Gluon Evolution at small-x:
Extending the PT Domain of QCD

DIMITRI COLFERAI

University of Firenze

In collaboration with:
- M. Ciafaloni
- G.P. Salam
- A.M. Stasto

Outline

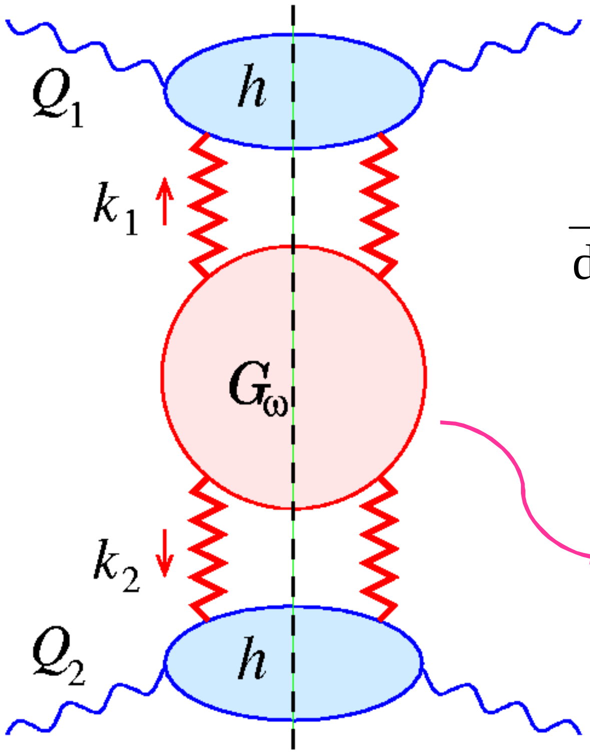
Definition of the RGI approach

Green's function

Splitting function

Extension of PT domain

BFKL vs DGLAP picture

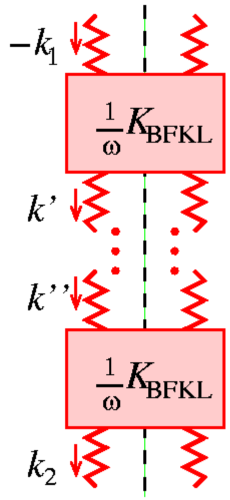


$$s >> Q_1, Q_2 \approx k_1, k_2$$

$$\frac{d\sigma}{dQ_1^2 dQ_1^2} = \int \frac{d^2 k_1}{k_1^2} \frac{d^2 k_2}{k_2^2} \frac{d\omega}{2\pi i} \left(\frac{s}{Q_1 Q_2} \right)^{\omega} h(Q_1, k_1)\, G_\omega(k_1, k_2)\, h(Q_2, k_2)$$

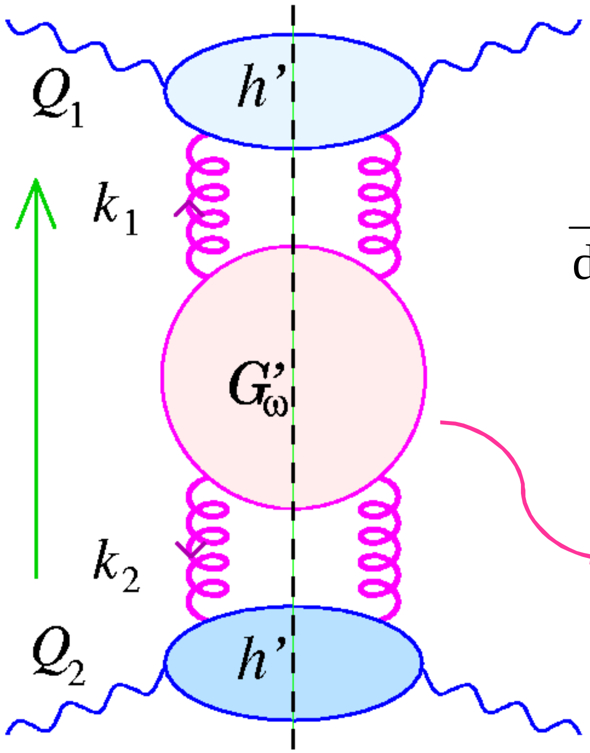
s_0

$$= \sum_{n=0}$$



BFKL vs DGLAP picture

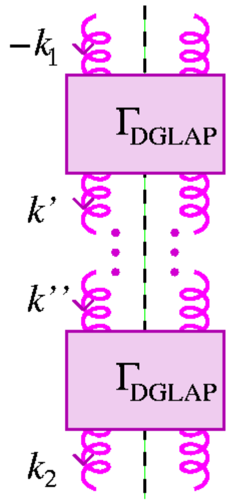
$$s \approx Q_1 >> k_1 >> k_2 >> Q_2$$



$$\frac{d\sigma}{dQ_1^2 dQ_1^2} = \int \frac{d^2k_1}{k_1^2}\frac{d^2k_2}{k_2^2}\frac{d\omega}{2\pi i}\left(\frac{s}{Q_1^2}\right)^{\omega} h'(Q_1,k_1)\, G'_\omega(k_1,k_2)\, h'(Q_2,k_2)$$

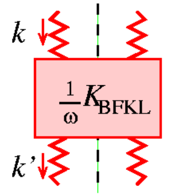
s_0

$$= \sum_{n=0}$$



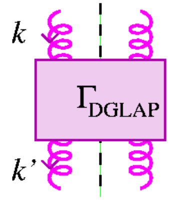
BFKL vs DGLAP kernel in the collinear limit $k >> k'$

Simplified case: $N_f = 0 \;\Rightarrow\; \gamma^{(+)} = \gamma_{gg} = \frac{1}{\omega} + A(\omega)\,;\;\; \alpha_s$ fixed $\;\Rightarrow\; K(k,k') \xleftrightarrow{\left(\frac{k'^2}{k^2}\right)^{\gamma}} \chi(\gamma)$



$s_0 = Q_1 Q_2 \Longrightarrow$

$$\frac{1}{\omega}K(\gamma) = \frac{\alpha_s}{\omega}\left[\chi_0(\gamma) + \alpha_s \chi_1(\gamma)\right] \xrightarrow[\gamma\simeq 0]{k>>k'} \frac{\alpha_s}{\omega}\left[\frac{1}{\gamma} + \alpha_s\left(-\frac{1}{2\gamma^3} + \frac{A(0)}{\gamma^2}\right) + \text{reg}\right]$$



$s_0 = Q_1^2 \Longrightarrow$

$$\Gamma'(\gamma) = \alpha_s \gamma_{gg}(\omega)\frac{1}{\gamma} = \frac{\alpha_s}{\omega}\left[\frac{1+\omega A(\omega)}{\gamma}\right]$$

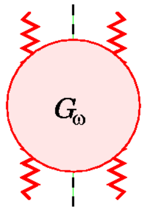
Scale change: $s_0 = Q_1^2 \to Q_1 Q_2$

$h' \to h \qquad G' \to G \qquad h' \to h$

$$\left(\frac{s}{Q_1^2}\right)^{\omega} = \left(\frac{s}{Q_1 Q_2}\right)^{\omega}\left(\frac{k_1}{Q_1}\right)^{\omega}\left(\frac{k_2}{k_1}\right)^{\omega}\left(\frac{Q_2}{k_2}\right)^{\omega}$$

$$\Gamma(k,k') = \left(\frac{k'^2}{k^2}\right)^{\omega/2}\Gamma'(k,k') \;\Rightarrow\; \Gamma(\gamma) = \Gamma'(\gamma + \omega/2) = \frac{\alpha_s}{\omega}\,\frac{1+\omega A(\omega)}{\gamma + \omega/2}$$

$$= \frac{\alpha_s}{\omega}\left[\frac{1}{\gamma} + \omega\left(-\frac{1}{2\gamma^2} + \frac{A(0)}{\gamma}\right) + O(\omega^2)\right] \approx \frac{1}{\omega}K(\gamma)$$



LLA $\Longrightarrow$

$$G_\omega(\gamma) \xrightarrow{\gamma\simeq 0} \sum_{n=0}\left(\frac{\alpha_s}{\omega}\frac{1}{\gamma}\right)^n = \left(1 - \frac{\alpha_s}{\omega\gamma}\right)^{-1} \;\Rightarrow\; \omega \approx \frac{\alpha_s}{\gamma}$$

What have we learnt?

- Position of collinear γ-poles of evolution kernels depends on s_0

$$\begin{array}{lll} s_0 = Q_1^2 & \gamma = 0 & \gamma = 1+\omega \\ s_0 = Q_1 Q_2 & \gamma = -\omega/2 & \gamma = 1+\omega/2 \\ s_0 = Q_2^2 & \gamma = -\omega & \gamma = 1 \end{array}$$

- Origin of cubic poles in $\chi_1(\gamma)$:

  expansion around $\omega = 0$ of shifted poles ~$1/(\gamma + \omega/2)$

- Non-singular part $A(\omega)$ of anomalous dimension $\gamma_{gg}(\omega) = 1/\omega + A(\omega)$ can be taken into account by a subleading kernel in 2 ways:

$$\alpha_s\text{-correction:} \qquad \alpha_s \chi_1 \approx \alpha_s \frac{A(\omega)}{\gamma^2} \qquad \text{(double pole)}$$

$$\omega\text{-correction:} \qquad \omega\, \chi_{\text{coll}} \approx \omega \frac{A(\omega)}{\gamma} \qquad \text{(single pole)}$$

(remember $\omega \approx \frac{\alpha_s}{\gamma}$)

Definition of RG Improved Approach

Adapting the procedure for running α_s and with $s_0 = Q_1 Q_2$:

$$K_\omega(k,k') \equiv \alpha_s(\vec{k}-\vec{k}')K_\omega^{(0)}(k,k') + \omega\alpha_s(k_>)K_\omega^{(\mathrm{coll})}(k,k') + \alpha_s^2(k_>)\tilde{K}_\omega^{(1)}(k,k')$$

$$\chi_\omega^{(0)}(\gamma) = 2\psi(1) - \psi(\gamma + \omega/2) - \psi(1+\omega/2-\gamma)$$

resums scale dependent terms of a pure BFKL approach

$$\tilde{\chi}_\omega^{(1)}(\gamma) = [\chi_1(\gamma) - \text{subtractions}] + \text{shift of } \gamma\text{-poles (NNL)}$$

includes left-over exact NL BFKL infos

$$\chi_\omega^{(\mathrm{coll})}(\gamma) = A(\omega)\left[\frac{1}{\gamma+\omega/2} + \frac{1}{1+\omega/2-\gamma}\right]$$

resums exact 1-loop gluon anomalous dimension (note ω-dependence of A)

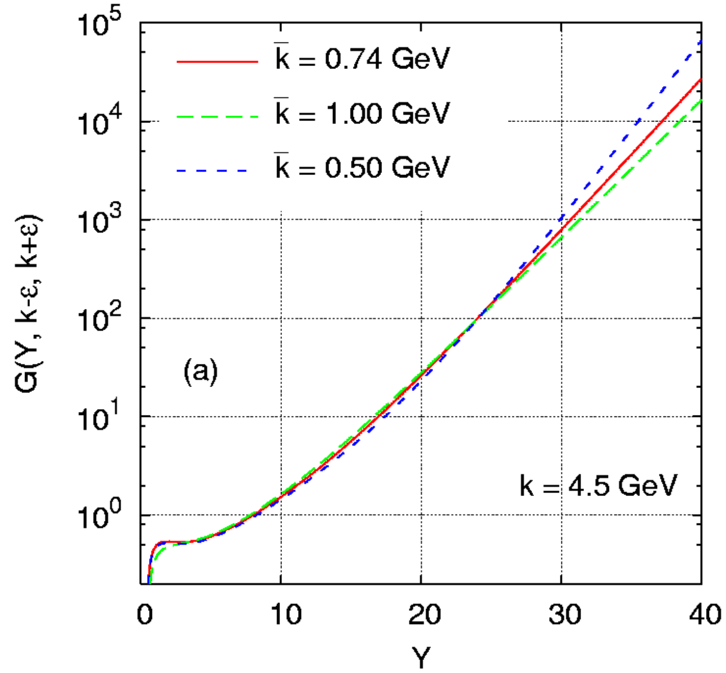
This procedure implements kinematical constraints

$$\frac{1}{\omega}K_\omega(\gamma) \approx \frac{\alpha_s}{\omega}[1+\omega A(\omega)]\left(\frac{1}{\gamma+\omega/2} + \frac{1}{1+\omega/2-\gamma}\right) \Rightarrow \frac{1}{\omega}K_\omega(k,k') \approx \frac{\alpha_s(k_>)}{k_>^2}\gamma_{gg}(\omega)\left(\frac{k_<}{k_>}\right)^\omega$$

$$\Rightarrow K(z;k,k') \approx \frac{\alpha_s(k_>)}{k_>^2}P_{gg}\left(z\frac{k_>}{k_<}\right) \Rightarrow \boxed{z\,k < k' < \frac{k}{z}}$$

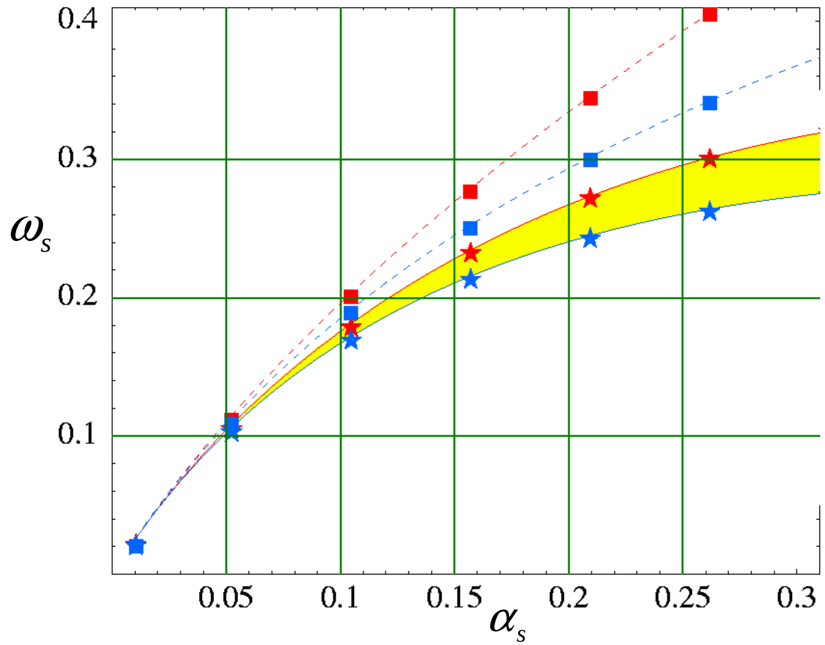
Y-evolution of Green's function

$$G(Y;k,k) \approx \frac{1}{\sqrt{Y}} \exp\left[\omega_s(\alpha_s(k))Y + \Delta(\alpha_s Y)\right]$$



Diffusion corrections:

$$\Delta(\alpha_s Y) = \frac{Y^3}{24}\left[\frac{\partial}{\partial \log k^2}\,\omega_s(\alpha_s(k))\right]^2 \chi''_{\rm eff}\left(\tfrac{1}{2}\right)$$



Hard Pomeron

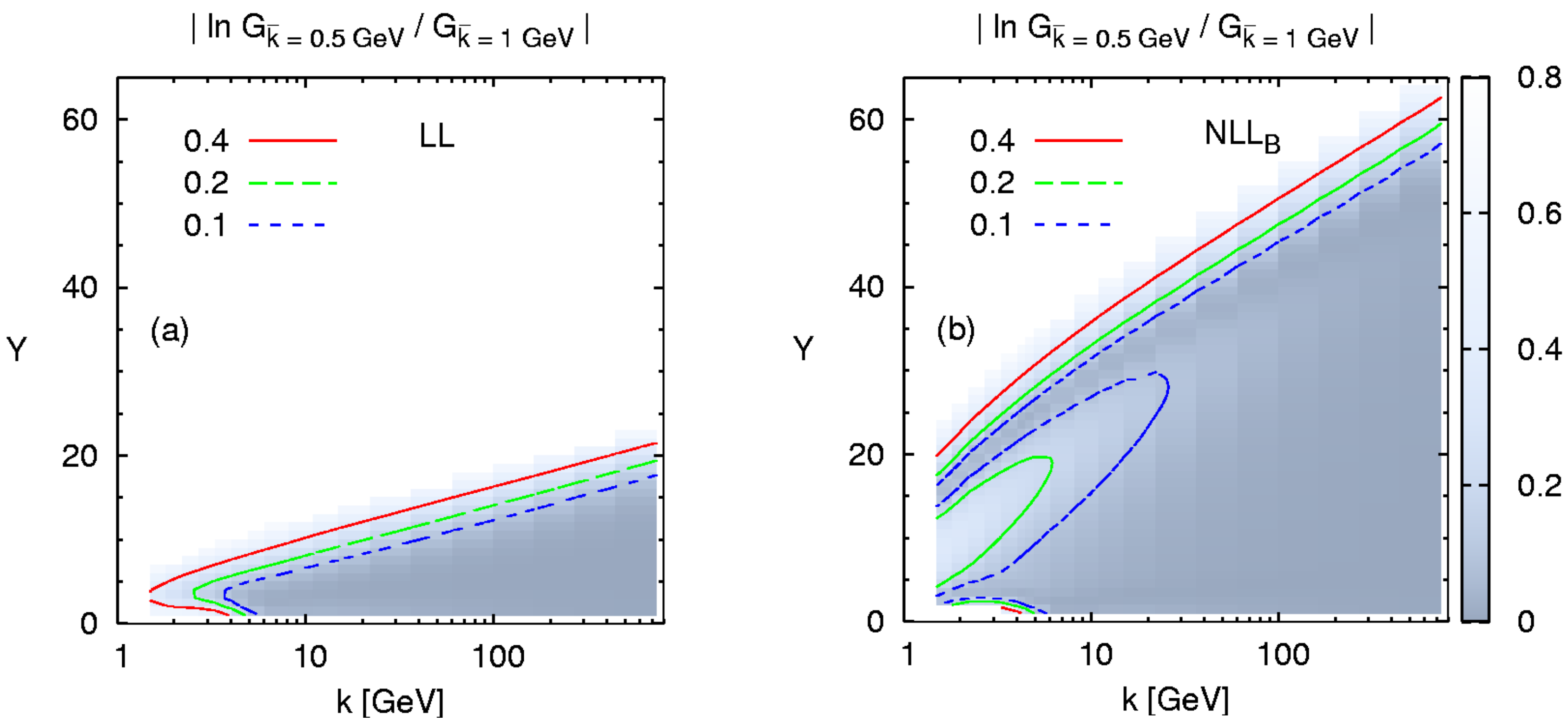
	Scheme A	Scheme B
fixed α_s	dashed	dashed
running α_s	solid	solid
numeric	■ ★	■ ★

Domain of applicability of perturbative QCD

$$Y < Y_c(k) \;\Rightarrow\; G(Y;k,k) \approx \exp\left[\omega_s\left(\alpha_s(k)\right)Y + \cdots\right]$$

$$Y > Y_c(k) \;\Rightarrow\; G(Y;k,k) \approx \left(\frac{\bar{k}^2}{k^2}\right)^{1+\omega_P} \exp\left[\omega_P\, Y\right]$$

$$Y_c(k) \approx \frac{(1+\omega_P)\log(k^2/\bar{k}^2)}{\omega_P - \omega_s\left(\alpha_s(k)\right)}$$

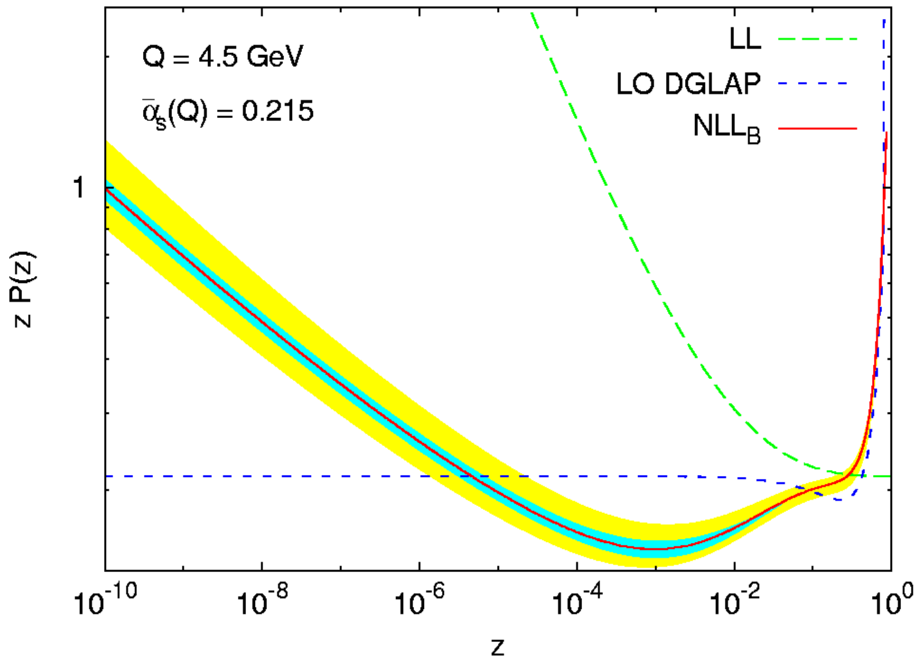


Resummed splitting function

Gluon density $$x\,g(x,Q^2) \equiv \int^{\log Q^2} \mathrm{d}t\; G(\log 1/x\,;k,k_0)$$

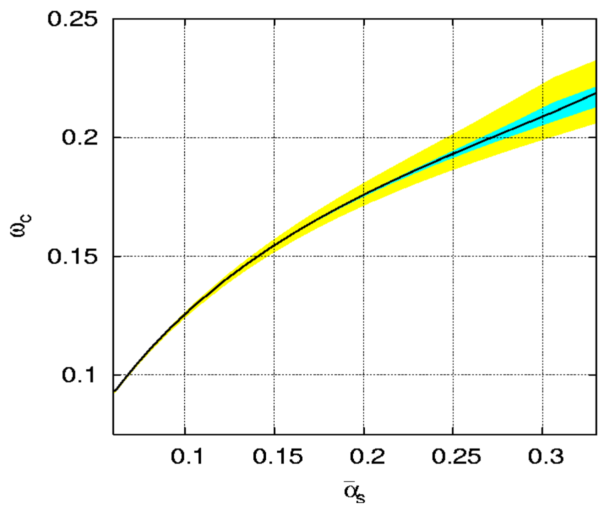
Evolution eqn. $$\frac{\partial g(x,Q^2)}{\partial \log Q^2} \equiv \int_x^1 \frac{\mathrm{d}z}{z}\; P_{\mathrm{res}}(z,Q^2)\, g\left(\frac{x}{z},Q^2\right)$$

P_{eff} is independent of k_0: important check of RG factorization



High-energy exponent $\omega_c(\alpha_s)$

$$P_{\mathrm{res}}(z,Q^2) \approx z^{-\omega_c(\alpha_s(Q))}$$



Conclusions

- 2 main problems of high-energy perturbative QCD:
  - occurrence of large leading log s contributions and of subleading ones of comparable size and opposite sign
  - increasing importance of wee partons, whence a strong coupling non-perturbative Pomeron regime
- RGI approach tames both problems:
  - through an understanding and consequent resummation of the most important subleading contributions
  - Subleading corrections and running coupling effects lower high-energy exponents and diffusion/tunneling into NP region
- We expect a large domain of applicability of PT QCD
- We have the tools to make reliable physical predictions:

  γ^*-γ^*, forward jets, Mueller-Navelet jets, …