γ^*-γ^* Scattering: Saturation and Unitarization

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Goal: Develop pQCD approach for saturation in the γ^*-γ^* scattering.

Advantages:

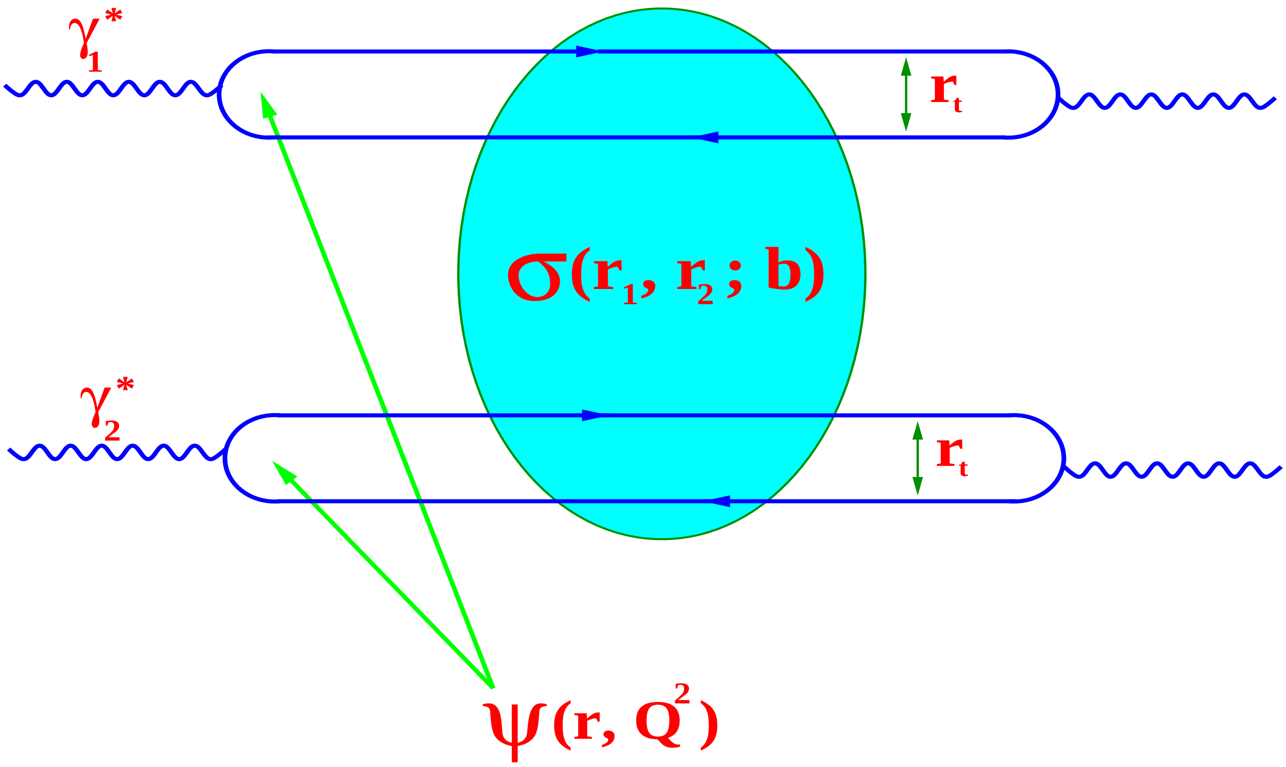
- **Simplicity of the initial state and for large** $Q_1^2\ Q_2^2 \rightarrow$ **pQCD approach;**

- $Q_1^2 \approx Q_2^2 > \mu^2$ **- soft scale** $\rightarrow$ **BFKL dynamics ;**

- $Q_1^2 > Q_2^2 > \mu^2$ **- soft scale** $\rightarrow$ **DGLAP dynamics ;**

- **Screening corrections in pQCD;**

γ^*-γ^* Scattering Picture



Parameterization of the $\sigma_{\gamma^*\gamma^*}$

- $\boldsymbol{\sigma_{\gamma^*\gamma^*} = \int |\Psi_{\gamma_1}|^2 |\Psi_{\gamma_2}|^2 \sigma(dipole + dipole)}$

Characteristic distances and times (Gribov 1970)

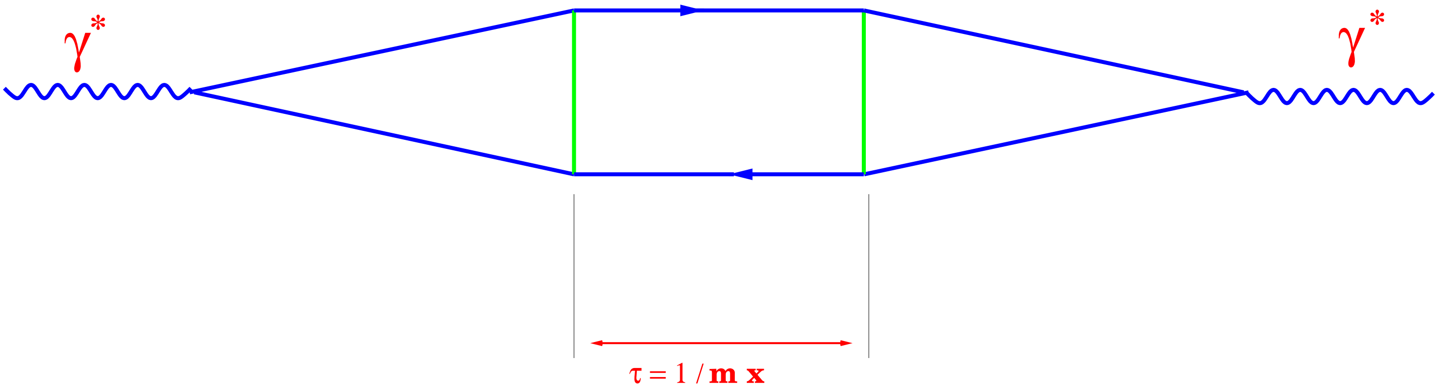


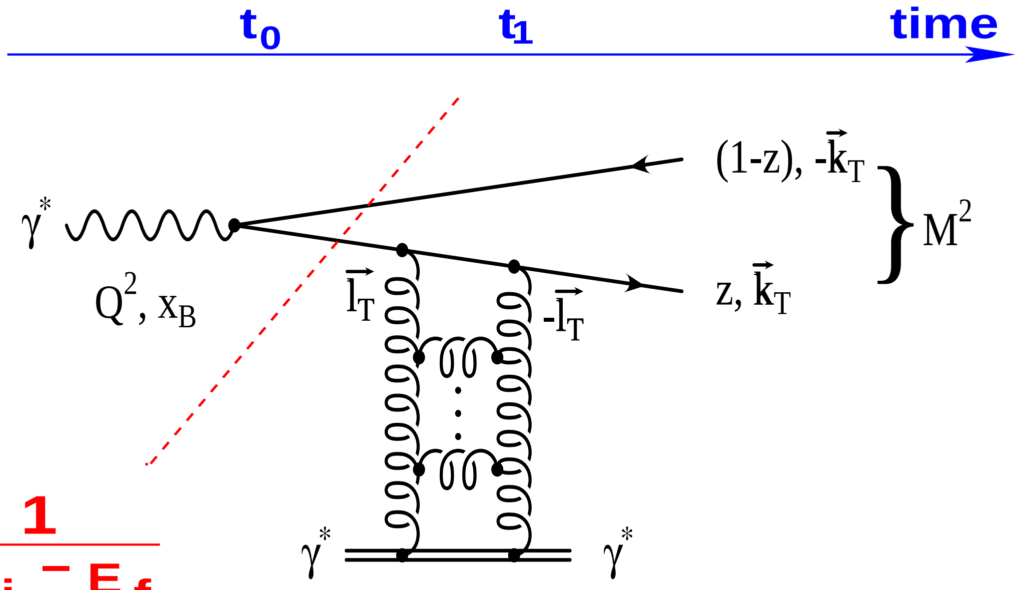
Figure 1: $\gamma^* \longrightarrow$ **hadron system ($q\bar{q}$ - pair)**

- E **is large** $\quad \rightarrow \quad \tau \propto \frac{E}{Q^2} = \frac{1}{2m_q \cdot x_B}$
- Q^2 **is large** $\quad \rightarrow \quad r_\perp \propto \frac{1}{Q^2} \quad \rightarrow \quad$ **pQCD**

Topics:

- **Wave function of the γ^*;**
- **Born $\sigma_{\gamma^*\gamma^*}$ and its b-dependence;**
- **$\sigma(dipole + dipole)$ in BFKL approach;**
- **Revision of Glauber-Mueller formula;**
- **Unitarity bound for $\sigma_{\gamma^*\gamma^*}$;**
- **Numerical estimations for $\sigma_{\gamma^*\gamma^*}$;**

Wave function of the γ^*



$$\Psi \sim \frac{1}{E_i - E_f}$$

Figure 2: *Diffractive production of a quark - antiquark pair.*

It is easier to calculate the photon wave function in the Light Cone (LC) approach.

[S.Brodsky and P.Lepage, 1980]

- Wave function in the coordinate space:

$$|\Psi_T(r_\perp)|^2 = \frac{6\alpha_{em}}{\pi^2}\sum_1^{N_f} Z_f^2[(1-z)^2+z^2]Q^2 z(1-z)K_1^2(Q\sqrt{z(1-z)}r_\perp)$$

Born Cross Section

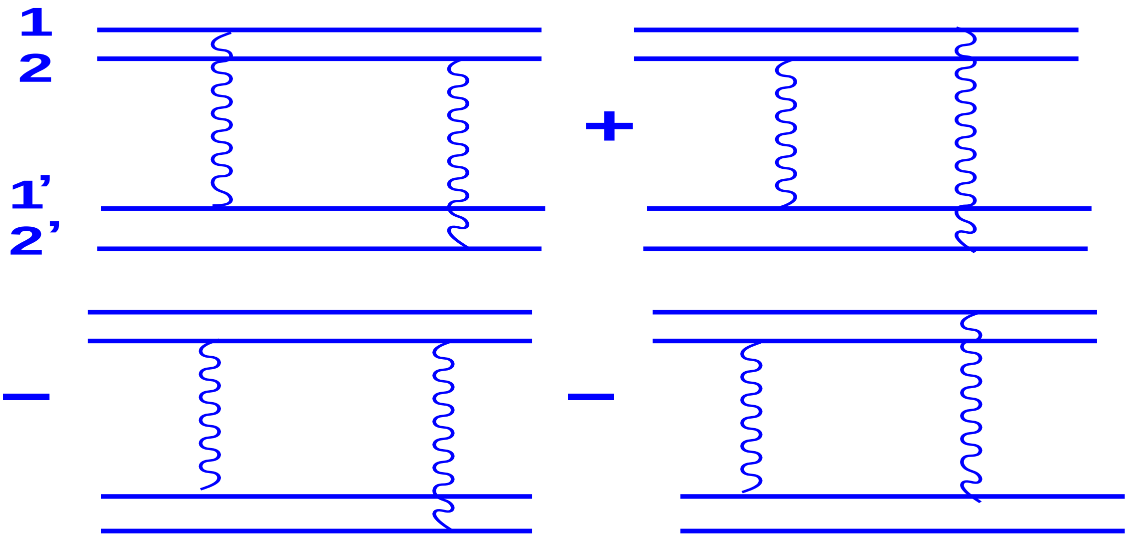


Figure 3: *Dipole-Dipole interaction in the Born approximation.*

- **In notations:** $r_{\perp,i}$; z_i ; $\bar{z}_i = z_i - 1$

$$N^{BA}(r_{\perp,1}, r_{\perp,2}, b_{\perp})$$

$$= \pi\alpha_S^2 \frac{N_C^2-1}{2N_C^2}\left(\ln\frac{(\vec{b}-z_1\vec{r_1}-z_2\vec{r_2})^2(\vec{b}-\bar{z}_1\vec{r_1}-\bar{z}_2\vec{r_2})^2}{(\vec{b}-\bar{z}_1\vec{r_1}-z_2\vec{r_2})^2(\vec{b}-z_1\vec{r_1}-\bar{z}_2\vec{r_2})^2}\right)^2$$

- **In notations:** $\vec{\rho}_i$; $\rho_{i,k} = \vec{\rho}_i - \vec{\rho}_k$

$$N^{BA}(\vec{\rho}_1, \vec{\rho}_2, \vec{\rho}_{1'}, \vec{\rho}_{2'}) = \pi\alpha_S^2 \frac{N_C^2-1}{2N_C^2}\left(\ln\frac{\rho^2_{1,1'}\ \rho^2_{2,2'}}{\rho^2_{1,2'}\ \rho^2_{2,1'}}\right)^2$$

Large $b_\perp$ behavior

- t-channel dispersion relation

$$N^{BA}(r_{1,\perp},\ r_{2,\perp};\ t = -q^2)$$

$$= C(r_{1,\perp}, r_{2,\perp})\ \frac{1}{2a}\ \int_{2m_\pi}^{\infty} \frac{J_1(\kappa\ a)\ \kappa^2\ d\kappa}{\kappa^2 - t}$$

- Where $a = z_2\, \overline{z_2}\, r_{2,\perp}^2$

- For $b_\perp \gg \frac{1}{2\, m_\pi}$

$$N^{BA} \propto e^{-2\, m_\pi\, b_\perp}$$

- For $\frac{1}{2\, m_\pi} > b_\perp > r_{1,\perp}$ and/or $r_{2,\perp}$

$$N^{BA} \propto \frac{1}{b_\perp^4}$$

N^{BA} for Numerical Estimations

- **Born Cross Section:**

$$N^{BA}(r_{\perp,1}, r_{\perp,2}, b_{\perp})$$

$$= \pi\alpha_S^2 \frac{N_C^2 - 1}{2N_C^2} \left(\ln \frac{(\vec{b} - z_1\vec{r_1} - z_2\vec{r_2})^2 (\vec{b} - \bar{z}_1\vec{r_1} - \bar{z}_2\vec{r_2})^2}{(\vec{b} - \bar{z}_1\vec{r_1} - z_2\vec{r_2})^2 (\vec{b} - z_1\vec{r_1} - \bar{z}_2\vec{r_2})^2} \right)^2$$

- **For Large b:** $\quad b_{\perp} \gg r_{2,\perp} \gg r_{1,\perp}$

$$N^{BA}(r_{\perp,1}, r_{\perp,2}, b_{\perp}) \rightarrow \pi\alpha_S^2 \frac{N_c^2 - 1}{N_c^2} \frac{r_{1,t}^2 \, r_{2,t}^2}{b_t^4}$$

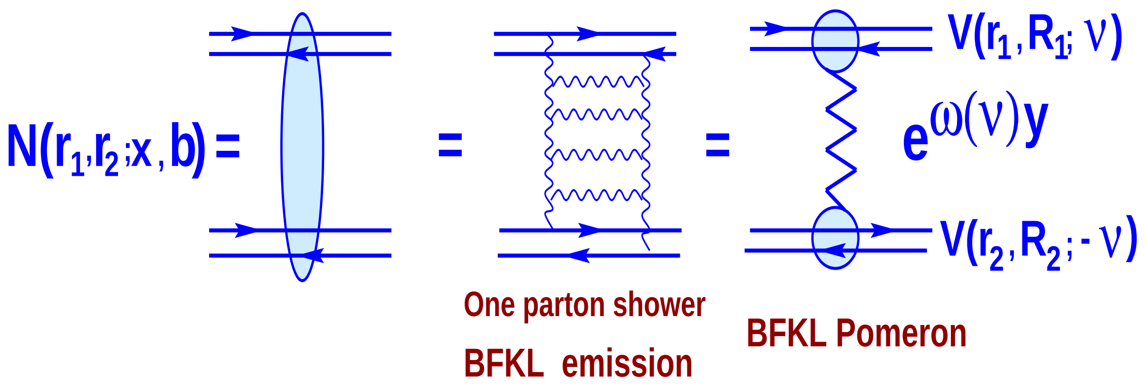
- **For Small b:** $\quad b_{\perp} \ll r_{1,\perp} \ll r_{2,\perp}$

$$N^{BA}(r_{\perp,1}, r_{\perp,2}, b_{\perp}) \rightarrow \pi\alpha_S^2 \frac{N_c^2 - 1}{N_c^2} \frac{r_{1,t}^2 \, r_{2,t}^2}{z_2^2 \bar{z}_2^2 \, r_{2,t}^4}$$

- **For Arbitrary b:**

$$N^{BA}(r_{\perp,1}, r_{\perp,2}, b_{\perp}) = \pi\alpha_S^2 \frac{N_c^2 - 1}{N_c^2} \frac{r_{1,t}^2 \, r_{2,t}^2}{(\, b_t^2 + z_2 \, \bar{z}_2 \, r_{2,t}^2 \,)^2}$$

From the Born amplitude to gluon (BFKL) emission



- **In diffusion approximation ($\nu \ll 1$):**

$$N^{DF}(x, r_{1,\perp}, r_{2,\perp}; b_\perp)$$

$$= \int \frac{d\nu}{2\pi i}\, \phi_{in}(\nu; b_\perp)\; e^{\omega(\nu)\, y} \left(\frac{r^2_{1,\perp}\, r^2_{2,\perp}}{b^4_\perp}\right)^{\frac{1}{2}+i\,\nu}$$

- **Where:**

$$\phi_{in}(\nu; b_\perp)$$

$$= \pi\, \alpha_S\, \frac{N_c^2-1}{3\, N_c^2}\, (m_\pi\, b_\perp)^4\; K_4(2\, m_\pi\, b_\perp)\, \frac{1}{\frac{1}{2} - i\,\nu}$$

- **At small ν:**

$$\omega(\nu) \;=\; \omega_L \;-\; D\,\nu^2$$

- **Where:**

$$\omega_L \;=\; \frac{\alpha_S\,N_c}{\pi}\,4\;\ln 2$$

$$D \;=\; \frac{\alpha_S\,N_c}{\pi}\;14\,\zeta(3)$$

- **The dipole amplitude:**

$$N^{DF}(x, r_{1,\perp}, r_{2,\perp}; b_\perp)$$

$$= \pi\alpha_S \frac{2(N_c^2-1)}{3\;N_c^2}\,(r_{1,\perp}\,r_{2,\perp}\,m_\pi^4\,b_\perp^2)\,K_4(2m_\pi\,b_\perp)$$

$$\cdot\sqrt{\frac{\pi}{D\,y}}\;e^{\omega_L\,y\;-\;\frac{\ln^2\frac{r_{1,\perp}^2\,r_{2,\perp}^2}{b_\perp^4}}{4\,D\,y}}$$

- **For** $\frac{1}{2\,m_\pi} > b_\perp > r_{1,\perp} \approx r_{2,\perp}$

$$N^{DF}(r_{1,\perp}\,,\,r_{2,\perp}\,;\,b_\perp)$$

$$\rightarrow\quad \pi\,\alpha_S\,\frac{2(N_c^2-1)}{3\,N_c^2}\,\frac{r_{1,\perp}\,r_{2,\perp}}{b_\perp^2}\,\sqrt{\frac{\pi}{D\,y}}\,e^{\omega_L\,y\,-\,\frac{\ln^2\frac{r_{1,\perp}^2\,r_{2,\perp}^2}{b_\perp^4}}{4\,D\,y}}$$

- **For** $b_\perp > \frac{1}{2\,m_\pi}$

$$N^{DF}(r_{1,\perp}\,,\,r_{2,\perp}\,;\,b_\perp)$$

$$\rightarrow\quad \pi\,\alpha_S\,\frac{2(N_c^2-1)}{3\,N_c^2}\cdot r_{1,\perp}\cdot r_{2,\perp}\cdot m_\pi^4\cdot b_\perp^2$$

$$\cdot\,\sqrt{\frac{\pi}{2\,m_\pi\,b_\perp}}\sqrt{\frac{\pi}{D\,y}}\,e^{\omega_L\,y\,-\,\frac{\ln^2\frac{r_{1,\perp}^2\,r_{2,\perp}^2}{b_\perp^4}}{4\,D\,y}\,-\,2\,m_\pi\,b_\perp}$$

The Glauber-Mueller formula

- S-channel unitarity constraint for the dipole-target amplitude:

$$2Im\{A_{dip}(x, r_\perp, b_\perp)\} = |A_{dip}(x, r_\perp, b_\perp)|^2 + G_{in}(x, r_\perp, b_\perp)$$

- Where:

$A_{dipole}(x, r_\perp, b_\perp)$ - elastic amplitude

$G_{in}(x, r_\perp, b_\perp)$ - the contribution of all inelastic process

- The solution of this unitary constraint at high energy ($A_{dipole} \approx Im\{A_{dipole}\}$):

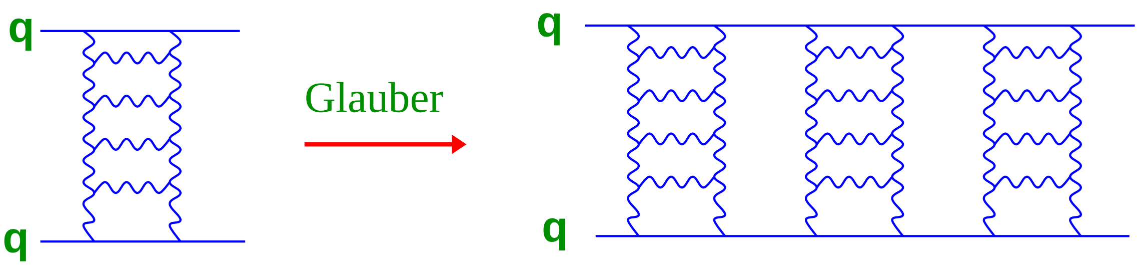
$$A_{dipole}(x, r_\perp, b_\perp) \ = \ i \ \{1 - e^{-\frac{1}{2}\,\Omega(x, r_\perp, b_\perp)}\}$$

$$G_{in}(x, r_\perp, b_\perp) \ = \ \{1 - e^{-\,\Omega(x, r_\perp, b_\perp)}\}$$

- Optical theorem:

$$\sigma^{tot}_{dipole} \ = \ 2 \int d^2 b_\perp \ Im\{A_{dipole}(x, r_\perp, b_\perp)\}$$
$$= \ 2 \int d^2 b_\perp \{1 - e^{-\frac{1}{2}\,\Omega(x, r_\perp, b_\perp)}\}$$

- **The Glauber-Mueller formula:**



$$\sigma_{tot} = 2 \int d^2 b_\perp \left(1 \; - \; e^{-\frac{1}{2}\Omega(r_{1,\perp}\,,\, r_{2,\perp}\,,\, b_\perp)}\right)$$

- **Where:**

$$\Omega(r_{1,\perp}\,,\; r_{2,\perp}\,,\; b_\perp) \;=\; 2 \cdot N^{DF}(r_{1,\perp}\,,\; r_{2,\perp}\,,\; b_\perp)$$

$$N^{DF}(r_{1,\perp}\,,\; r_{2,\perp}\,,\; b_\perp)$$

$$= \pi \alpha_S \frac{2(N_c^2-1)}{3\;N_c^2}\,(r_{1,\perp}\, r_{2,\perp}\, m_\pi^4\, b_\perp^2)\, K_4(2 m_\pi\, b_\perp)$$

$$\cdot \sqrt{\frac{\pi}{D\,y}}\; e^{\omega_L\, y \;-\; \frac{\ln^2 \frac{r_{1,\perp}^2\, r_{2,\perp}^2}{b_\perp^4}}{4\,D\,y}}$$

Unitarity Bound

- **The Optical theorem:**

$$\sigma_{tot} = 2 \int d^2 b_\perp \;\; Im\ A(b_\perp)$$

- **For inelastic interactions:**

$$\begin{aligned} 2 Im A(b_\perp) &= |A(b_\perp)|^2 \; + \; G_{in}(b_\perp) \\ &= (Im\ A(b_\perp))^2 + (Re\ A(b_\perp))^2 + G_{in}(b_\perp) \end{aligned}$$

- **At high energies:**

$$Im\ A(b_\perp) \;\; >> \;\; Re\ A(b_\perp)$$

- **Unitarity Bound:**

$$A(b_\perp) \leq 1$$

Unitarity Bound For $\sigma(dipole - dipole)$

- **The dipole-dipole total cross section:**

$$\sigma^{dd}_{tot} = 2 \int d^2b \, N^{GM}(b)$$
$$= 2 \int d^2b \, \left(1 - e^{-N^{DF}(b)}\right)$$
$$= 2\pi \int_0^{b_0^2} db^2 \, N^{GM}(b) + \int_{b_0^2}^{\infty} db^2 \, N^{GM}(b)$$

- b_0 **is defined from:** $N^{DF}(b_0) = 1$

For $b < b_0$**:** $N^{GM} \leq 1$ **since** $N^{DF} > 1$

For $b \geq b_0$ **and** $N^{DF} < 1$**:** $N^{GM} \leq N^{DF}$

- **Unitarity Bound:**

$$\sigma^{dd}_{tot} \leq 2\,\pi \left\{ b_0^2 + \int_{b_0}^{\infty} db^2 \, N^{GM}(b) \right\}$$

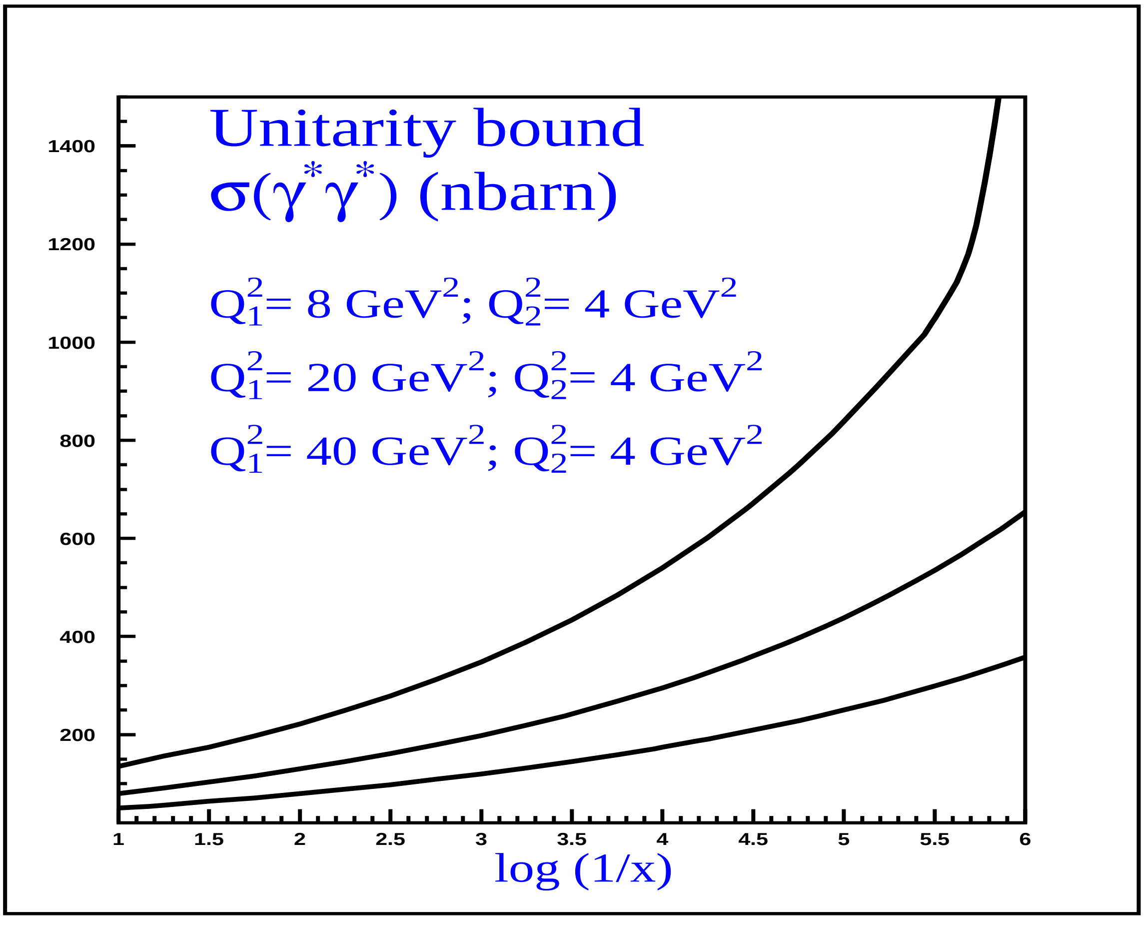
- **For** $\frac{1}{2\, m_\pi} > b_\perp > r_{1,\perp} \approx r_{2,\perp}$

$$b_0^2(x) = r_{1,\perp} \cdot r_{2,\perp} \cdot e^{\,y\,\sqrt{D\,\omega_L}}$$

- **Consequently:**

$$\sigma_{tot} \leq 2\pi b_0^2(x) \cdot \{1 + 1\}$$

$$= 4\pi \cdot r_{1,\perp} \cdot r_{2,\perp} \cdot e^{\,y\,\sqrt{D\,\omega_L}}$$



- **For** $b_{\perp} > \frac{1}{2\, m_{\pi}}$

$$b_0(x) = \frac{\omega_L}{2m_{\pi}}\, y$$

- **Consequently:**

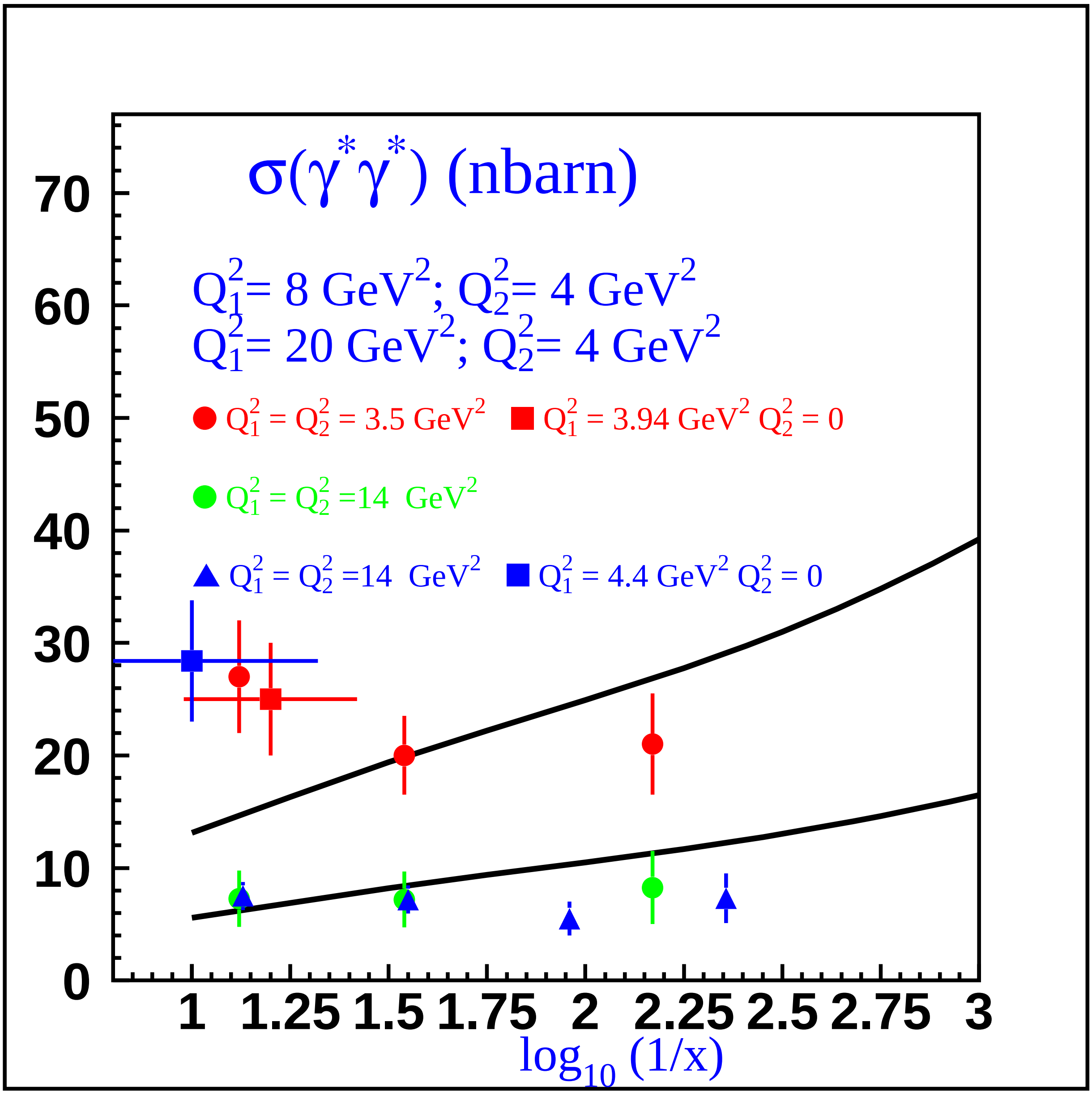
$$\sigma_{tot} \leq 2\pi b_0^2(x) \cdot \{1 + 0\}$$

$$= 2\pi\, \frac{\omega_L^2}{4\, m_{\pi}^2}\, y^2$$

[Levin & Riskin (1990)]

[Ferreiro, Iancy, Itakura & McLerran (2002)]

Numeric Estimations and Experimental Data



Results:

- b-dependence of Born cross section ;
- npQCD have to be introduced for large values of b ;
- Glauber-Mueller formula in pQCD ;
- It is enough to take into account npQCD in Born approximation ;

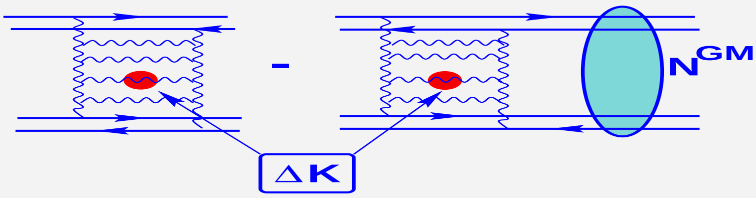


Figure 4: *First order corrections with respect to* $\boldsymbol{\Delta K}$

- $\sigma_{\gamma^*\gamma^*}$ increases with energy $> \log(\frac{1}{x})^2$ for wide range of energies ;
- Numerical estimations for $\sigma_{\gamma^*\gamma^*}$;