic colliders

⚠ allow ‘traditional’ measures of α_s and of **colour factors**

⚠ require investigation of **non-perturbative** effects
➡ insight into **hadronisation** mechanism and **soft collisions** (underlying event)

⚠ test our knowledge of strong interaction dynamics in a **very involved** environment

⚠ constitute the **QCD background** for the search of **new physics**

Event shape variables in hadronic dijet production

Event shape variables (V) describe the geometry of the final state in high energy hadronic processes

Event shapes in **hh collisions** can be constructed in analogy with e^+e^- and DIS

Transverse Thrust: particle alignment in the transverse plane

$$T_T \equiv \frac{\max_{\vec{n}_T} \sum_i |\vec{p}_i \cdot \vec{n}_T|}{\sum_i |p_{ti}|}$$

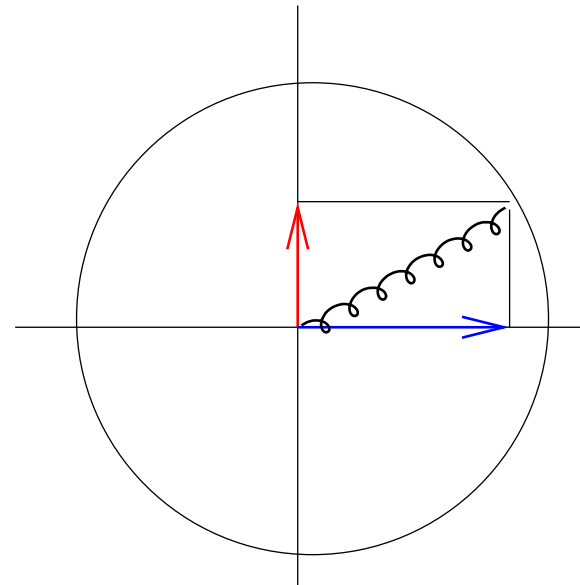
Thrust minor: out of event plane energy flow

$$T_m \equiv \frac{\sum_i |p_i^{\text{out}}|}{\sum_i |p_{ti}|}$$

Ideal testing ground for an **automated resummation**

- complicated **phase space cuts**
- non-trivial **flavour** and **colour** structure
- ➡ analytical resummation **cumbersome**

— Transverse Thrust
— Thrust Minor



Transverse plane

Resummation master formula

NLL resummation in the N -jet limit for $\Sigma(v)$ (fraction of events with $V < v$)

[AB, Salam, Zanderighi hep-ph/0304148]

$$V(k) \simeq d_\ell \left(\frac{k_t}{Q} \right)^a e^{-b_\ell \eta} g_\ell(\phi)$$

\Downarrow

$$\Sigma(v) = \prod_{\ell=1}^{n_{in}} \underbrace{f_\ell(v^{\frac{2}{a+b_\ell}} \mu_F^2)}_{\alpha_s^n L^n} \otimes \prod_{\ell=1}^N \underbrace{J_\ell(L, a, b_\ell)}_{\alpha_s^n L^{n+1}} \cdot \underbrace{S(T(L/a)) \cdot \mathcal{F}(R')}_{\alpha_s^n L^n}$$

✌ collinear emissions \Rightarrow LL jet function $J_\ell(L, a, b_\ell)$ with $L \equiv \ln 1/v$

✌ initial state hard collinear splitting \Rightarrow evolution of the pdf's

✌ QCD coherence \Rightarrow geometry dependence through the NLL function $S(T)$

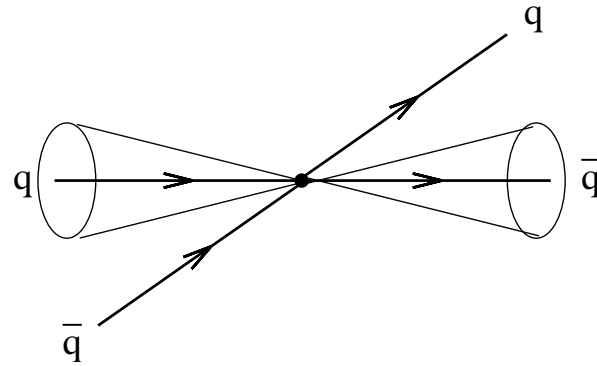
[Botts, Sterman NPB 352 (1989) 62]

✌ multiple emission effects $V(k_1, \dots, k_n) \neq V(k_1) \Rightarrow$ NLL function $\mathcal{F}(R')$

Computed automatically by the numerical program **CAESAR**. The program needs only a routine that computes the observable given a set of four-momenta

Theoretical problems in hadronic collisions

Experiments in hadronic collisions involve necessarily measures only in a part of the phase space, namely one imposes a **rapidity cut** around the beam



emission in the beam region **does not affect** the observable

➡ **non global logarithms** arise, CAESAR stops warning the user that the observable is non-global

Non global logarithms are **problematic** for two reasons

- ❖ non-trivial geometry \Rightarrow only **numerical** resummation
- ❖ complicated colour structure \Rightarrow only **large N_c** limit



emission in the beam region **affects** the observable through **recoil**

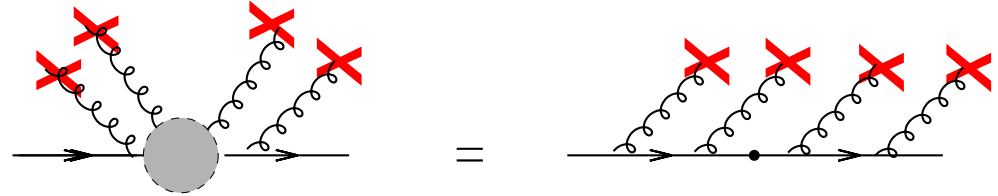
➡ CAESAR proceeds but the function $\mathcal{F}(R')$ has **divergences** for $R' = R'_c$

One needs to introduce observables that are **computable** at the current accuracy

Independent soft gluon emission

The basic assumption of the master formula is the **independent emission** picture of soft radiation (QED)

$$w(k_1, \dots, k_n) = \prod_i w(k_i)$$



The basis of independent emission is that **secondary gluon branching** can be **neglected** within NLL accuracy

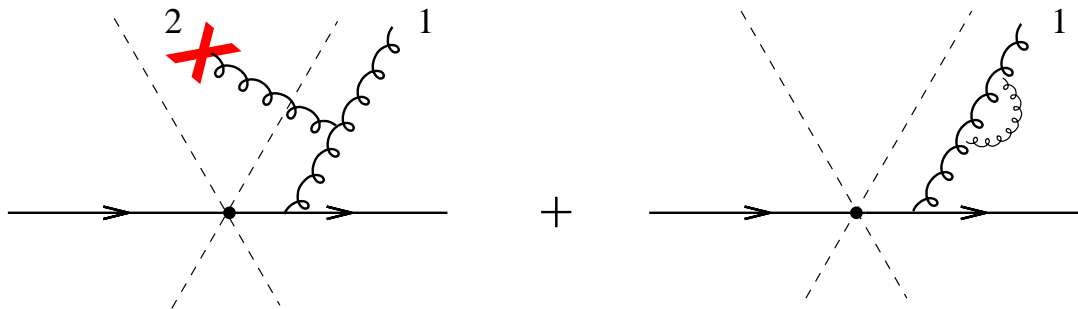
Consider for instance the **Transverse Thrust**

$$\tau \equiv 1 - T_T \simeq \sum_i \omega_i / Q \quad (\text{large angles})$$

$$+ \quad = \alpha_s^2 \int_{\tau Q}^{\tau Q} \frac{d\omega_1}{\omega_1} \int_{\tau Q}^{\omega_1} \frac{d\omega_2}{\omega_2} = \mathcal{O}(\alpha_s^2 L)$$

Non-global logarithms

In the case of a **non-global observable** such a real-virtual cancellation does not work any more



The diagram shows two Feynman diagrams for a process with two incoming particles (solid lines) and two outgoing particles (wavy lines). The first diagram has a red 'X' over the second outgoing particle, labeled '2', indicating it is not in the measure region. The second diagram has the first outgoing particle, labeled '1', in the measure region. The diagrams are summed and equated to an integral expression.

$$= \alpha_s^2 \int_{\tau Q}^Q \frac{d\omega_1}{\omega_1} \int_{\tau Q}^{\omega_1} \frac{d\omega_2}{\omega_2} = \mathcal{O}(\alpha_s^2 L^2)$$

SL contributions arise when only the **softest gluon** is emitted in the measure region

$$\Sigma(\tau) = \underbrace{\Sigma_g(\tau)}_{\text{independent}} \cdot \underbrace{S_{ng}(t)}_{\text{non-global}} \quad t = \int_{\tau Q}^Q \frac{d\omega}{\omega} \alpha_s(\omega)$$

Two main approaches to resum non global logarithms in the **large N_c** limit

● Monte Carlo simulation

[Dasgupta, Salam JHEP 0203(2002)017]

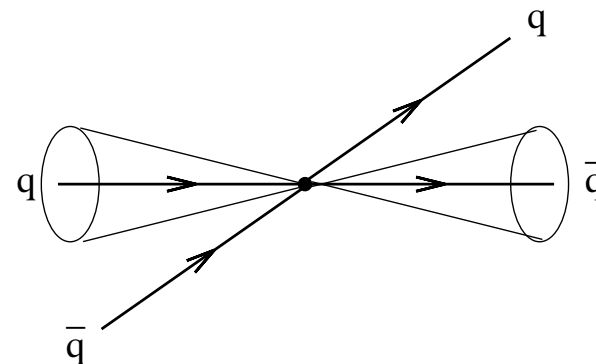
● Non-linear evolution equation

[AB, Marchesini, Smye JHEP 0208(2002)006]

Divergences in the function $\mathcal{F}(R')$

Thrust minor as an example

$$T_m = \frac{1}{E_T} \left\{ \sum_{|\eta_i| < \Delta} |p_i^{\text{out}}| + \left| \sum_{i \in U} p_i^{\text{out}} \right| + \left| \sum_{i \in D} p_i^{\text{out}} \right| \right\}$$



Consider an emission from the incoming legs: there are two main mechanisms that make the out-of-plane momentum p_{out} small

- Real radiation suppression \Rightarrow probability $\sim p_{\text{out}}^{-R'_{in}}$
- Transverse momentum cancellation \Rightarrow probability $\sim p_{\text{out}}$

Transverse momentum cancellation dominant for $R'_{in} \geq 1 \Rightarrow$ no LL jet function

\Rightarrow divergence in the function $\mathcal{F}(R')$ for $R'_{in} = 1$

The intuitive position of the divergence should be taken with care: complications may arise due to kinematics

Directly and indirectly global observables

Directly global observables:

Δ as large as possible

● Transverse thrust

$$T_T = \frac{1}{E_T} \max_{\vec{n}_T} \sum_i |\vec{p}_{ti} \cdot \vec{n}_T|$$

● Thrust minor

$$T_m = \frac{1}{E_T} \sum_i |p_i^{out}|$$

Predictions valid as long as

$$|\log v| \lesssim (a + b_\ell) |\Delta|$$

Indirectly global observables:

$\Delta = \mathcal{O}(1) \Rightarrow$ recoil term added

● Transverse thrust

$$T_T = \frac{1}{E_{T,\Delta}} \left(\max_{\vec{n}_T} \sum_{|\eta_i| < \Delta} |\vec{p}_{ti} \cdot \vec{n}_T| - \left| \sum_{|\eta_i| < \Delta} \vec{p}_{ti} \right| \right)$$

● Thrust minor

$$T_m = \frac{1}{E_{T,\Delta}} \left(\sum_{|\eta_i| < \Delta} |p_i^{out}| + \left| \sum_{|\eta_i| < \Delta} \vec{p}_{ti} \right| \right)$$

Predictions valid everywhere but

divergence for $R'_c = 2$

Directly global two-jet rate

For Δ large we can define a directly global jet rate

1. One chooses a particular frame (two-jet c.o.m. frame)

$$\sum_i p_{ti} \eta_i = 0$$

2. For each pair of hadrons and for each combination of particle i and beam direction b one defines

$$y_{ij} \equiv \frac{2 \min\{E_i^2, E_j^2\}}{E_T^2} (1 - \cos \theta_{ij}) \quad y_{ib} \equiv \frac{2E_i^2}{E_T^2} (1 - \cos \theta_{ib})$$

3. One finds $y_{\min} = \min\{y_{ij}, y_{ib}\}$ and if $y_{\min} < y_c$ either recombines particles i and j or include particle i in the beam jet
4. If all $\{y_{ij}, y_{ib}\} > y_c$ one stops, otherwise goes back to 2

We study the 3-jet resolution y_{23} , the maximum value of y_c for which the event is clustered as a 3-jet event

This algorithm is the generalisation of the Durham algorithm in DIS in the Breit frame

Other indirectly global observables



Jet invariant mass: total mass of the **U hemisphere** and the **D hemisphere**

$$\rho = \frac{1}{E_T^2} \left[\left(\sum_{i \in U} p_i \right)^2 + \left(\sum_{i \in D} p_i \right)^2 \right] + \left| \sum_{|\eta_i| < \Delta} \vec{p}_{ti} \right|$$



Two-jet rate: one computes $y_{23,\Delta}$ restricted to particles with $|\eta| < \Delta$ and adds the recoil term

$$y_{23} = y_{23,\Delta} + \left| \sum_{|\eta_i| < \Delta} \vec{p}_{ti} \right|^2$$



The two recoil terms have **different powers** to make the observables **continuously global**

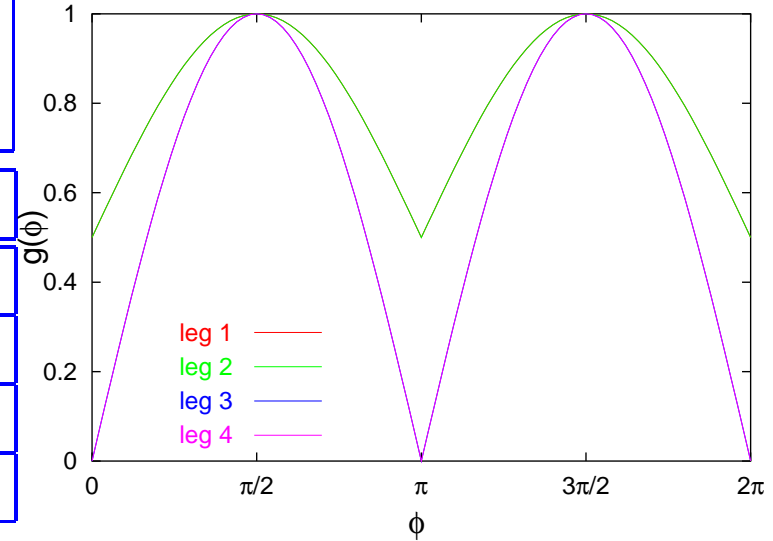
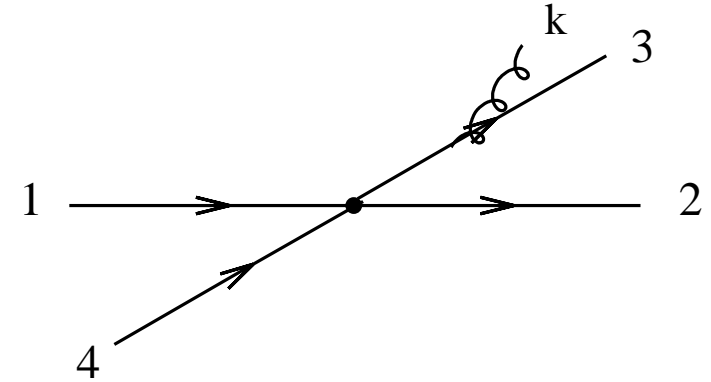
Analysis of the indirectly global thrust minor

Soft/collinear emission from 2+2 partons at an angle $\cos\theta = 0.2$ in their c.o.m. frame

Tables and plots generated automatically by CAESAR, picture generated by AB

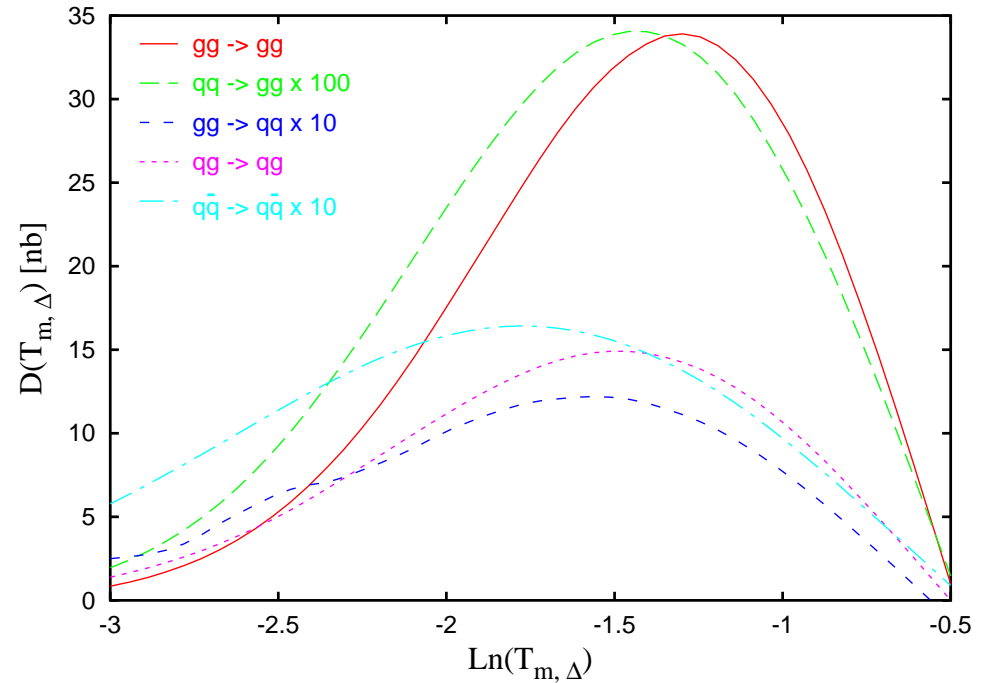
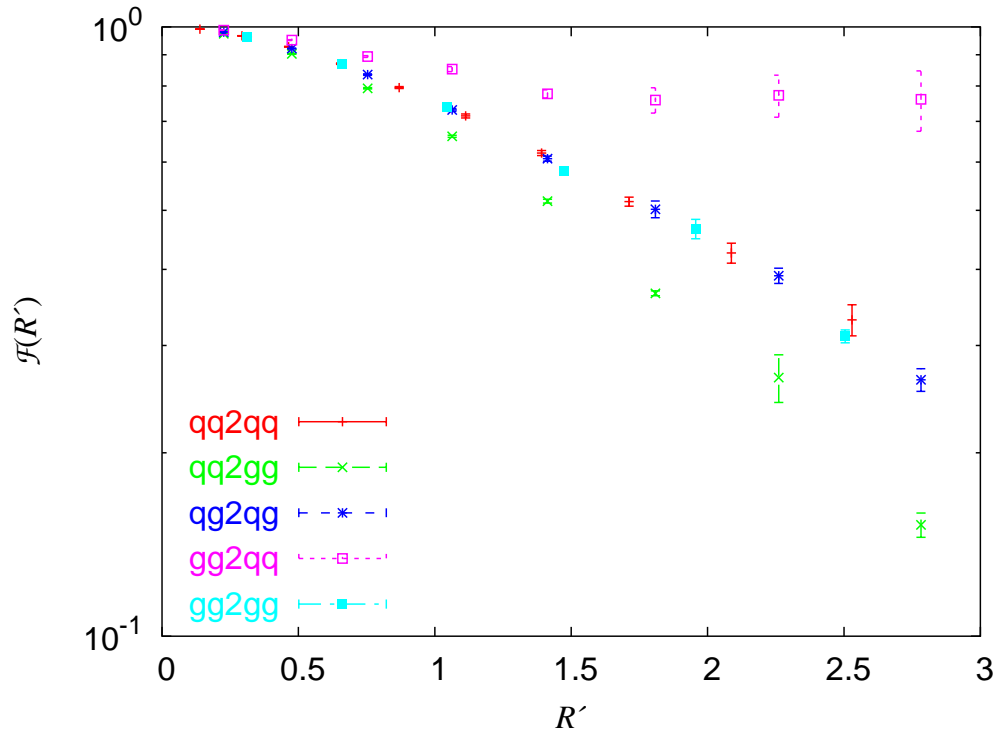
Test	performed	result
check number of jets	YES	T
all legs positive	YES	T
globalness	YES	T
continuously global	YES	T
additivity	YES	F
exponentiation (preliminary)	YES	T
eliminate subleading effects	YES	T
opt. probe region exists	YES	T

leg ℓ	a_ℓ	b_ℓ	$g_\ell(\phi)$	d_ℓ	$\langle \ln g_\ell(\phi) \rangle$
1	1	0	tabulated	2.04124	-0.22005
2	1	0	tabulated	2.04124	-0.22005
3	1	0	$ \sin(\phi) $	2.04124	$-\ln(2)$
4	1	0	$ \sin(\phi) $	2.04124	$-\ln(2)$



Resummation of the indirectly global thrust minor

Dijet events at Tevatron run II $\sqrt{s} = 1.96\text{TeV}$ with $E_T > 50\text{GeV}$ and $\Delta = 1$



☺ The divergences in $\mathcal{F}(R')$ occur at the **left** of the peak
➡ NLL resummation still **sensible**

☺ Clean **separation** of the different incoming channels
➡ useful information for fits of **parton distributions**

Only hadronic collisions? Of course not!



y_3 in DIS with 1+1 jets

Test	result
check number of jets	T
all legs positive	T
globalness	T
continuously global	T
additivity	F
exponentiation (preliminary)	T
eliminate subleading effects	F
opt. probe region exists	F

leg ℓ	a_ℓ	b_ℓ	$g_\ell(\phi)$	d_ℓ
1	2	0	1	0.00063
2	2	0	1	0.00063



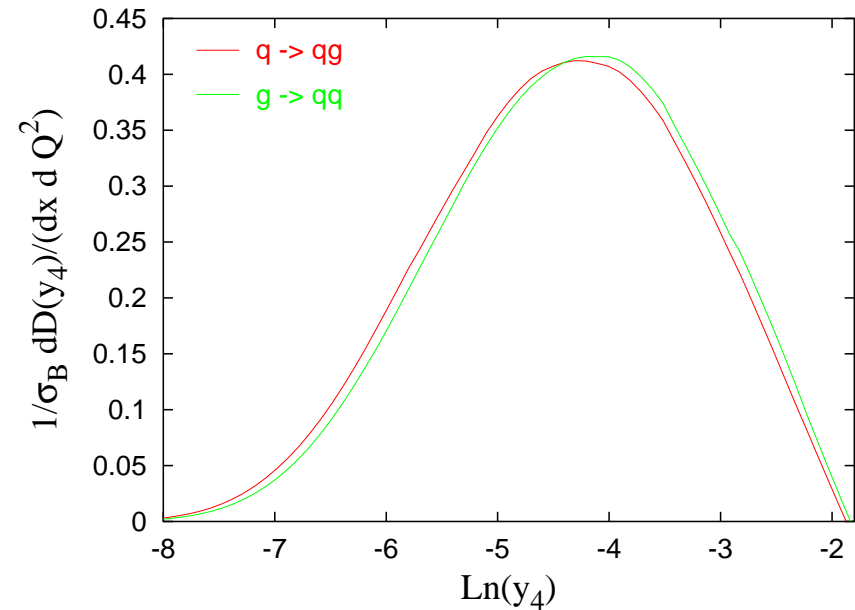
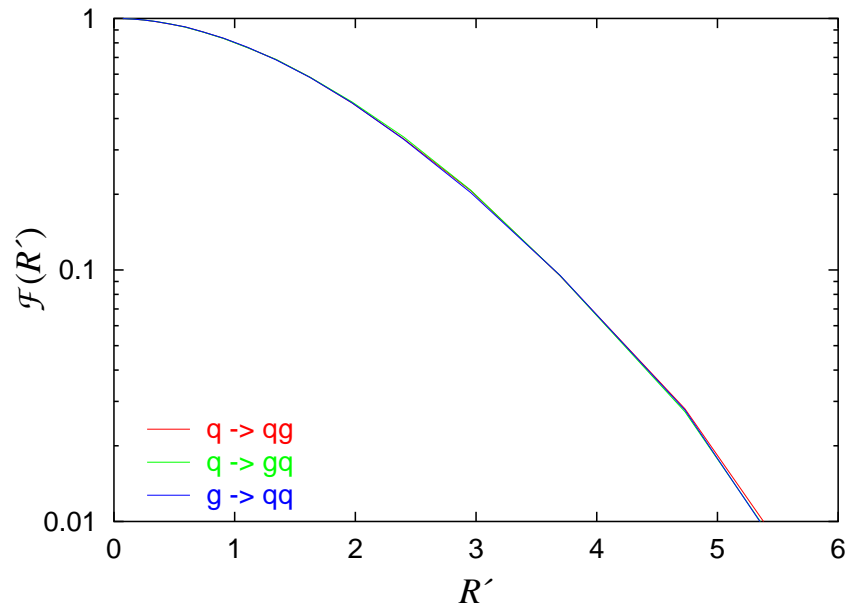
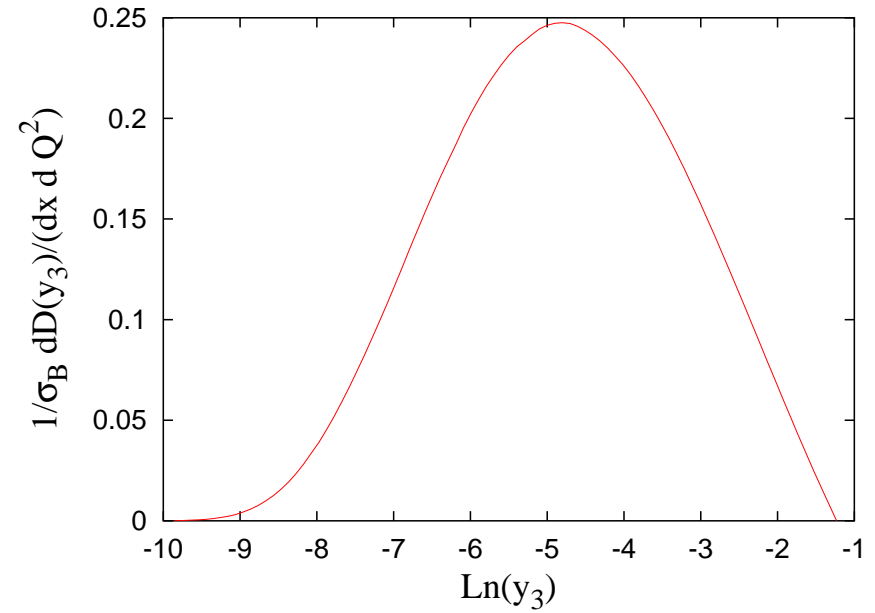
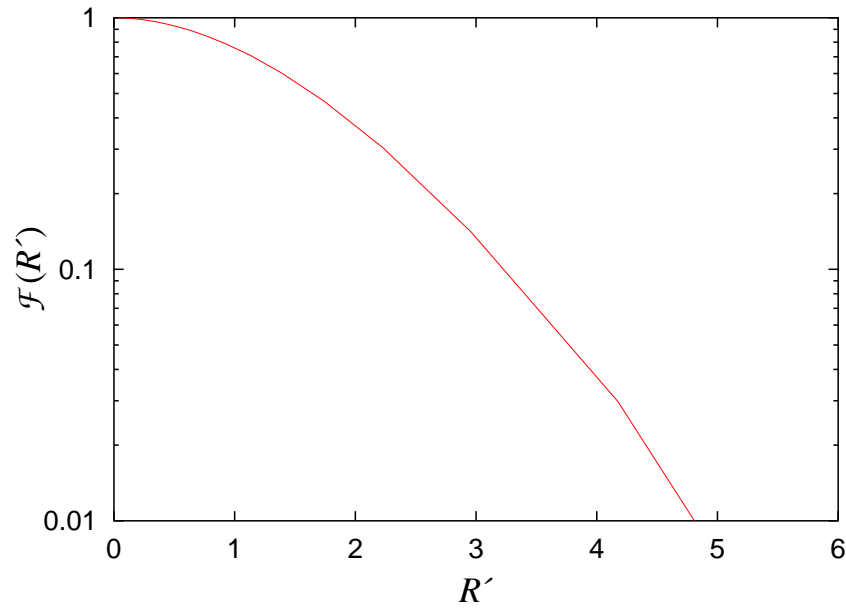
y_4 in DIS with 1+2 jets

Test	result
check number of jets	T
all legs positive	T
globalness	T
continuously global	T
additivity	F
exponentiation (preliminary)	T
eliminate subleading effects	F
opt. probe region exists	F

leg ℓ	a_ℓ	b_ℓ	$g_\ell(\phi)$	d_ℓ
1	2	0	1	0.00063
2	2	0	1	0.00063
3	2	0	1	0.00063

Resummation of jet rates in DIS

DIS jet events at HERA with $Q = 40\text{GeV}$ and $x_B = 0.039$ (dijets with $y_3 > 0.05$)



Conclusions and outlook

- 😊 We have a numerical program which **fully automates** the resummation of '**any**'-jet suitable event shape in an **arbitrary** hard process
- 😊 First-ever prediction for event shapes in **hh collisions**
 - ✌ Soft radiation from four emitters, including **colour interference**
 - ✌ Multiple emission effects computed **generally**
 - ✌ Separation of **different** partonic channels

For the future . . .

- ⚡ Extension to **non-global** observables
- ⚡ Eventually include leading NP corrections (**LAPPSE**)
- ⚡ Comparison with experimental data