Leptoproduction of Vector Mesons at Small x

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1 Introduction

- Vector meson leptoproduction at $x << 1$ in GPD approach.
- Gluon contribution is predominated here.
- LO amplitude $\rightarrow$ cross section comes out to large by order of magnitude.

Brodsky et al '94; Mankiewicz et al '98;....

- transverse degrees of freedom can improve situation.

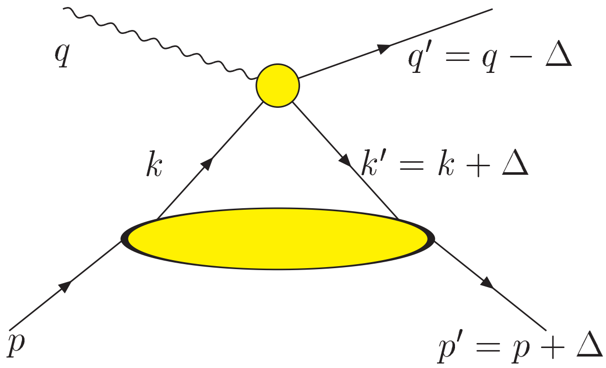
Frankfurt, Koepf, Strikman '95...

- Modified PA
  - transverse degrees of freedom
  -Sudakov suppressions in the subprocess
  -» Quantitative description of light and
  heavy vector meson production.

[1] now in SLAC

2 Leptoproduction of Vector Mesons in GPD approach

The process of VM production is shown in figure



The gluon contribution to the helicity amplitudes for leptoproduction of longitudinally polarized vector mesons looks as

$$\begin{aligned}
\mathcal{M}^{V(g)}_{0+,0+} &= e \sum_a e_a B^V_a \sqrt{1-\zeta} \int_0^1 \frac{dX}{(X - i\hat{\varepsilon})(X - \zeta + i\hat{\varepsilon})} \\
&\times \left\{ \mathcal{H}^{(g)}_{0+,0+} \mathcal{F}^g_\zeta(X,t) \right\} \qquad (1)
\end{aligned}$$

For the ρ^0 production $\Sigma_a e_a\ B^{\rho^0}_a = 1/\sqrt{2}$.
The parton-level amplitudes have the form

$$\mathcal{H}^{V(g)}_{0+,0+} = \frac{2\pi\alpha_s(Q/2) f_V}{N_c} \int_0^1 d\tau\, \Phi_V(\tau)\ f_0(g)\,. \qquad (2)$$

and

$$f^{(g)}_0 = \frac{2}{Q} \frac{1}{\tau\bar{\tau}}\,. \qquad (3)$$

We use double distributions for GPDs and take a factorising ansatz for the double distributions $f(x, y, t)$ which define the SPDs $\mathcal{F}_\zeta(X)$ by

Radyushkin '99

$$\mathcal{F}^a_\zeta(X, t) = \int_0^1 dx \int_0^{1-x} dy\, f^a(x, y, t)\, \delta(X - x - \zeta\, y)\,, \quad (4)$$

where

$$f(x, y, t) = xg(x) \cdot h(x, y)/h(x) \cdot f(t)\,. \quad (5)$$

We use

- $h(x, y) = 6y(1 - x - y)$ for the construction of the double distribution integral
- $h(x) \equiv \int_0^{(1-x)} dy\, h(x, y)$
- $f(t)$ determines t -dependence of GPD

Small x region.

We use simple form of the gluon parton distribution

$$xg(x, Q^2) \simeq 1.94 x^{\lambda(Q^2)}\,. \quad (6)$$

In the region of about $0.001 \leq x \leq 0.1$ the Q^2 dependence of λ is approximatively

Martin et al '96

$$\lambda(Q^2) = 0.17 + 0.05 \ln(Q^2/Q_0^2)\,. \quad (7)$$

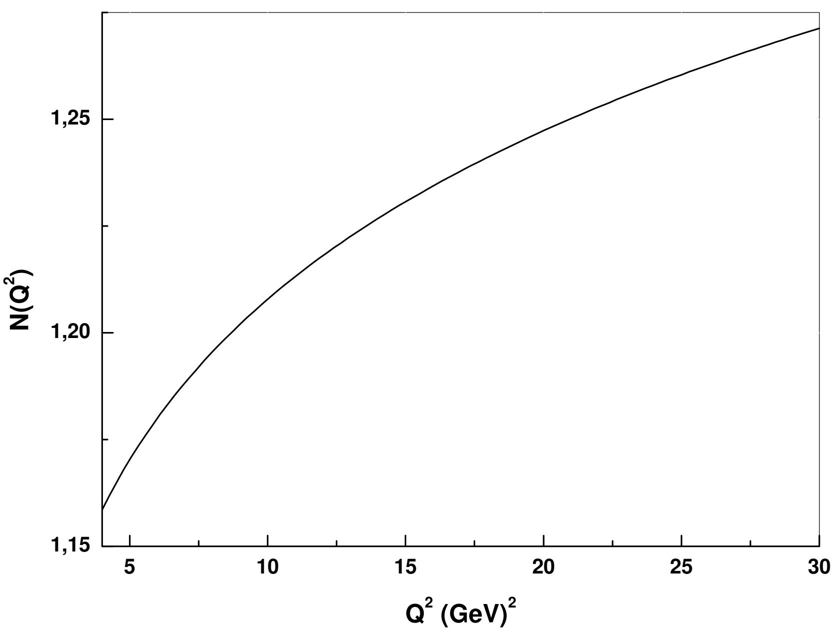
Direct calculation of the integral (4) for $X = \zeta$ gives at small x

$$F_\zeta(\zeta, t) = N(Q^2)(\zeta g(\zeta)) f(t), \tag{8}$$

where

$$N(Q^2) = \frac{6}{(3 - \lambda(Q^2))(2 - \lambda(Q^2))} \tag{9}$$

Similar to Martin and Riskin estimations '98



σ_L represents the integrated differential cross section assumed to behave as $d\sigma/dt = d\sigma/dt(t = t_{min})\, \mathrm{e}^{-B_V\, t}$ with $t_{min} = -4m^2\, \xi^2/(1-\xi^2) \sim 0$. Thus,

$$\begin{aligned} \sigma_L &= \int_{t_{min}}^{t_{max}} \frac{d\sigma}{dt} dt \\ &= \frac{1}{B_V} \frac{d\sigma}{dt}(t_{min})\, e^{-B_V t_{min}} \left(1 - e^{-B_V (t_{max} - t_{min})}\right). \end{aligned} \tag{10}$$

Here B_V is a diffraction peak slope. We use

Martin, Riskin ... '99

$$B_\rho = \frac{4}{0.71\ \mathrm{GeV}^2} + \frac{2}{Q^2+m_\rho^2} + 2\ \cdot\ 0.15\ \mathrm{GeV}^{-2} \ln\left(\frac{s\, m_\rho^2}{(Q^2+m_\rho^2)^2}\right) \tag{11}$$

For small $2\xi \simeq x_{Bj} = Q^2/(s+Q^2) \simeq Q^2/s \ll 1$ we find

$$\sigma_L = \frac{4\pi^4 \alpha_{\mathrm{elm}} \alpha_s^2 f_V^2}{B_V N_C^2 Q^6} \langle 1/\tau\rangle_V^2 \left[\sum e_a B_a^V\right]^2 \left|\xi g(\xi, Q^2)\right|^2 . \tag{12}$$

- This the result coincides with previous calculations.

Brodsky et al '94;....

- Far from experiment about 10 times larger.
- Can be improved by including of transverse degrees of freedom.

Frankfurt, Koepf, Strikman '95....

3 Modified PA to meson leptoproduction

We use so-called modified perturbative approach to meson leptoproduction in order to suppress the contributions from the soft end-point regions which give not small contribution.

Li, Sterman '92; Jakob, Kroll '93...

- Consider transverse momentum by a soft valence Fock state light-cone wave function

Kroll, Raulfs '96

$$\Psi_V(\tau,\mathbf{l}_\perp) \;=\; 8\pi^2\sqrt{2N_c}\, f_V\, a_V^2\, \exp\left[-a_V^2\,\frac{\mathbf{l}_\perp^2}{\tau\bar{\tau}}\right]. \qquad (13)$$

Transverse momentum integration of (13) leads back to the associated distribution amplitude $\Phi_V^{AS} = 6\tau\bar{\tau}$, the asymptotic form of a meson distribution amplitude

- Consider transverse momentum and mass terms in propagators. For example, the modification for one of the propagator looks like:

$$\frac{1}{\tau Q^2} \to \frac{1}{\tau Q^2 + \tau\bar{\tau}M_V^2 + m_Q^2 + k^2} \qquad (14)$$

After integration over τ we find for $m_Q = M_V/2$ that the amplitude approximately depends on the $Q^2 + M_V^2$ variable.

- Consider Sudakov suppression of large quark-antiquark separations. These effects suppress contributions from the end-point regions, in which one of the partons entering the meson wave function becomes soft and where factorization breaks down.

We work in the transverse separation space since the Sudakov factor only exponentiates there.

The final result of the gluon contributions to the helicity amplitudes for meson leptoproduction read

$$\mathcal{M}^{V(g)}_{0+,0+} = -\frac{e}{2\sqrt{2N_c}} \sum_a e_a\, B^V_a \sqrt{1-\xi^2}\,\frac{Q^2}{\xi^2}\, \int dX\, d\tau\, (\tau\bar{\tau})^2 f^{(g)}_0$$

$$F^g_\zeta(X,t) \int d^2\mathbf{b}\, \hat{\Psi}(\tau,-\mathbf{b})\, \hat{T}_g(\tau,Q,\mathbf{b})\, \alpha_s(\mu_R)\, \exp\left[-S(\tau,b,Q)\right] \quad (15)$$

The Sudakov exponent S in (15), comprising those gluonic radiative corrections not taken into account in the evolution of the wave function, is given by

$$S(\tau,b,Q) = s(\tau,b,Q) + s(\bar{\tau},b,Q) - \frac{4}{\beta_0} \ln \frac{\ln(\mu_R/\Lambda_{QCD})}{\ln(1/b\,\Lambda_{QCD})}, \quad (16)$$

where a Sudakov function s appears for each quark line entering the meson.

- μ_R is taken to be the largest mass scale appearing in the hard scattering amplitude, i.e. $\mu_R = \max(\tau Q, \bar{\tau}Q, 1/b)$.
- For small b there is no suppression from the Sudakov factor; as b increases the Sudakov factor decreases, reaching zero at $b = 1/\Lambda_{QCD}$. For $b > 1/\Lambda_{QCD}$ the Sudakov is set to zero.
-
  $$s(\beta,b,Q) = \frac{8}{3\beta_0}\left(\hat{q}\ln\left(\frac{\hat{q}}{\hat{b}}\right) - \hat{q} + \hat{b}\right) + \mathrm{NLL-terms} \quad (17)$$

  where

  $$\hat{q} = \ln\left(\beta Q/(\sqrt{2}\Lambda_{\mathrm{QCD}})\right), \quad \hat{b} = \ln\left(1/(b\Lambda_{\mathrm{QCD}})\right). \quad (18)$$

Botts, Sterman '1989

Estimations for the longitudinal cross sections in the modified perturbative approach for the the production of ρ mesons are obtained using the parameters $\Lambda_{QCD} = 0.22\,\mathrm{GeV}$ (3-loop), $a_\rho = 0.6\,\mathrm{GeV}^{-1}$. Calculations of Im and Re parts of amplitudes was done.

- The ratio real/imaginary part is approximately 0.3 for the amplitudes (i.e. 0.1 for the cross section).

$$
\begin{aligned}
Im(M) &\propto (xg(x)); \\
Re(M) &\propto -\frac{\pi}{2}\,\frac{d(xg(x))}{d(ln(x))}
\end{aligned}
\tag{19}
$$

This agrees with Frankfurt, Koepf, Strikman '95, Ryskin et al '97

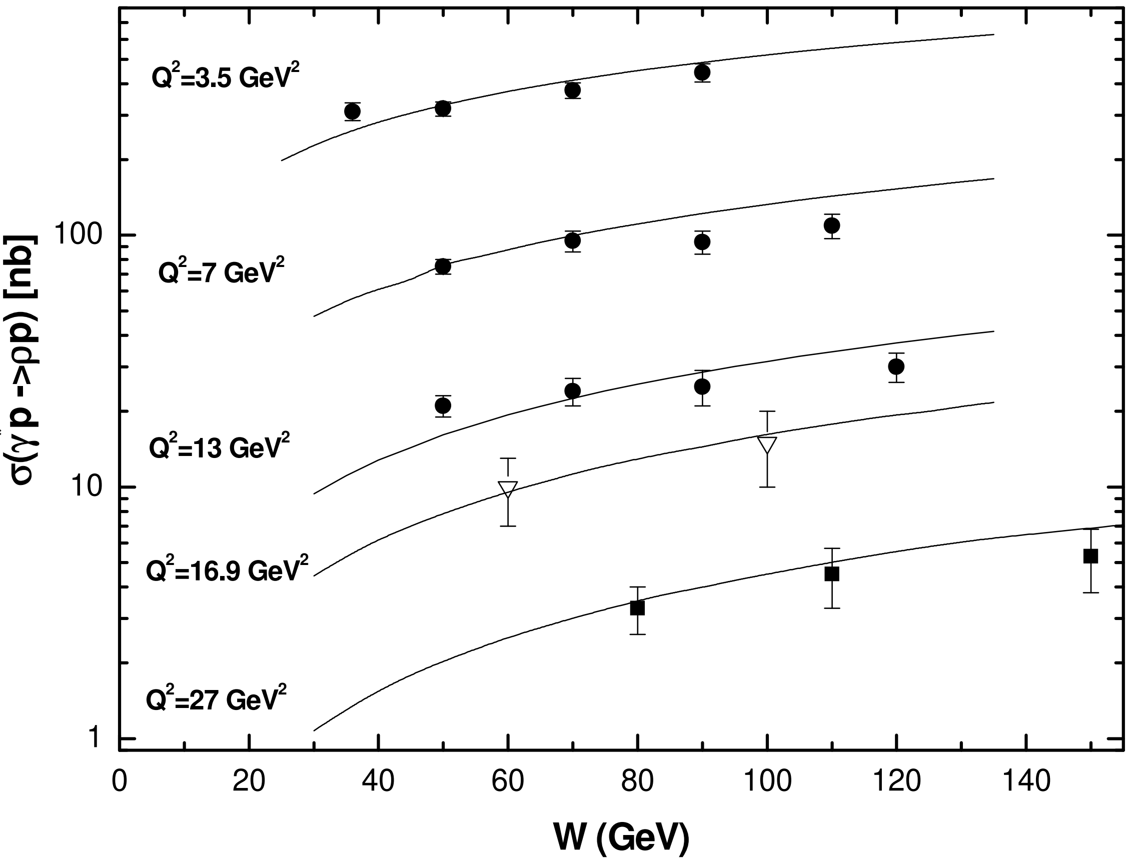


Figure 1: The cross section for $\gamma_L^* p \to \rho_L p$ vs. $W = \sqrt{s}$ (in GeV) for fixed values of Q^2. ZEUS95 data

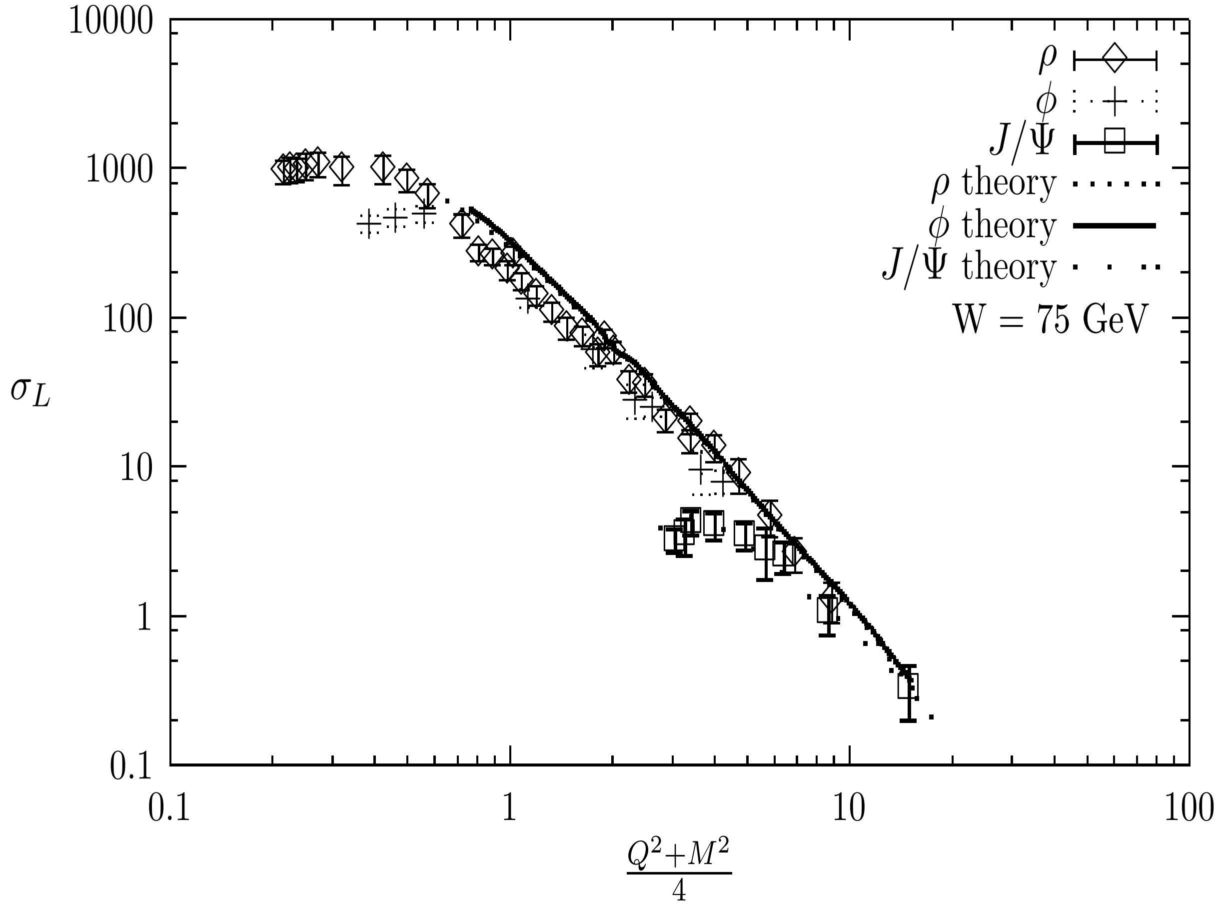


Figure 2: The cross section for ρ, ϕ, J/Ψ production vs. $\frac{Q^2+M_V^2}{4}$ variable. (B.Clerbaux graph)

4 Conclusion

- Modified PA which consider - transverse degrees of freedom and Sudakov suppressions in the subprocess give quantitative fit of σ_L cross section for ρ production
- good description of ϕ and J/Ψ meson production. For heavy meson nonrelativistic wave function is used.
- Sudakov factors & transverse motion effectively suppress suppress contributions from the end-point regions, where factorization breaks down.
- can regularize propely the infrared divergencies in the $\gamma_T^* \to V_L$ and $\gamma_T^* \to V_T$ amplitudes???