The Status of QCD

From the Perspective of the Lattice

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Outline

- **Introduction**
- **Heavy Quark Physics**
- **QCD Thermodynamics**
- **Hadron Structure**
  - Moments of Parton Distributions
  - Generalized Form Factors
- **Summary and Prospects**

Introduction

- QCD produces amazingly rich structure from beautifully simple Lagrangian

- After quarter century, much understood in perturbative regime

- Non-perturbative regime still a challenge

  Only tool to solve, rather than model, QCD is lattice field theory

Auspicious Time for Lattice Perspective

- **Emerging from development phase to era of definitive results**

- **Confluence of theoretical/algorithmic development and advances in computer technology**

- **Theoretical Developments**

    Improved action and operators

    Chiral fermions

    Partially quenched chiral perturbation theory

- **Computer technology**

    Special purpose machines: QCDOC, ApeNEXT

    Optimized commodity clusters

Resources

- **Current** (peak Tflops $\sim$ 50 % sustained)

  Japan: $\sim$ 2 Tflops

  Germany, Italy: $\sim$ 2 Tflops ApeMille

  US: 1 Tflops QCDSP

- **Future** (sustained Tflops)

  03 **1.5 Tflops Columbia QCDOC**

  03-04 **1.5 Tflops Fermilab, JLab clusters**

  04 **5 Tflops UKQCD QCDOC**

  10 Tflops BNL QCDOC

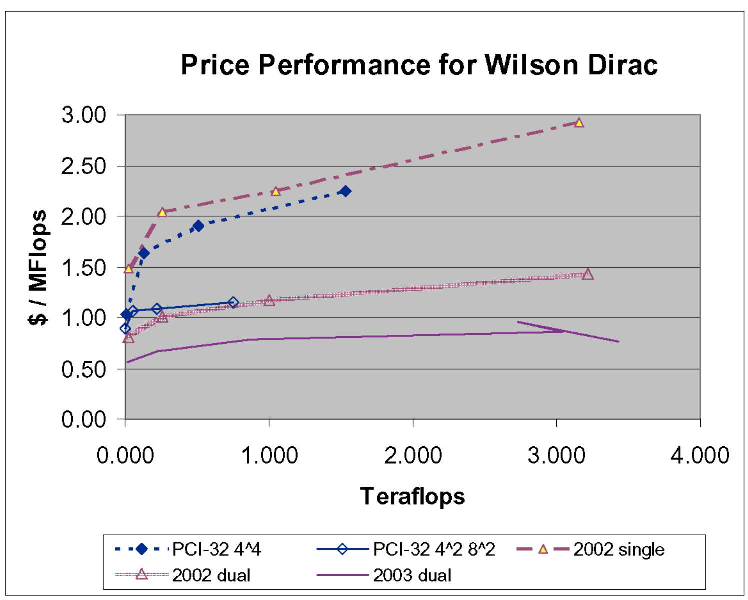
  25 ($\times \sim 50$ %) DESY,GSI ApeNEXT

- **Costs**

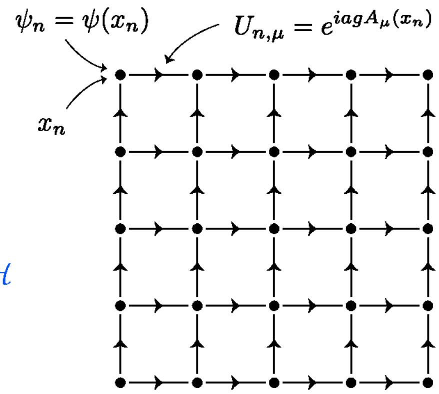
  QCDOC 1$ / sustained Tflops

  ApeMille $\sim$ 1-1.5 $ / sustained Tflops

  Clusters



Lattice QCD



Euclidean:

$$e^{i\int dt\, d^3x\, \mathcal{L}} \to e^{-\int d\tau\, d^3x\, \mathcal{H}}$$

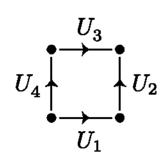
$$\langle T\, e^{-\beta H}\, \psi\psi\psi \cdots \bar{\psi}\bar{\psi}\bar{\psi}\rangle$$

$$= \frac{1}{Z}\int \mathcal{D}[\psi]\mathcal{D}[\bar{\psi}]\mathcal{D}[A] e^{-\int d^4x \left[\bar{\psi}(\partial\!\!\!/ + m + igA\!\!\!/)\psi + \frac{1}{4}F^2_{\mu\nu}\right]} \psi\psi\psi \cdots \bar{\psi}\bar{\psi}\bar{\psi}$$

$$\to \prod_n \frac{1}{Z}\int d\psi_n\, d\bar{\psi}_n\, dU_n\, e^{-\sum\limits_n \left[\bar{\psi}M(U)\psi + S(U)\right]} \psi\psi\psi \cdots \bar{\psi}\bar{\psi}\bar{\psi}$$

$$= \prod_n \int dU_n \underbrace{\frac{1}{Z} \det M(U) e^{-S(U)}}_{\text{Sample with M.C.}} \sum M^{-1}(U)M^{-1}(U)\cdots M^{-1}(U)$$

$$\to \frac{1}{N} \sum_{\substack{i=1 \\ U_i \in \frac{\det M(U)}{Z} e^{-S(U)}}}^{N} M^{-1}(U_i)M^{-1}(U_i)M^{-1}(U_i)$$



$$S(U) = \sum_{\Box} \frac{2N}{g^2}\left(1 - N^{-1}\mathrm{ReTr}U_{\Box}\right) \to \tfrac{1}{4}F^2_{\mu\nu} \qquad U_{\Box} \equiv U_1 U_2 U_3^\dagger U_4^\dagger$$

$$\bar{\psi}M(U)\psi = \sum_n \left[\bar{\psi}_n\psi_n + \kappa(\bar{\psi}_n(1-\gamma_\mu)U_{n,\mu}\psi_{n+\mu} + \bar{\psi}_{n+\mu}(1+\gamma_\mu)U^\dagger_{n,\mu}\psi_n\right]$$

Observables

$$\langle T\, e^{-\beta H} \hat{\bar\psi}\hat{\bar\psi}\hat{\bar\psi}\cdots\hat\psi\hat\psi\hat\psi\rangle$$
$$= Z^{-1}\int \mathcal{D}(U)\mathcal{D}(\bar\psi\psi)e^{-\bar\psi M(U)\psi - S(U)}\bar\psi\bar\psi\bar\psi\cdots\psi\psi\psi$$
$$= Z^{-1}\int \mathcal{D}(U)e^{\overbrace{\text{ln det}\, M(U)}_{②} - \overbrace{S(U)}_{③}} \sum \underbrace{M^{-1}(U)M^{-1}(U)\cdots M^{-1}(U)}_{①}$$

① $M^{-1} = (1+\kappa U)^{-1}$ connects $\bar\psi$ and ψ with line of U's

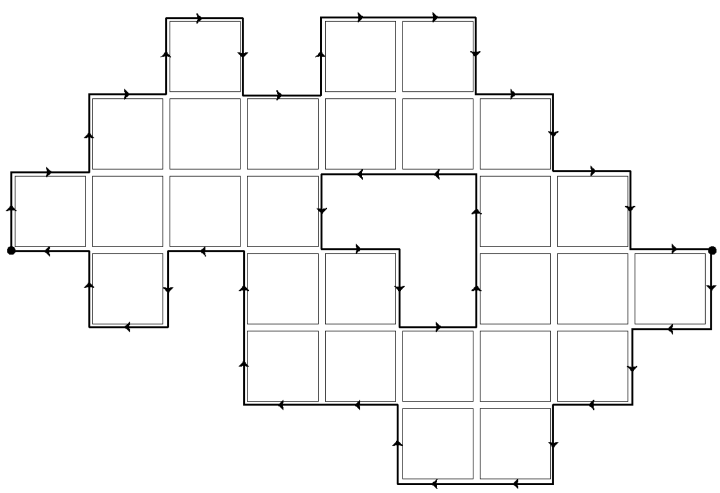
→ Sum over all valence quark paths.

② ln det M generates closed loops of U's

→ Sum over all $\bar q q$ excitations from sea
omit in quenched approximation

③ $S(U)$ tiles with plaquettes

→ sum all gluons



$32^3 \times 64$ lattice $\Longrightarrow 10^8$ gluon variables

Two Distinct Regions of QCD

- Heavy quark regime

  Confinement

  Flux tubes

  Adiabatic potential

  Isgur Wise function

- Light quark regime

  Chiral symmetry breaking

  Instantons

  Zero modes: $\langle \bar{\psi}\psi \rangle = \pi \rho(0)$

  Quarks propagate via 't Hooft interaction

  Zero modes dominate quark propagation

  Instantons alone yield observables similar to those from all gluons

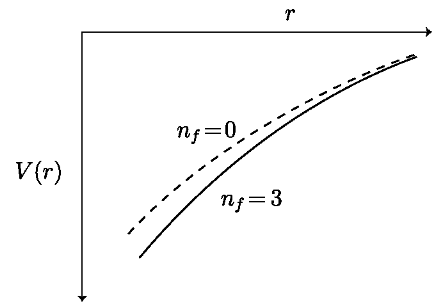
  Low energy effective theory - chiral perturbation theory

Major Issues

- **Include dynamical fermions in full QCD**

  Chiral logs

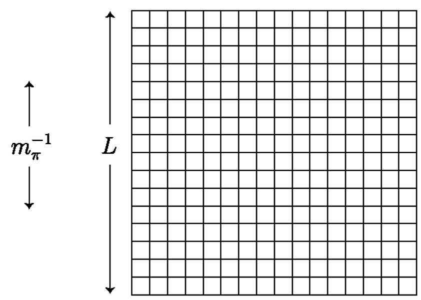
  Static potential



- **Calculate in physical regime of light quark masses**

$$\frac{1}{m_\pi} \leq \frac{L}{4}$$

L (fm)	m_π (MeV)
1.6	500
4.0	200
5.7	140



- **Finite baryon density**

Three Broad Areas of QCD Physics

Lattice calculations are essential to understanding physics in each area

- **Heavy Quark Systems**

  Extract fundamental CKM matrix elements from weak hadronic matrix elements

  Confirm standard model or elucidate physics beyond it

- **QCD Thermodynamics**

  Equation of state and QCD phase diagram

  Foundation for relativistic heavy ion physics at RHIC, LHC

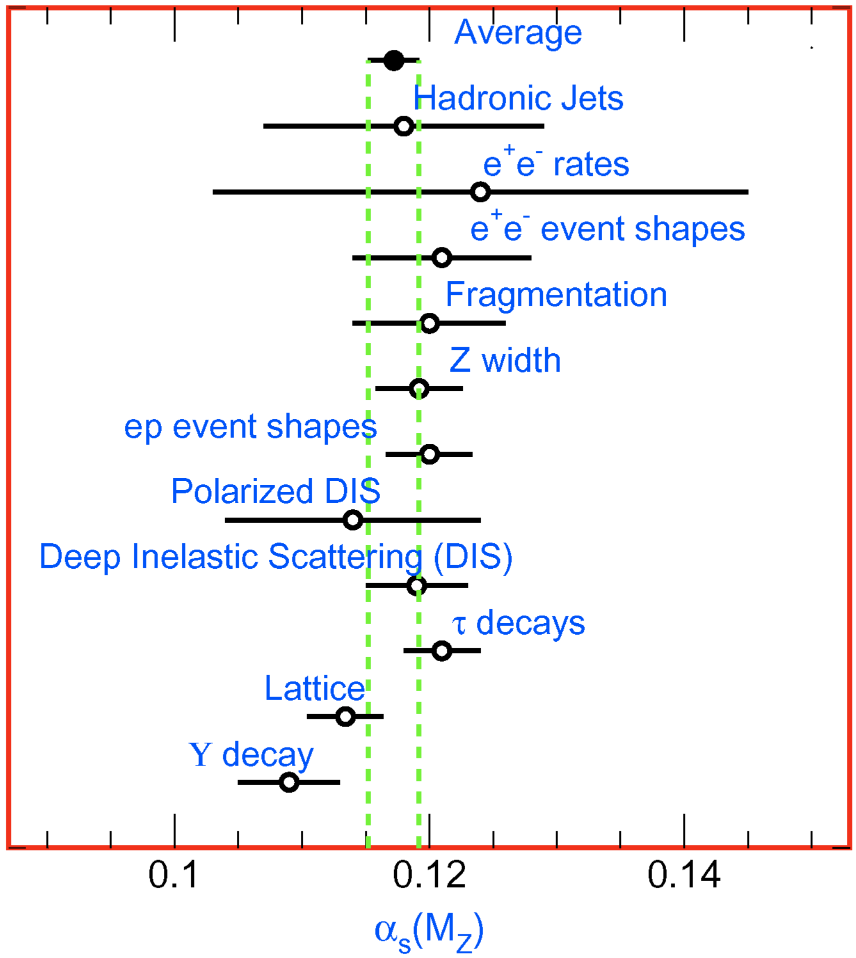
- **Hadron Structure**

  Quark and gluon structure of hadrons

  Structure functions and generalized parton distributions

Heavy Quark Systems

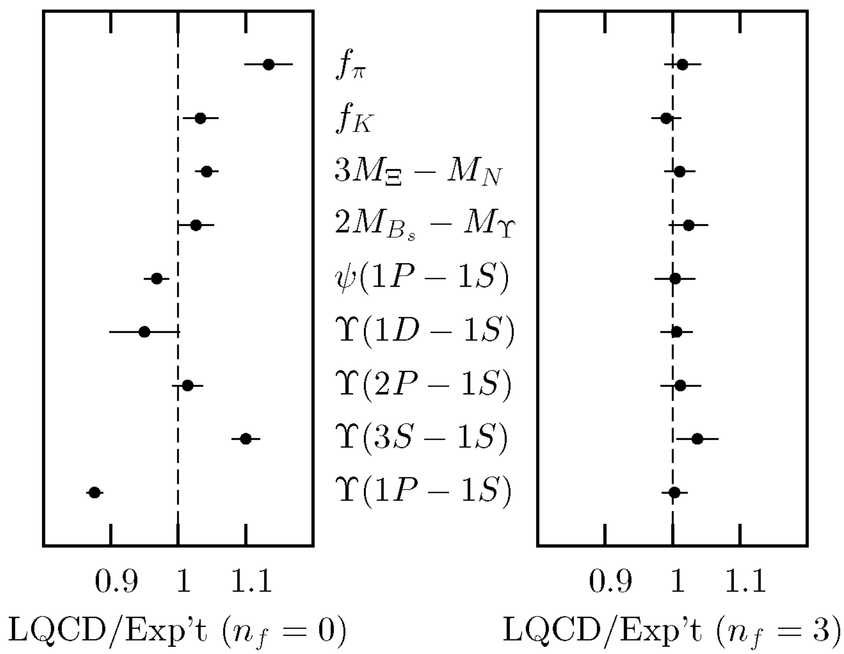
- **Determination of $\alpha_S(M_Z)$**



Summary of the determinations of $\alpha_S(M_Z)$ from the Particle Data Group

Precision Agreement in Full QCD

- **Gold-Plated Observables** Davies et al, hep-lat/0304004



Staggered quarks

Asqtad improved action

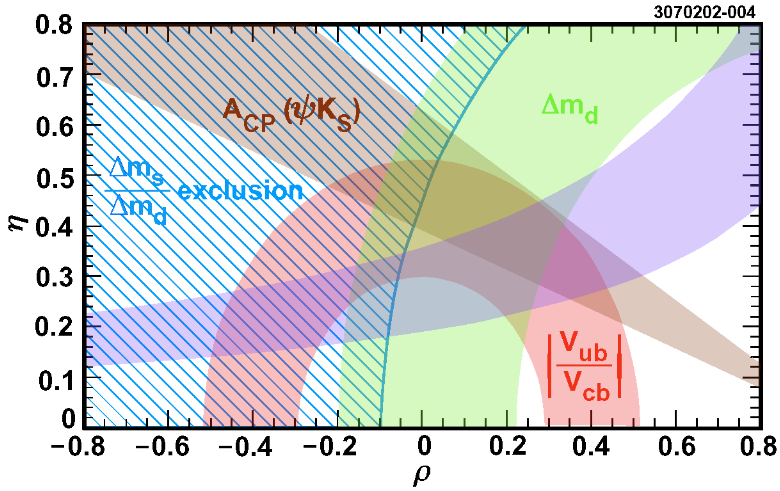
a = 0.13, 0.09 fm

Errors $\sim$ 3%

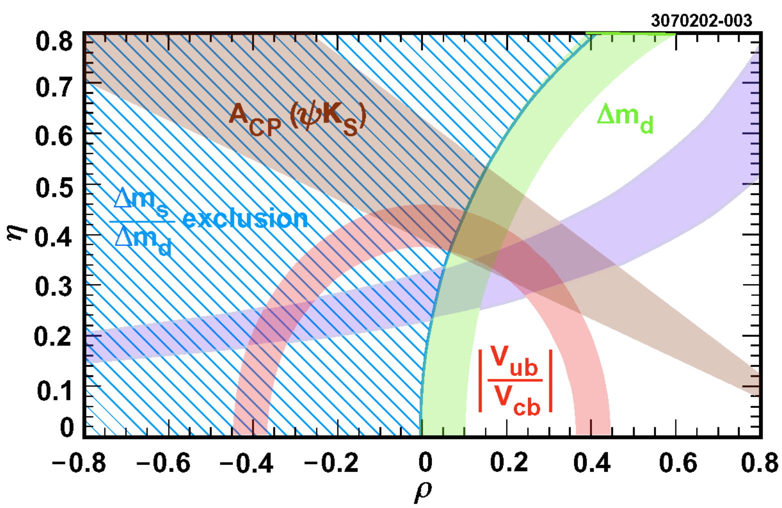
Gold-plated processes for 8/9 CKM elements

Impact on Standard Model Parameters

- **Current constraints on ρ and η**

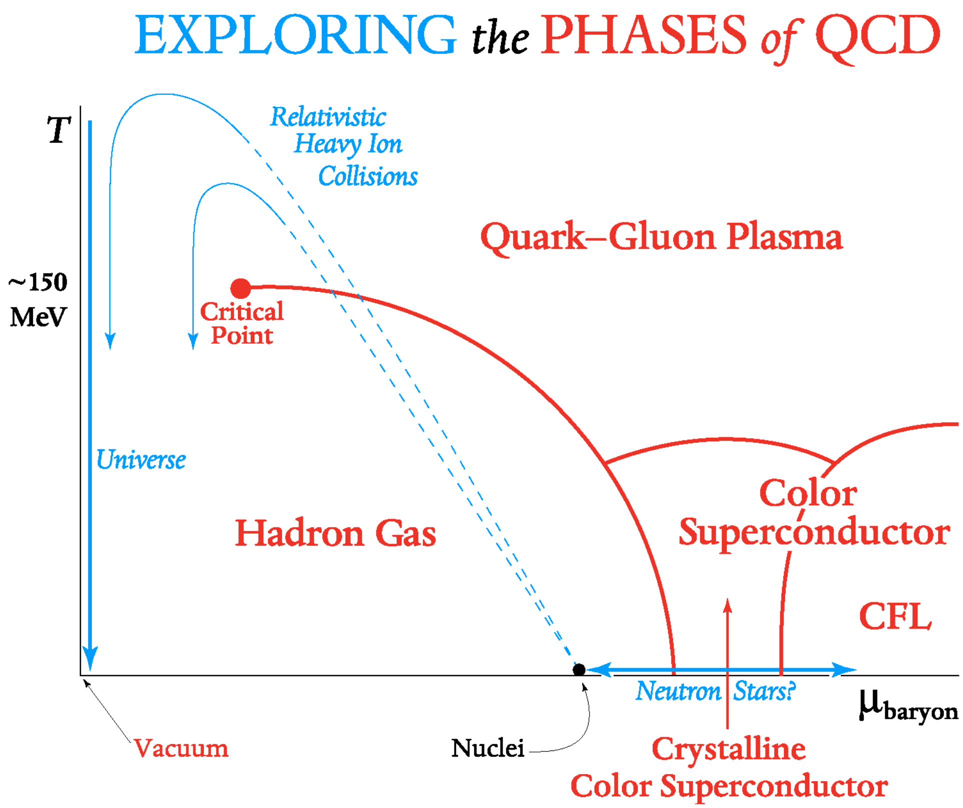


- **Future constraints with 3% errors** (R. Patterson)



QCD Thermodynamics

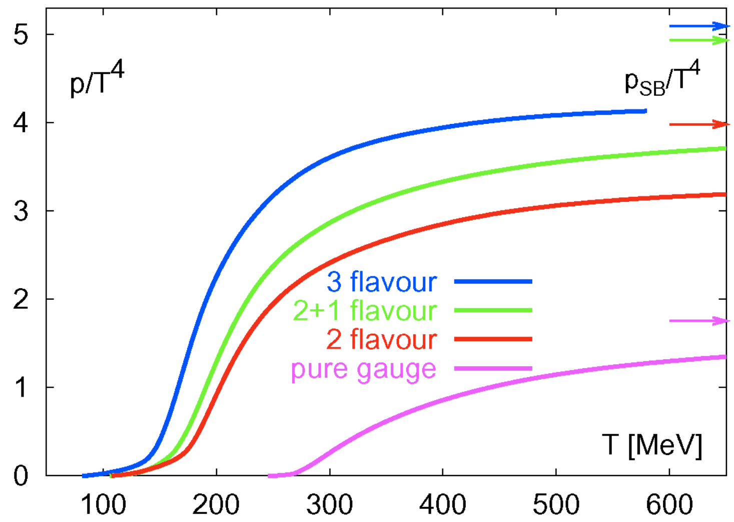
- QCD phase diagram



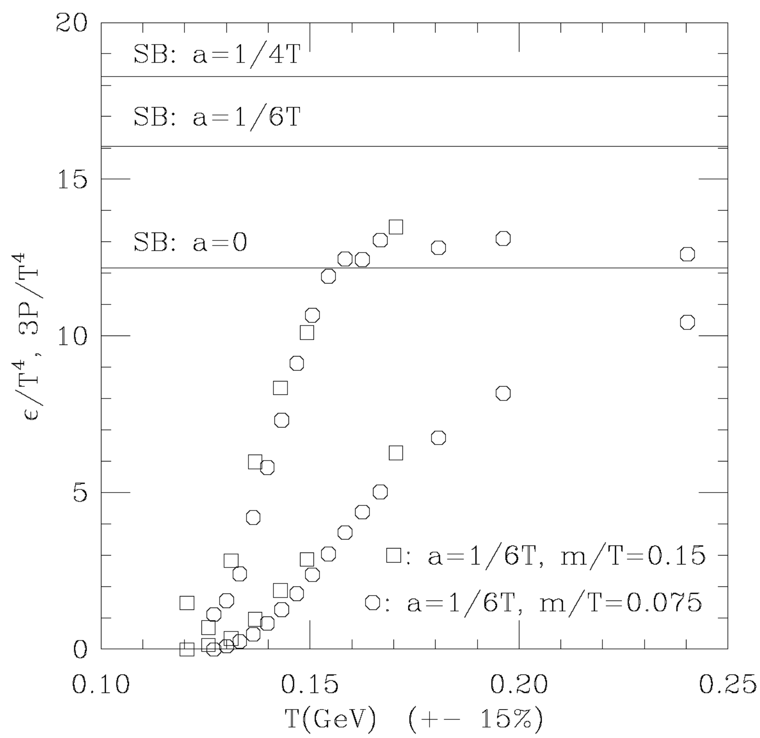
- Currently restricted to $\mu \sim 0$

Equation of State at $\mu = 0$

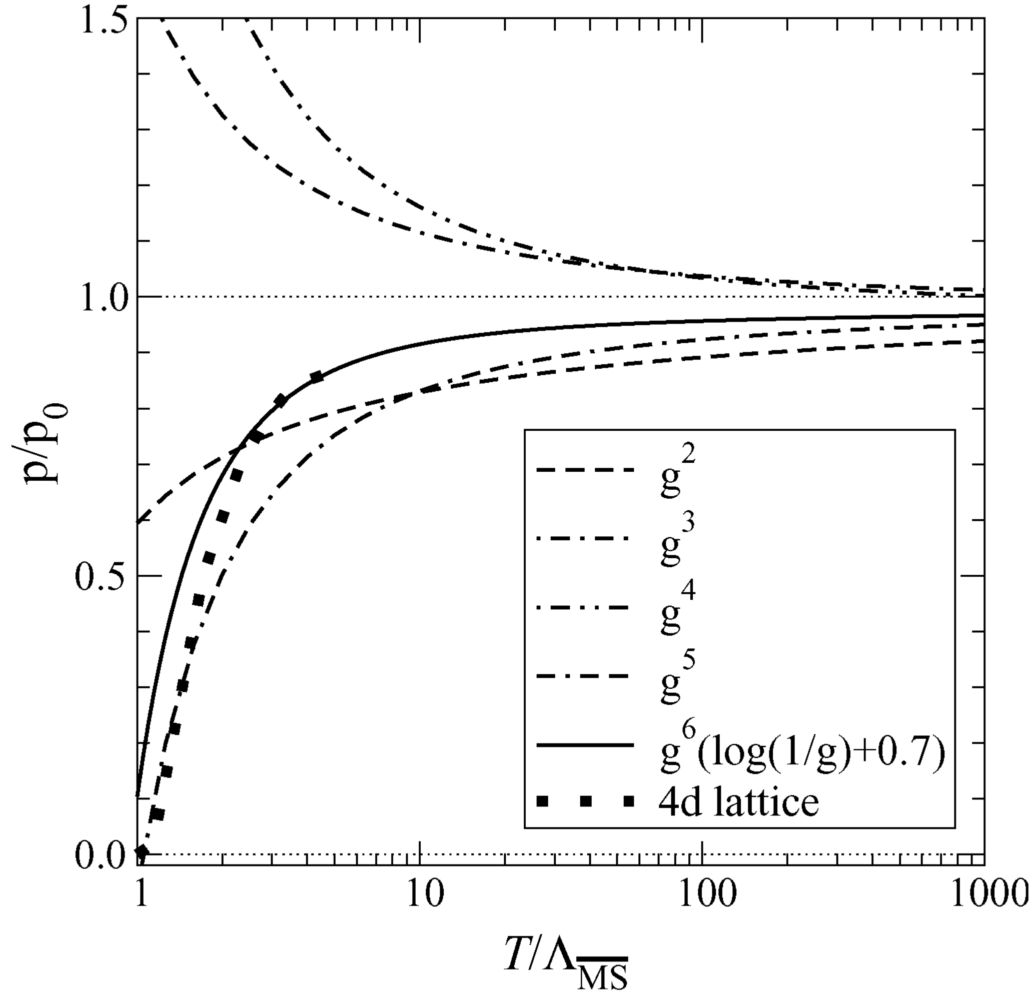
- **Equation of state** (F. Karsch hep-lat/0106019)



- **Pressure and energy density for 2 flavors** (MILC)



Matching to Perturbative QCD



(Kajantie, Laine, Rummukainen, Schroder hep-ph/0211321)

- Dimensionally reduced effective theory

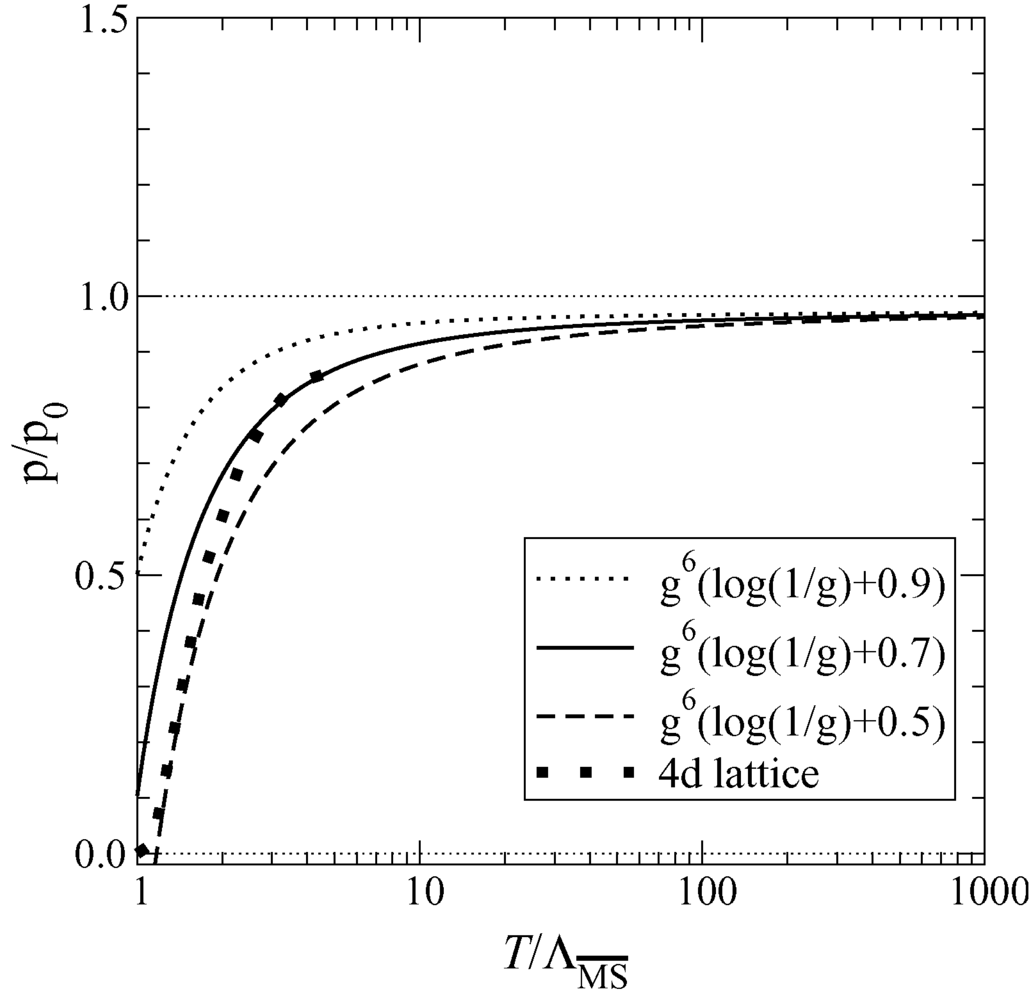
  Integrate out 'hard' modes $\sim \pi T$

  Integrate out 'soft' modes $\sim gT$

  Calculate p_G in 3-D

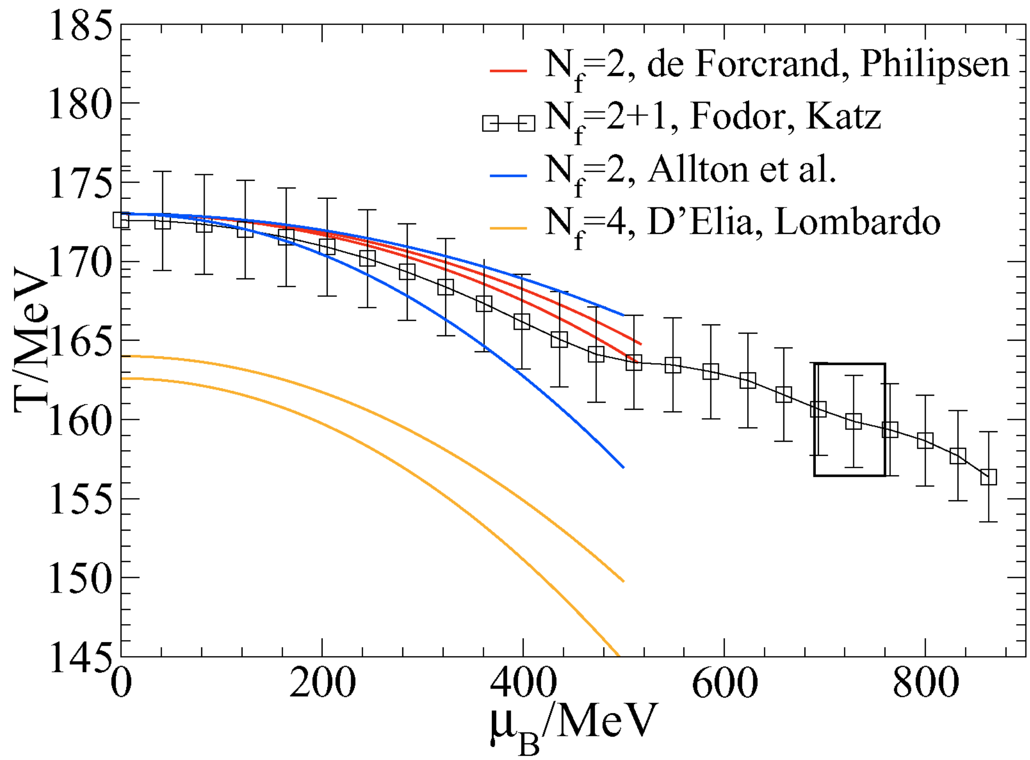
  $p = 1 + g^2 + g^3 + g^4 + g^5 + g^6 \ln g + \# g^6$

Sensitivity to g^6 Coefficient



- g^6 coefficient can be calculated in 3-d lattice QCD
- Successful joining would put equation of state on solid theoretical foundation

Finite Baryon Density



- **Three methods for calculation**

  Reweighting

  Expansion around $\mu = 0$

  Analytic continuation with complex μ

- **Open question - large** μ

  Cluster algorithms ?

Hadron Structure

- High energy scattering measures light cone correlation functions

$$O(x) = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \bar{\psi}(-\tfrac{\lambda}{2}n)\ \not{n} \mathcal{P} e^{-ig\int_{-\lambda/2}^{\lambda/2} d\alpha\, n\cdot A(\alpha n)} \psi(\tfrac{\lambda}{2}n)$$

Diagonal matrix elements

$$\langle P|O(x)|P\rangle = q(x)$$

Off-diagonal matrix elements

$$\langle P'|O(x)|P\rangle \rightarrow H(x,\xi,t), E(x,\xi,t)$$

- Expansion of $O(x)$ generates twist-2 operators

$$O_q^{\{\mu_1\mu_2\ldots\mu_n\}} = \overline{\psi}_q \gamma^{\{\mu_1} iD^{\mu_2} \ldots iD^{\mu_n\}} \psi_q$$

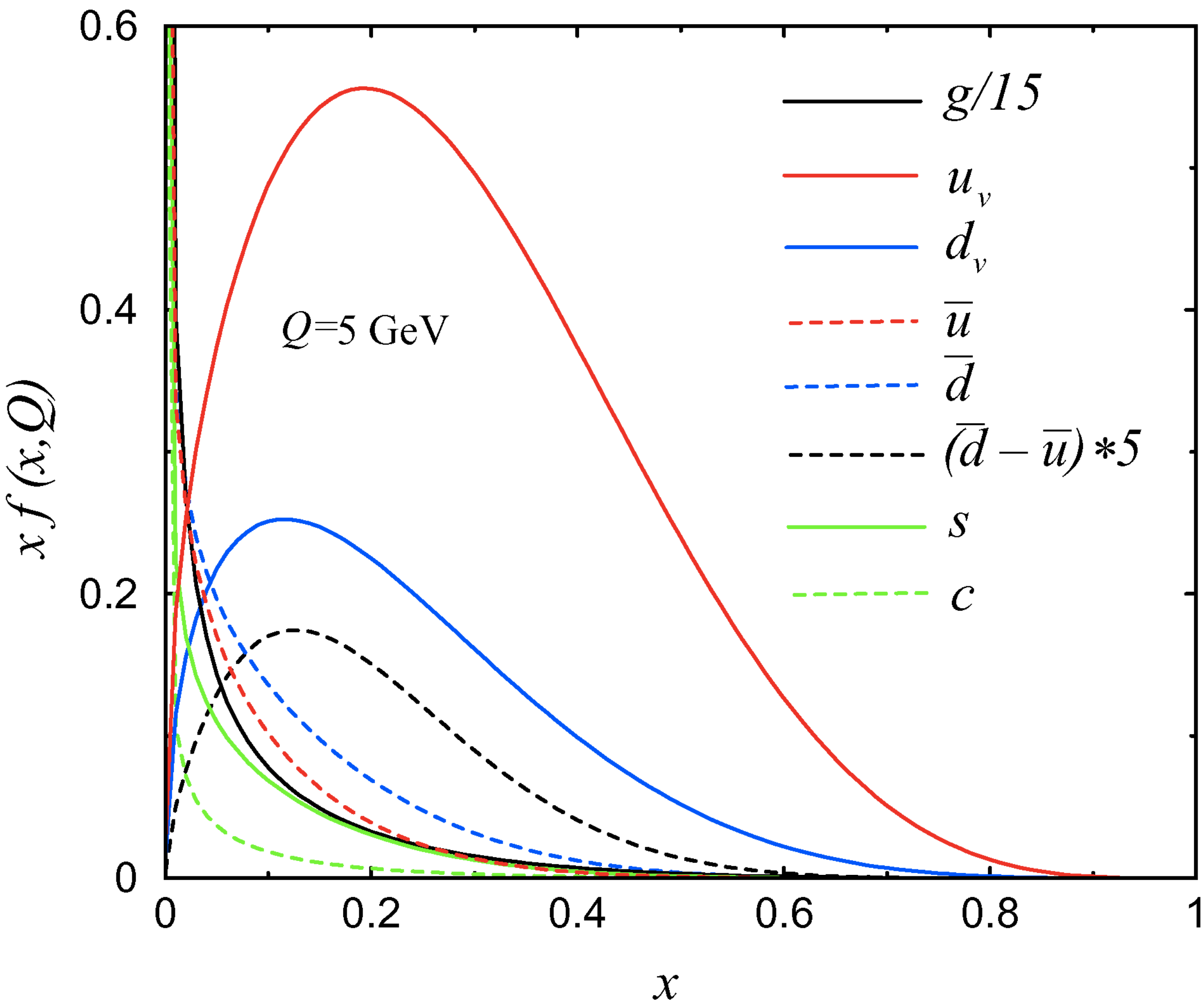
Diagonal matrix elements

$$\langle P|O_q^{\{\mu_1\mu_2\ldots\mu_n\}}|P\rangle = \int dx\, x^{n-1} q(x)$$

Off-diagonal matrix elements

$$\langle P'|O_q^{\{\mu_1\mu_2\ldots\mu_n\}}|P\rangle \rightarrow A_{ni}(t), B_{ni}(t), C_n(t)$$

Experimentally measured quark and gluon distributions at $Q^2 = 5$ GeV



Phenomenological fits to unpolarized data

CTEQ: http://www-spires.dur.ac.uk/hepdata/cteq.html

GRV: http://www-spires.dur.ac.uk/hepdata/grv.html

MRS: http://durpdg.dur.ac.uk/hepdata/mrs.html

Phenomenological fits to polarized data

GRSV: http://doom.physik.uni-dortmund.de/PARTON/index.html

GS:http://www.desy.de/ gehrt/pdf

Moments of quark and gluon distributions

Moments of quark distributions in the proton

$$\langle x^n \rangle_q = \int_0^1 \mathrm{d}x\, x^n \big(q(x) + (-1)^{n+1} \bar{q}(x)\big)$$
$$\langle x^n \rangle_{\Delta q} = \int_0^1 \mathrm{d}x\, x^n \big(\Delta q(x) + (-1)^{n} \Delta\bar{q}(x)\big)$$
$$\langle x^n \rangle_{\delta q} = \int_0^1 \mathrm{d}x\, x^n \big(\delta q(x) + (-1)^{n+1} \delta\bar{q}(x)\big)$$

where $\quad q = q_\uparrow + q_\downarrow \qquad \Delta q = q_\uparrow - q_\downarrow \qquad \delta q = q_\top + q_\perp$

are related to matrix elements of twist-2 operators

$$\frac{1}{2}\big\langle PS\big|\bar{\psi}\gamma^{\{\mu_1} iD^{\mu_2}\cdots iD^{\mu_n\}}\psi\big|PS\big\rangle = 2\langle x^{n-1}\rangle_q\, P^{\{\mu_1}\cdots P^{\mu_n\}}$$

$$\big\langle PS\big|\bar{\psi}\gamma^{\{\mu_1}\gamma_5 iD^{\mu_2}\cdots iD^{\mu_n\}}\psi\big|PS\big\rangle = \frac{2}{n}\langle x^{n-1}\rangle_{\Delta q}\, S^{\{\mu_1} P^{\mu_2}\cdots P^{\mu_n\}}$$

$$\big\langle PS\big|\bar{\psi}\sigma^{[\alpha\{\mu_1]}\gamma_5 iD^{\mu_2}\cdots iD^{\mu_n\}}\psi\big|PS\big\rangle = \frac{2}{M}\langle x^{n-1}\rangle_{\delta q}\, S^{[\alpha} P^{\{\mu_1]} P^{\mu_2}\cdots P^{\mu_n\}}$$

where $\{\,\}$ $\Rightarrow$ symmetrization, $[\,]$ $\Rightarrow$ antisymmetrization, and $S^2 = M^2$

Higher twist operators:

$$\big\langle PS\big|\bar{\psi}\gamma^{[\mu_1}\gamma_5 iD^{\{\mu_2]}\cdots iD^{\mu_n\}}\psi\big|PS\big\rangle = \frac{1}{n} d_{n-1}\, S^{[\mu_1} P^{\{\mu_2]}\cdots P^{\mu_n\}}$$

Lattice Operators

Use irreducible representations of hypercubic group with minimal operator mixing and minimal non-zero momentum components

$\langle x\rangle_q^{(a)}$	6_3^+	$\bar\psi\gamma_{\{1}\overleftrightarrow{D}_{4\}}\psi$	p
$\langle x\rangle_q^{(b)}$	3_1^+	$\bar\psi\gamma_4\overleftrightarrow{D}_4\psi-\frac{1}{3}\sum_{i=1}^{3}\bar\psi\gamma_i\overleftrightarrow{D}_i\psi$	0
$\langle x^2\rangle_q$	8_1^-	$\bar\psi\gamma_{\{1}\overleftrightarrow{D}_1\overleftrightarrow{D}_{4\}}\psi-\frac{1}{2}\sum_{i=2}^{3}\gamma_{\{i}\overleftrightarrow{D}_i\overleftrightarrow{D}_{4\}}\psi$	p,m
$\langle x^3\rangle_q$	2_1^+	$\bar\psi\gamma_{\{1}\overleftrightarrow{D}_1\overleftrightarrow{D}_4\overleftrightarrow{D}_{4\}}\psi+\bar\psi\gamma_{\{2}\overleftrightarrow{D}_2\overleftrightarrow{D}_3\overleftrightarrow{D}_{3\}}\psi-\{3\leftrightarrow 4\}$	p,m^*
$\langle 1\rangle_{\Delta q}$	4_4^+	$\bar\psi\gamma^5\gamma_3\psi$	0
$\langle x\rangle_{\Delta q}^{(a)}$	6_3^-	$\bar\psi\gamma^5\gamma_{\{1}\overleftrightarrow{D}_{3\}}\psi$	p
$\langle x\rangle_{\Delta q}^{(b)}$	6_3^-	$\bar\psi\gamma^5\gamma_{\{3}\overleftrightarrow{D}_{4\}}\psi$	0
$\langle x^2\rangle_{\Delta q}$	4_2^+	$\bar\psi\gamma^5\gamma_{\{1}\overleftrightarrow{D}_3\overleftrightarrow{D}_{4\}}\psi$	p
$\langle 1\rangle_{\delta q}$	6_1^+	$\bar\psi\gamma^5\sigma_{34}\psi$	0
$\langle x\rangle_{\delta q}$	8_1^-	$\bar\psi\gamma^5\sigma_{3\{4}\overleftrightarrow{D}_{1\}}\psi$	p
d_1	6_1^+	$\bar\psi\gamma^5\gamma_{[3}\overleftrightarrow{D}_{4]}\psi$	$0,M$
d_2	8_1^-	$\bar\psi\gamma^5\gamma_{[1}\overleftrightarrow{D}_{\{3]}\overleftrightarrow{D}_{4\}}\psi$	p,M

where $(\bar\psi\overleftrightarrow{D}\psi)_n=\bar\psi_nU_{n,\mu}\psi_{n+\mu}-\bar\psi_{n-\mu}U^\dagger_{n-\mu,\mu}\psi_n$

m ⇒ mixing with same dimension operators

m^* ⇒ no mixing for Wilson or overlap

M ⇒ mixing with lower dimension for Wilson, not for overlap

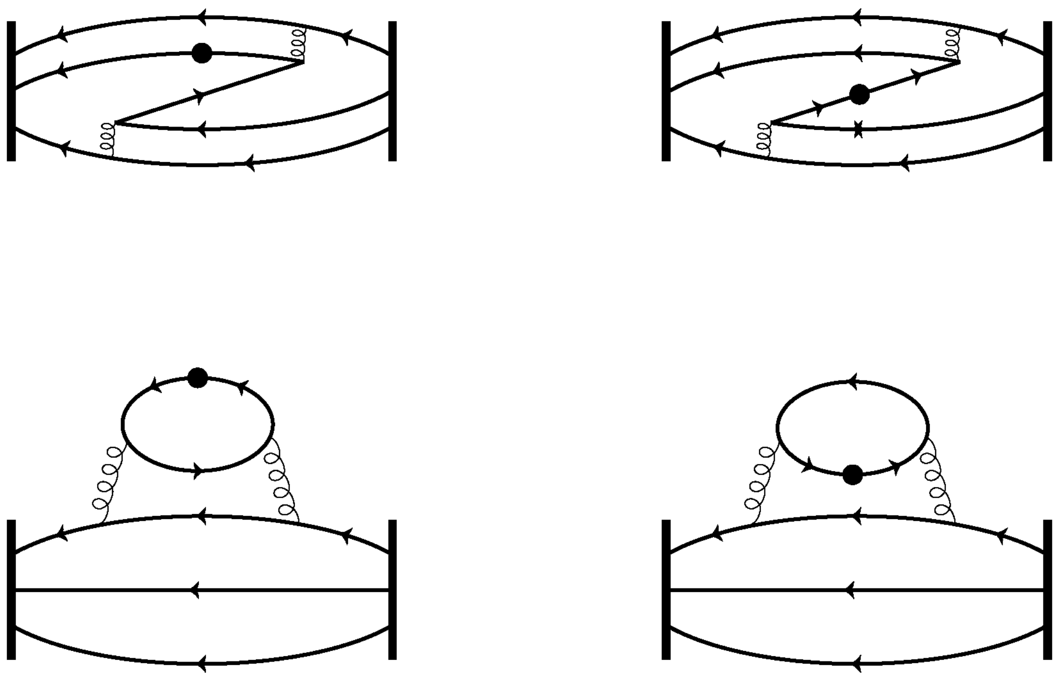
Perturbative Renormalization

$$O_i^{\overline{MS}}(Q^2) = \sum_j \left(\delta_{ij} + \frac{g_0^2}{16\pi^2} \frac{N_c^2 - 1}{2N_c} \left(\gamma_{ij}^{\overline{MS}} \log(Q^2 a^2) - (B_{ij}^{LATT} - B_{ij}^{\overline{MS}}) \right) \right) \cdot O_j^{LATT}(a^2)$$

observable	γ	B^{LATT}	$B^{\overline{MS}}$	$Z(\beta = 6.0)$	$Z(\beta = 5.6)$
$\langle x \rangle_q^{(a)}$	8/3	-3.16486	-40/9	0.9892	0.9884
$\langle x \rangle_q^{(b)}$	8/3	-1.88259	-40/9	0.9784	0.9768
$\langle x^2 \rangle_q$	25/6	-19.57184	-67/9	1.1024	1.1097
$\langle x^3 \rangle_q$	157/30	-35.35192	-2216/225	1.2153	1.2307
$\langle 1 \rangle_{\Delta q}$	0	15.79628	0	0.8666	0.8571
$\langle x \rangle_{\Delta q}^{(a)}$	8/3	-4.09933	-40/9	0.9971	0.9969
$\langle x \rangle_{\Delta q}^{(b)}$	8/3	-4.09933	-40/9	0.9971	0.9969
$\langle x^2 \rangle_{\Delta q}$	25/6	-19.56159	-67/9	1.1023	1.1096
$\langle 1 \rangle_{\delta q}$	1	16.01808	-1	0.8563	0.8461
$\langle x \rangle_{\delta q}$	3	-4.47754	-5	0.9956	0.9953
d_1	0	0.36500	0	0.9969	0.9967
d_2	7/6	-15.67745	-35/18	1.1159	1.1242

Note mixing of $d_n \propto \gamma_5 \gamma_{[\sigma} \overleftrightarrow{D}_{\{\mu_1]} \cdots \overleftrightarrow{D}_{\mu_n\}}$ with $\frac{1}{a} \gamma_5 \gamma_{[\sigma} \gamma_{\{\mu_1]} \cdots \overleftrightarrow{D}_{\mu_n\}}$ for Wilson fermions.

Hadron Matrix Elements on the Lattice



- Measure $\langle \mathcal{O} \rangle$, for m_q, a, L
- Connected diagrams

  $p = 0$

  $p \neq 0$

- Disconnected diagrams
- Extrapolate

  $m_q : m_\pi \rightarrow 140$ MeV

  $a \rightarrow \sim 0.05$ fm

  $L \rightarrow \sim 5.0$ fm

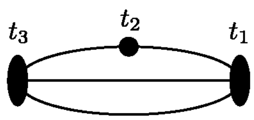
- Note: For $\langle \mathcal{O} \rangle_u - \langle \mathcal{O} \rangle_d$, disconnected diagrams cancel

Calculation of Matrix Elements on Euclidean Lattice

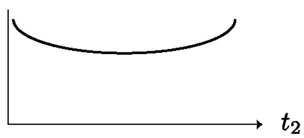
$J^\dagger$: Current with quantum numbers of proton

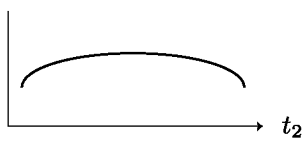
$|\psi_J\rangle = J^\dagger|\Omega\rangle$ Trial function

$$\langle T J(t_3)\, \mathcal{O}(t_2)\, J^\dagger(t_1)\rangle = \sum_{m,n} \langle\psi_J|n\rangle\langle n|\mathcal{O}|m\rangle\langle m|\psi_J\rangle e^{-E_n(t_3-t_2)-E_m(t_2-t_1)}$$



$$\xrightarrow[\substack{t_3-t_2\gg 1\\ t_2-t_1\gg 1}]{} \left|\langle\psi_J|0\rangle\right|^2\langle 0|\mathcal{O}|0\rangle e^{-E_0(t_3-t_1)}$$





want $\left|\langle\psi_J|n\rangle\right|^2 \sim \delta_{n0}$

for best plateau

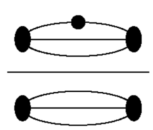
Normalize:

$$\langle T J(t_3)\, J^\dagger(t_1)\rangle = \sum_n \left|\langle\psi_J|n\rangle\right|^2 e^{-E_n(t_3-t_1)}$$

$$\xrightarrow[t_3-t_1\gg 1]{} \left|\langle\psi_J|0\rangle\right|^2 e^{-E_0(t_3-t_1)}$$

$\Longrightarrow$

$$\langle 0|\mathcal{O}|0\rangle = \frac{\langle J\mathcal{O}J^\dagger\rangle}{\langle JJ^\dagger\rangle} =$$

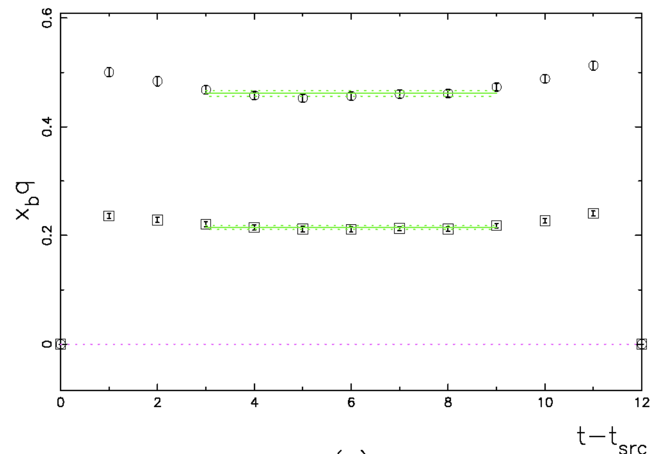


Sequential source options:

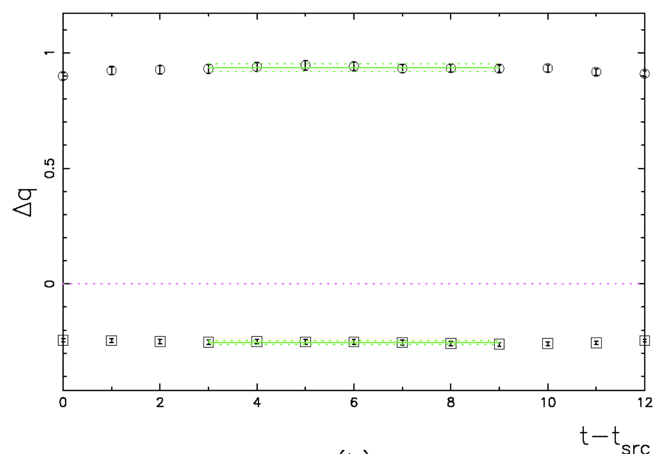
Each $\mathcal{O}(2)S(2,1)$ generates $S(2,3)$ to all t_3

Sink at fixed t_3 generates $S(3,2)$

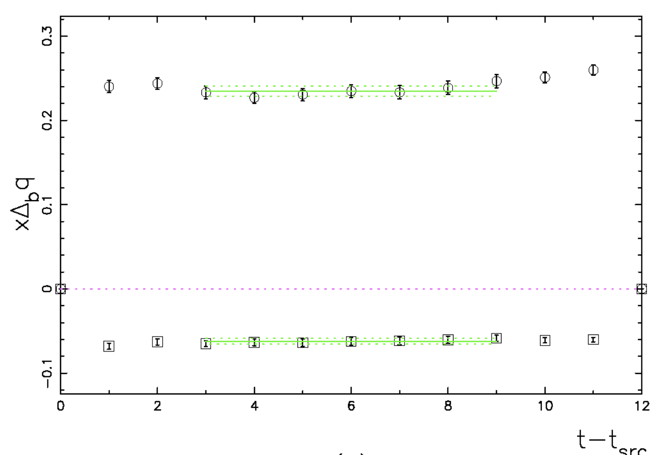
Plateaus in full QCD for operators with $p = 0$



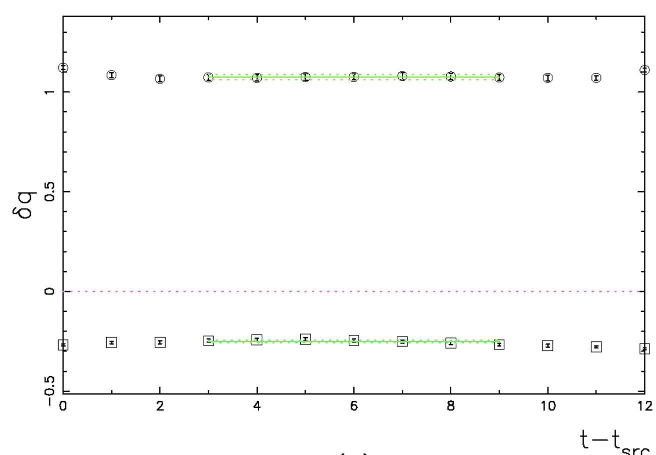
(a)



(b)

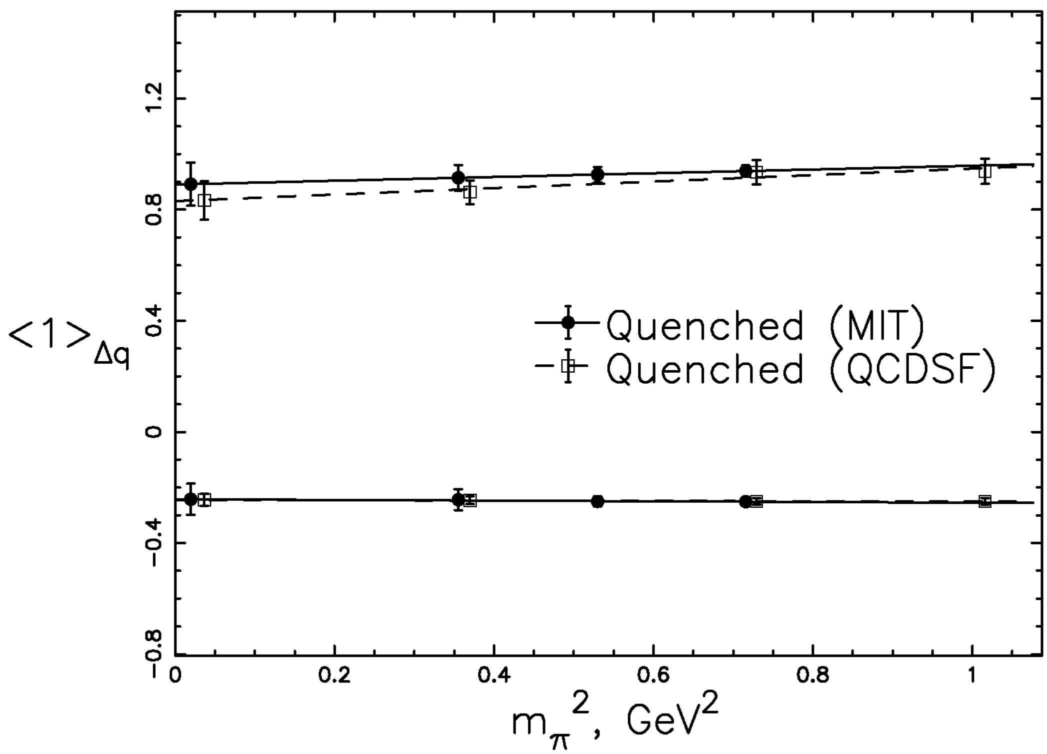


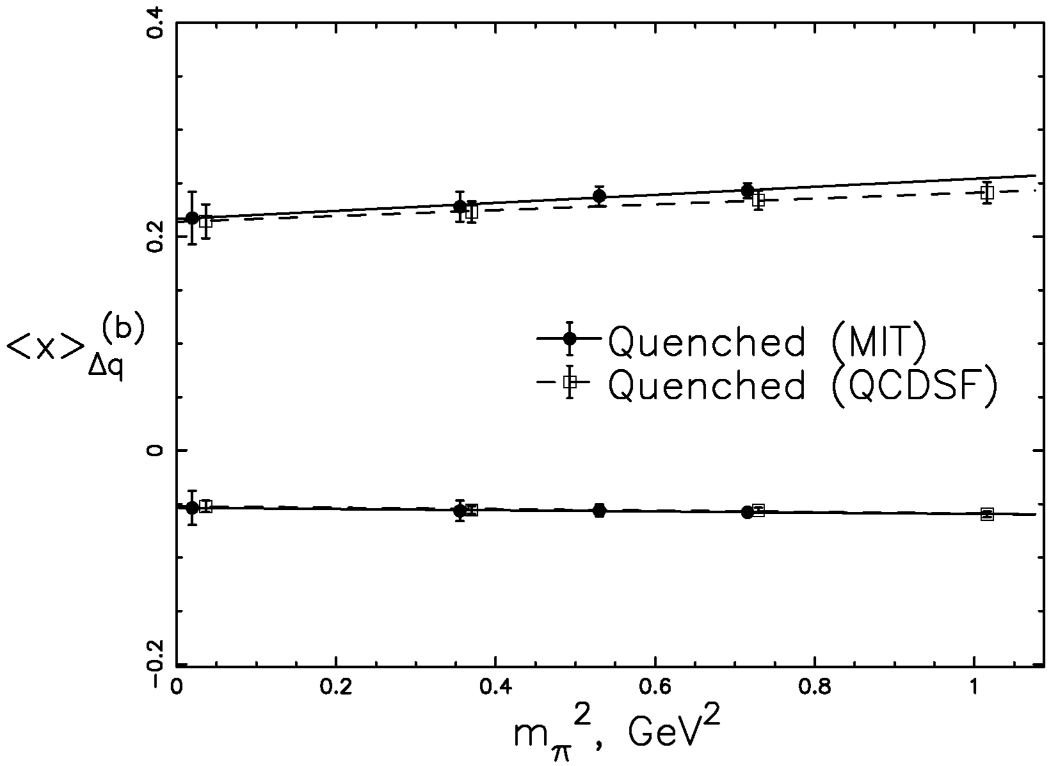
(c)



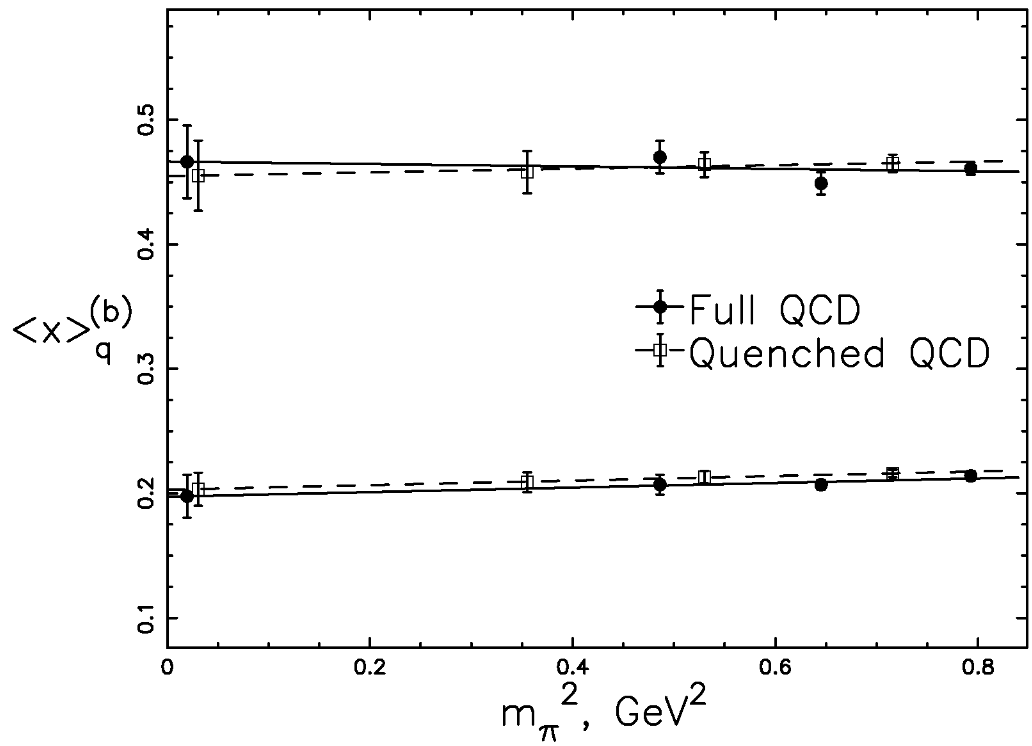
(d)

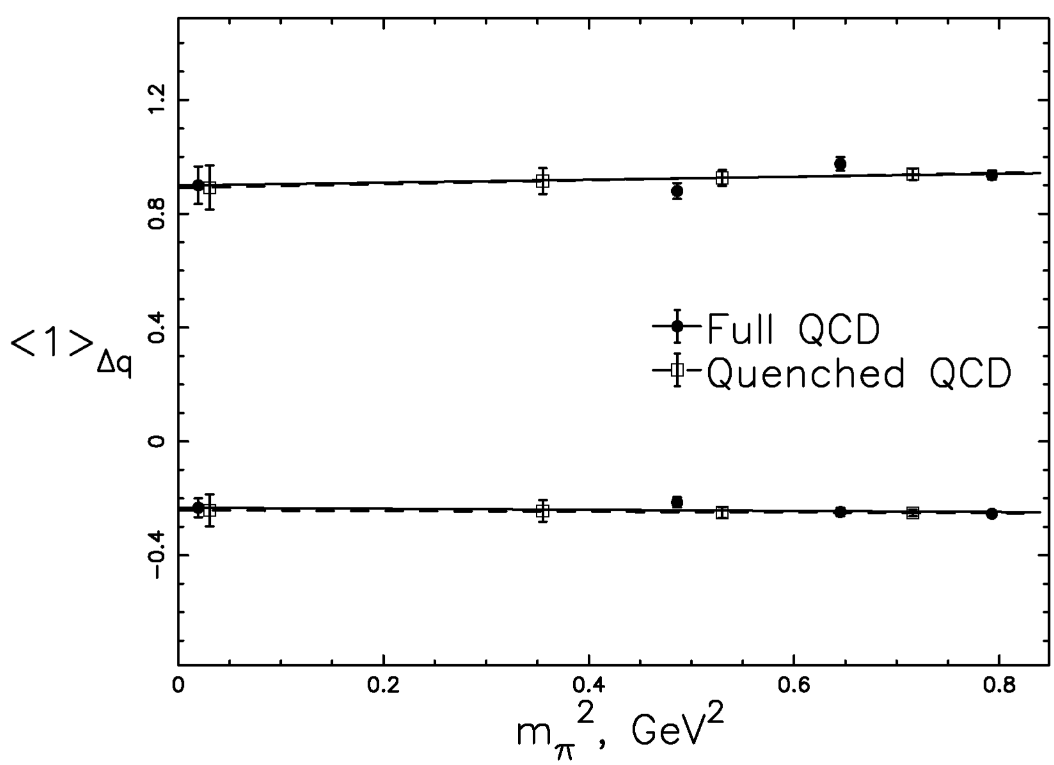
Comparison of Quenched Calculations



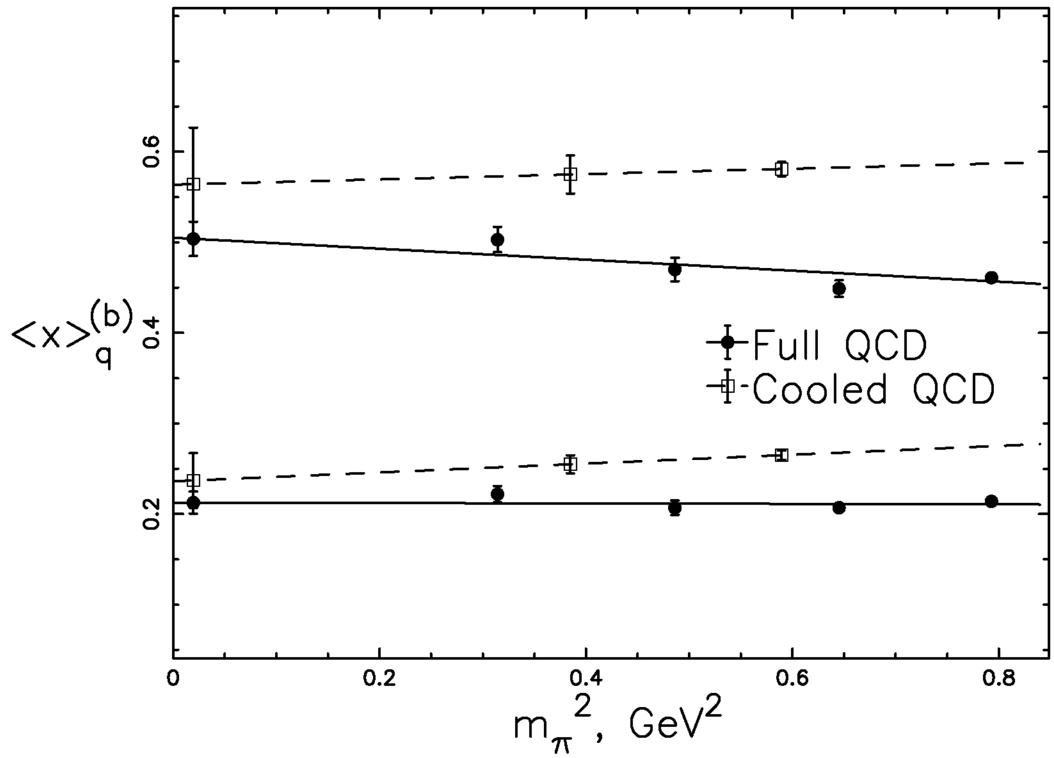


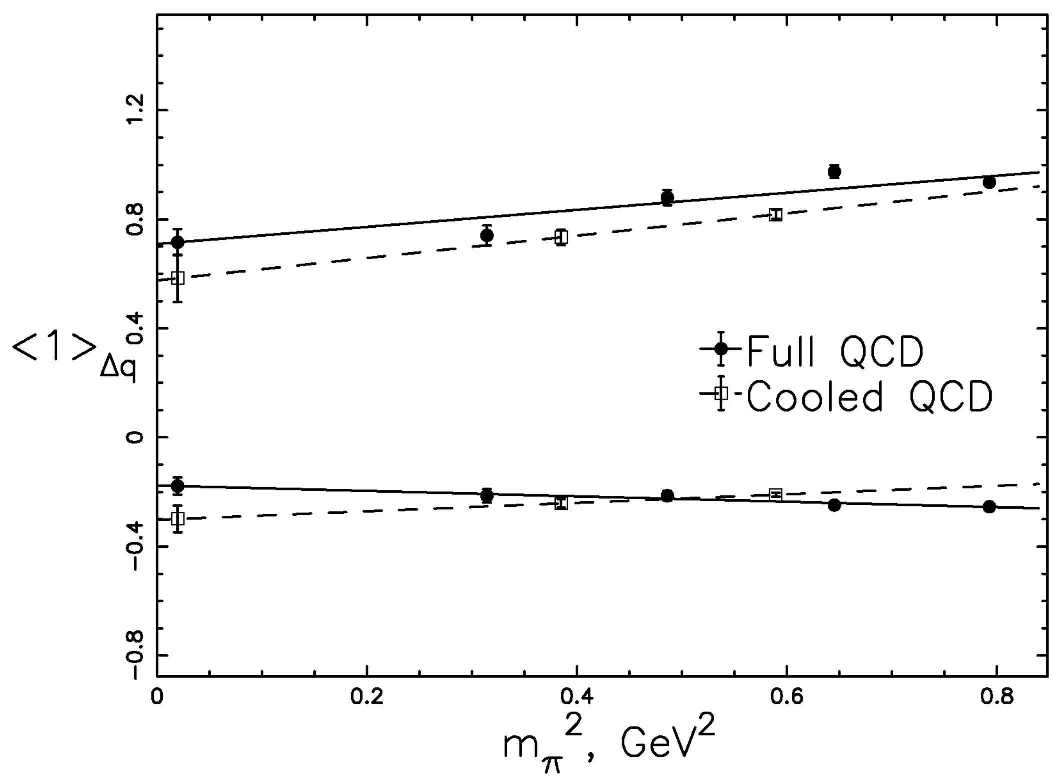
Comparison of Full and Quenched QCD





Qualitative Behavior from Instantons





Chiral Extrapolation - Physics of the Pion Cloud

- Long-standing puzzle: Linear extrapolation in m_q yields serious discrepancies

$$\langle x \rangle_u - \langle x \rangle_d \sim 0.24 - 0.28 \quad (0.16)$$

$$g_A = \langle 1 \rangle_{\Delta u} - \langle 1 \rangle_{\Delta d} \sim 1.0 - 1.1 \quad (1.26)$$

- Pion cloud essential component of nucleon

  μ_N, g_A

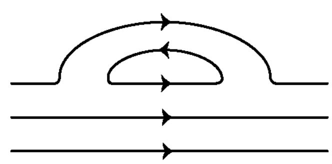
  Suppressed with heavy quarks in small volume

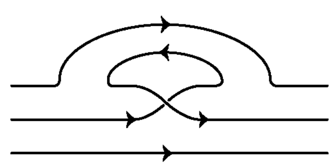
  Require:

  Light quarks

  Large volume: $L \geq 4 \frac{1}{m_\pi}$

  Full QCD





Chiral Perturbation Theory

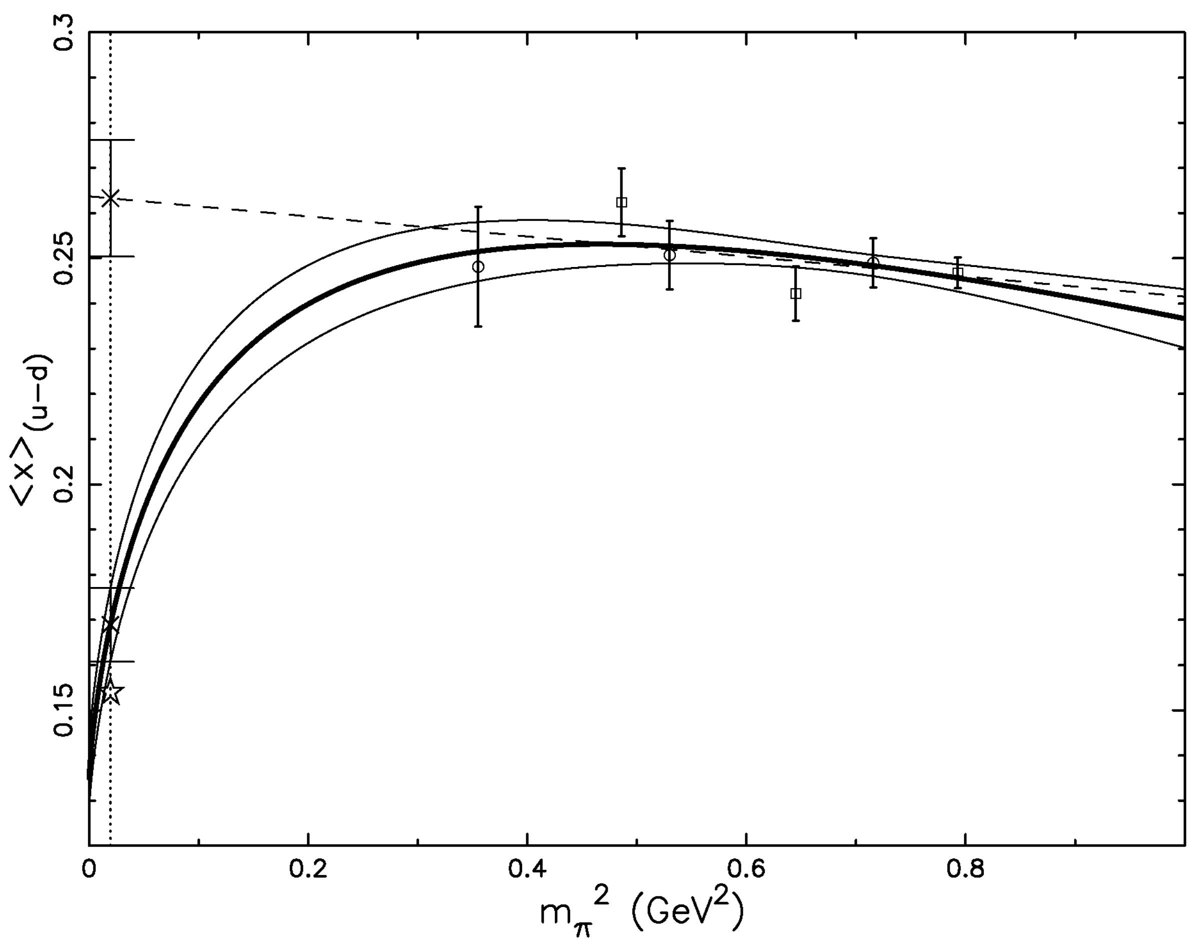
Heavy baryon chiral perturbation theory for nucleon parton distributions

Chen & Ji, Arndt & Savage, Chen & Savage

$$\langle x^n\rangle_u - \langle x^n\rangle_d \sim a_n \left[1 - \frac{(3{g_A}^2+1)m_\pi^2}{(4\pi f_\pi)^2}\ln(m_\pi^2)\right] + \text{analytic terms}$$

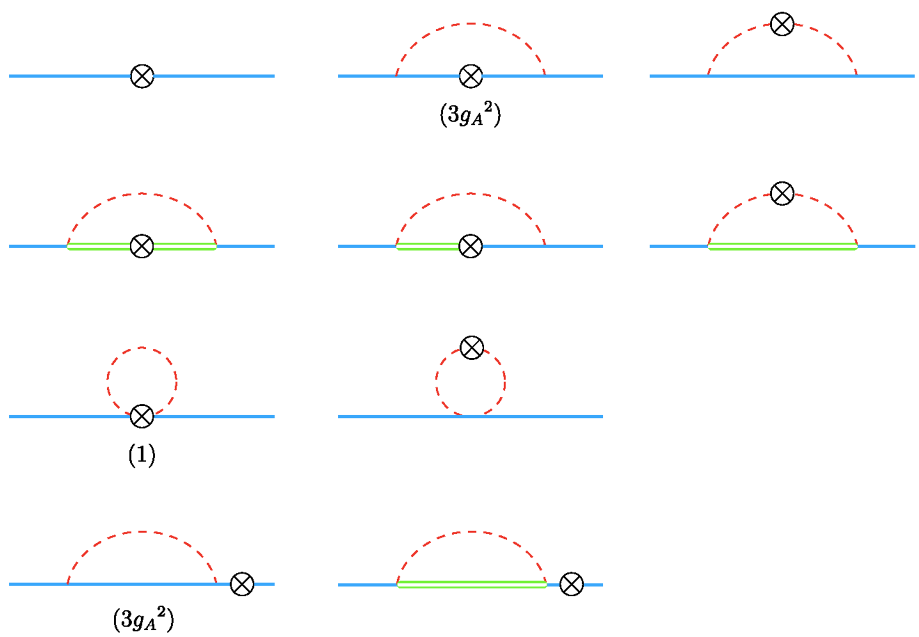
- Physical chiral extrapolation formula hep-lat/0103006

$$\langle x^n\rangle_u - \langle x^n\rangle_d \sim a_n \left[1 - \frac{(3{g_A}^2+1)m_\pi^2}{(4\pi f_\pi)^2}\ln\left(\frac{m_\pi^2}{m_\pi^2+\mu^2}\right)\right] + b_n m_\pi^2$$



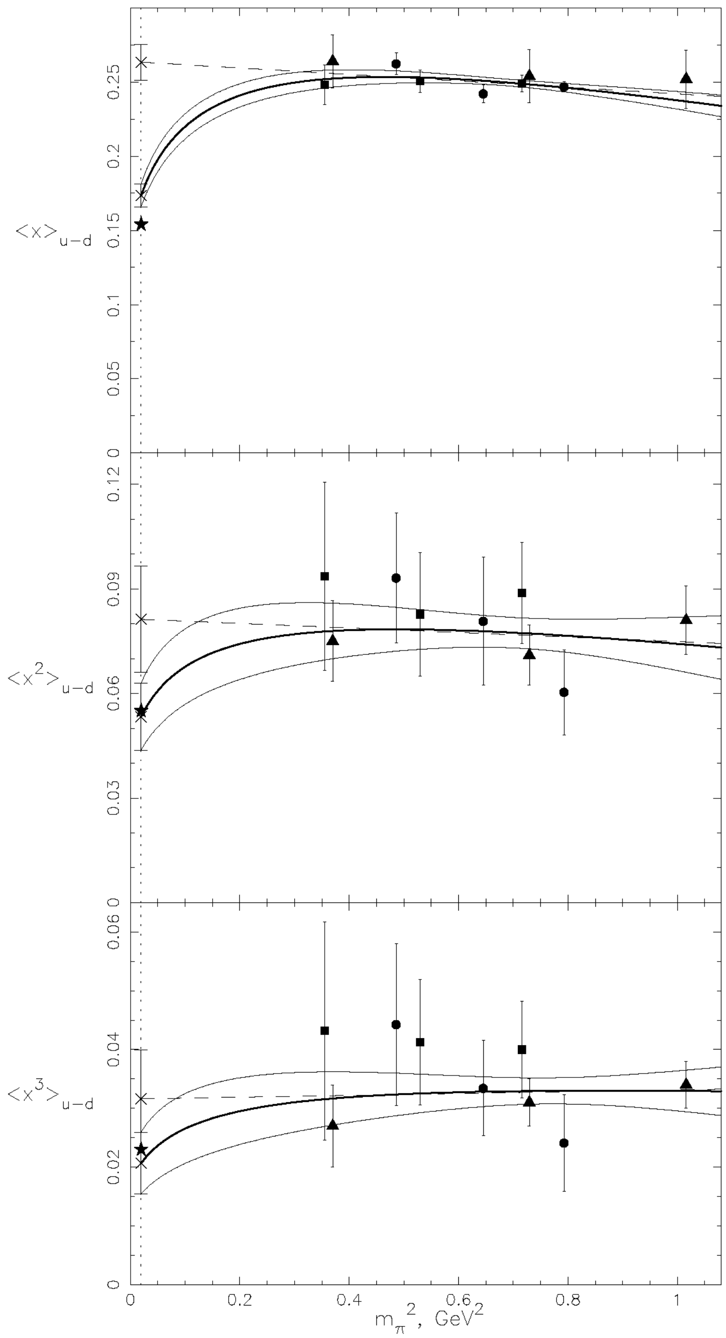
Squares full QCD, circles quenched, $\mu = 550MeV$

Chiral Perturbation Theory



Consistent Results for Three Moments

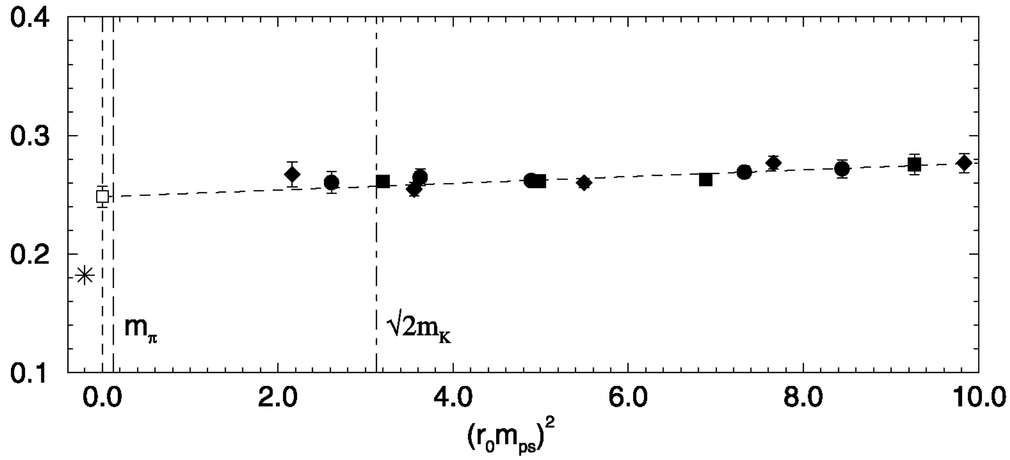
$$\langle x \rangle_u - \langle x \rangle_d \ , \quad \langle x^2 \rangle_u - \langle x^2 \rangle_d \ , \quad \langle x^3 \rangle_u - \langle x^3 \rangle_d$$



Diamonds full QCD, squares MIT quenched, triangles QCDSF quenched

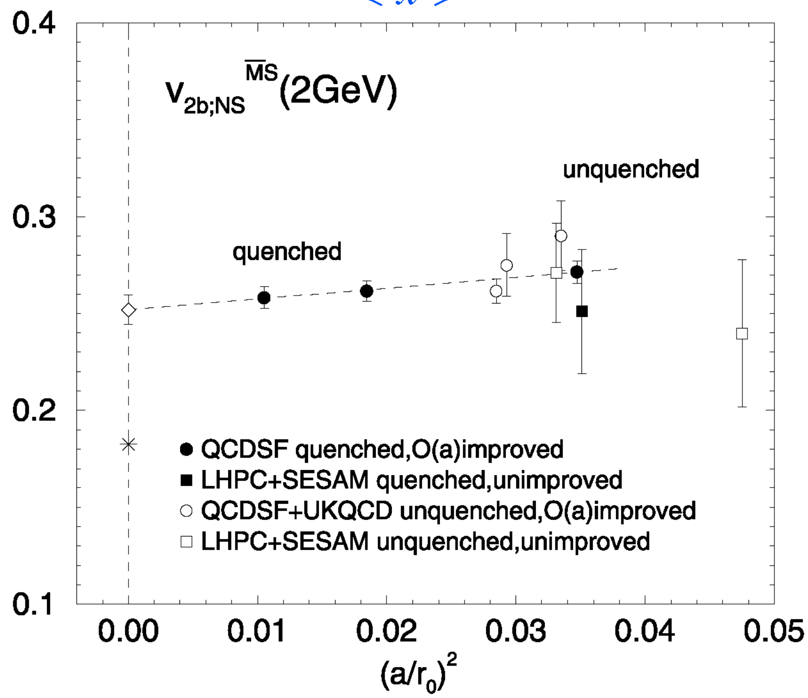
Chiral Extrapolation hep-lat/0209160

$<x>$ quenched improved Wilson - QCDSF

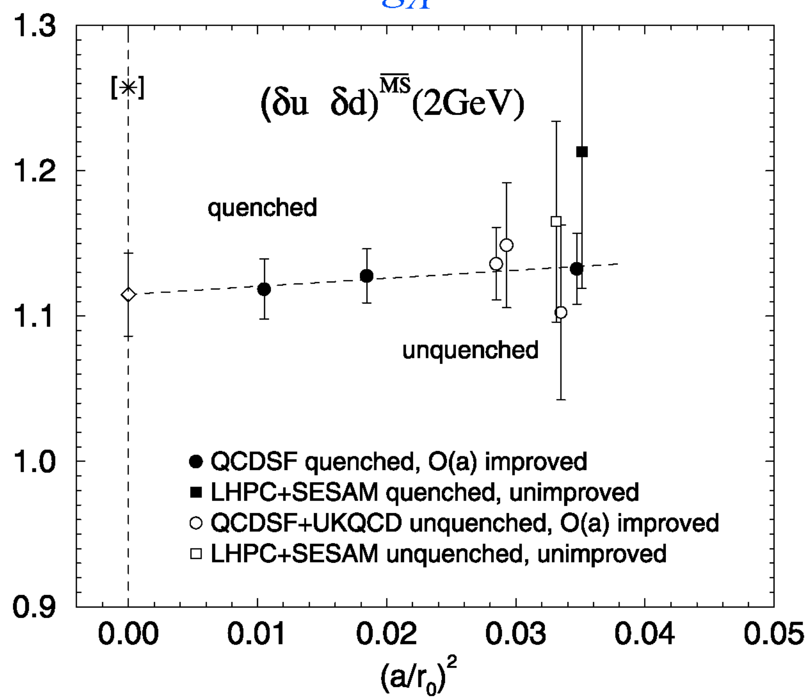


Continuum Extrapolation hep-lat/0209160

$< x >$



g_A



Chiral Perturbation Theory *(cont.)*

Spin distribution

$$\langle x^n \rangle_{\Delta u} - \langle x^n \rangle_{\Delta d} \sim a_n \left[1 - \frac{(2{g_A}^2 + 1) m_\pi^2}{(4\pi f_\pi)^2} \ln(m_\pi^2) \right] + \text{analytic terms}$$

Significant contributions from Δ excitation

Large N_C estimate: cancel $\sim$ 60 % of chiral log

Quenched QCD

Ghost loops cancel sea quark loops

Alters coefficient of chiral logs

Introduces spurious double pole from η'

Partially Quenched QCD

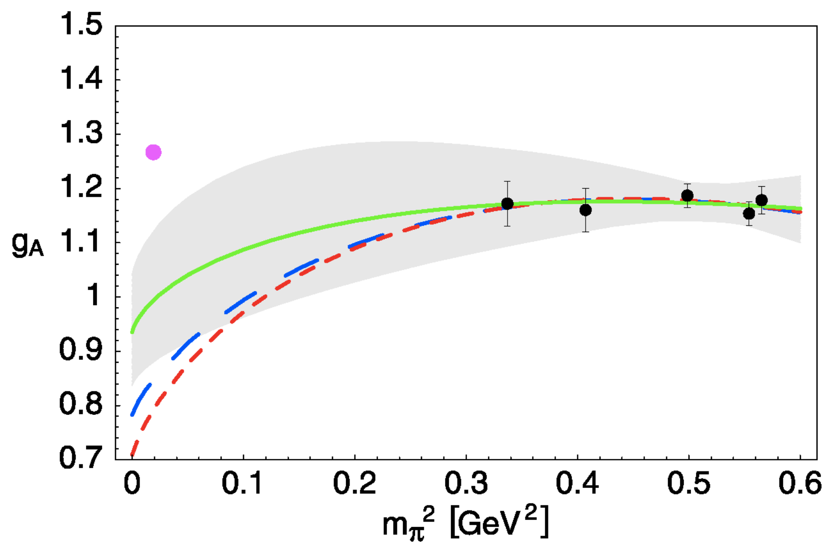
Chiral logs for general m_{sea} and $m_{valence}$

Tool to extrapolate to $m_{sea} = m_{valence} = m_{physical}$

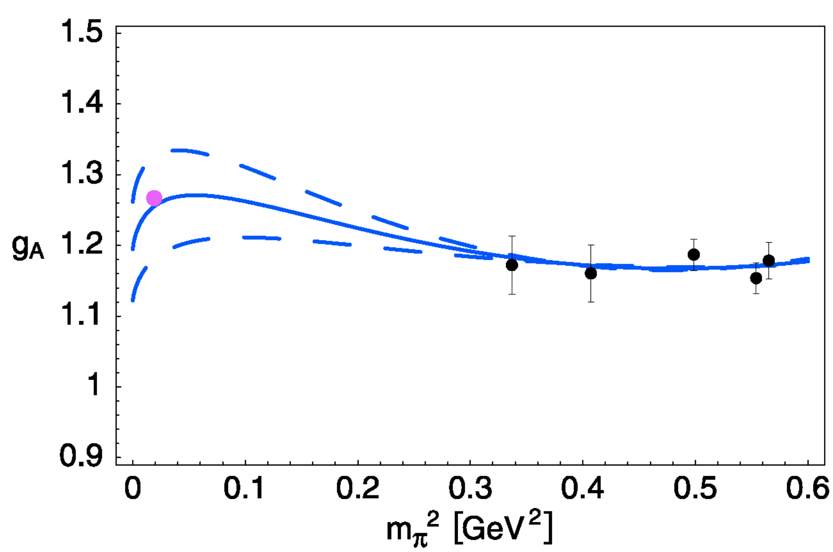
Incorporate finite volume, finite Q corrections

Chiral Extrapolation of $g-A$

- **Free fit with Δ** hep-lat/0303002

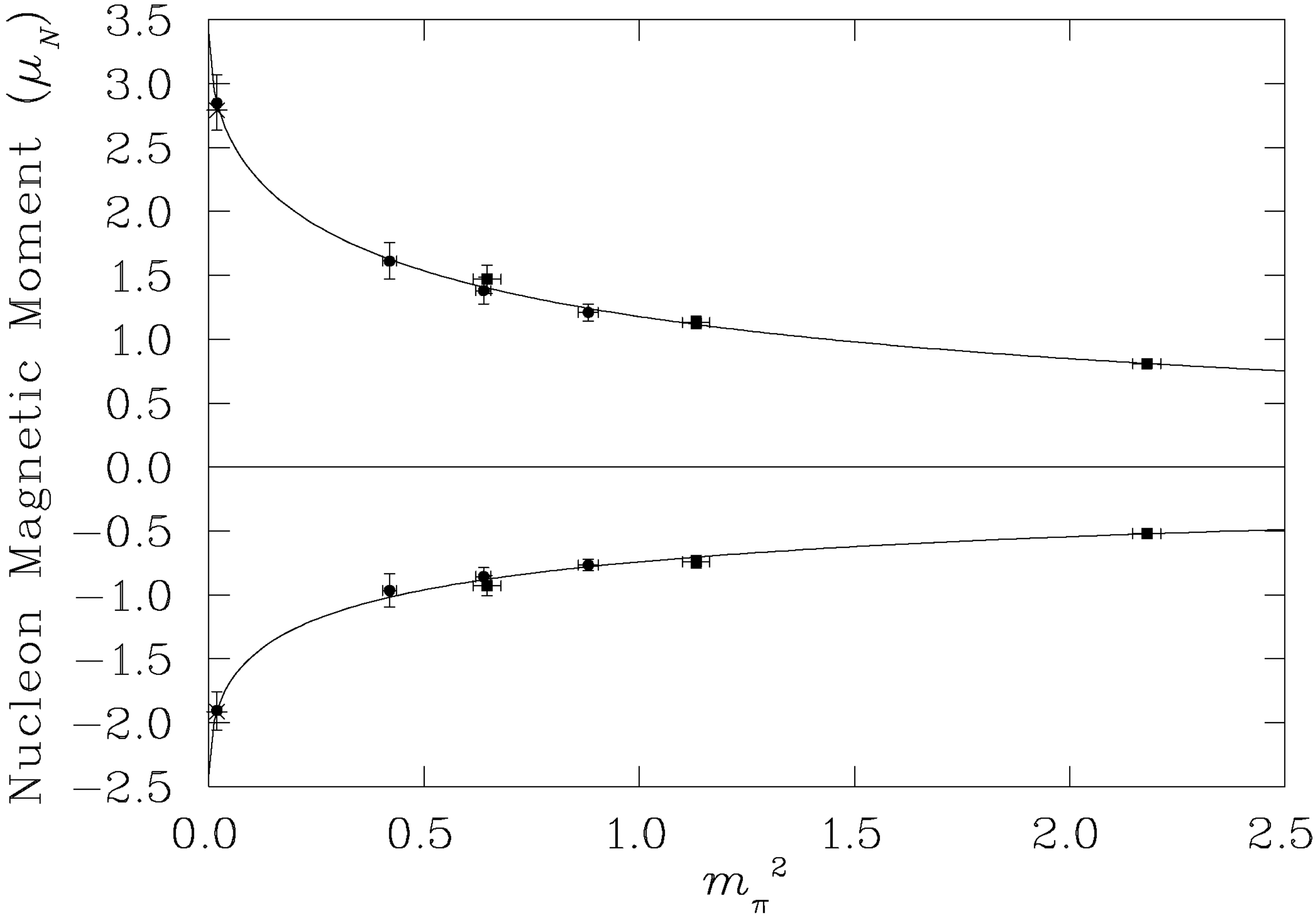


- **Fit constrained by $\pi N \to \pi\pi N$**



Analogous Result for Magnetic Moment

D. Leinweber, D. Lu, and A. Thomas hep-lat/0103006



Definitive Result Requires Terascale Calculation

5% measurement at m_π^2 =0.05 GeV^2

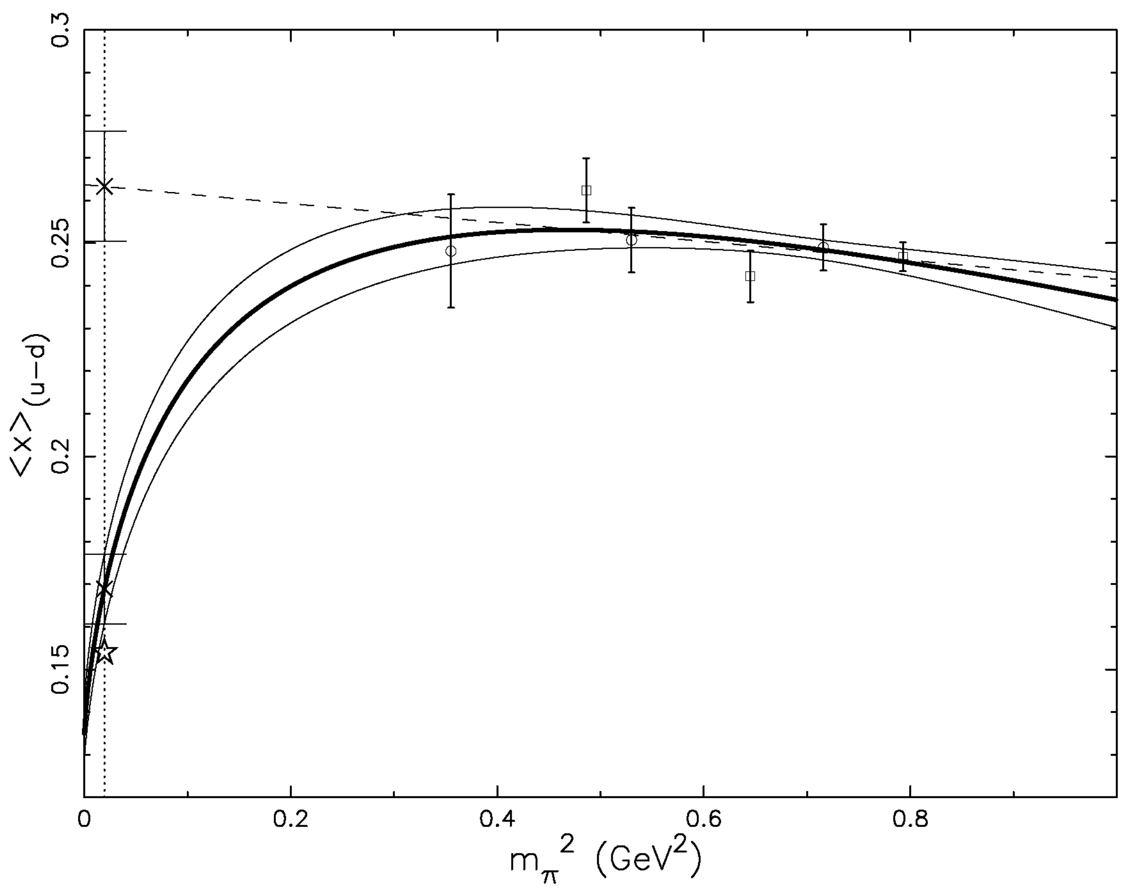
$$m_\pi \sim 230 MeV$$

$$L \sim 4.3 fm.$$

SESAM cost function

$$N_{\rm OPS} \sim 0.38 \left[\frac{L}{4}\right]^{4.55} \left[\frac{0.8}{a}\right]^{7.25} \left[\frac{0.3}{m_\pi/m_\rho}\right]^{2.7}$$

~ 8 Tflops-years

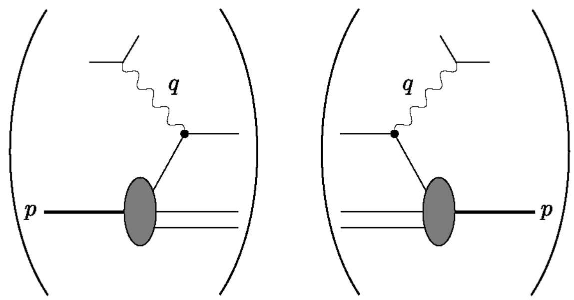


Generalized Parton Distributions

- **Consider twist 2 operators**

$$O^{\mu_1 \dots \mu_n} = \bar{\psi} \gamma^{\{\mu_1} iD^{\mu_2} \dots iD^{\mu_n\}} \psi$$

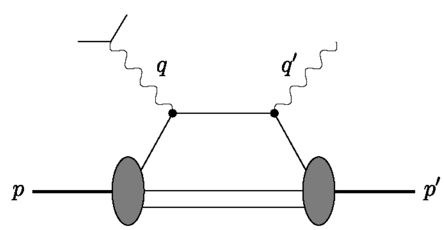
- **Deep inelastic scattering**



Forward M.E. $\longrightarrow$ moment of quark distribution

$$\langle p | O^{\mu_1 \dots \mu_n} | p \rangle \sim \int dx x^{n-1} q(x) p^{\{\mu_1} \dots p^{\mu_n\}}$$

- **Deeply virtual Compton scattering**



$$\langle p | O^{\mu_1 \dots \mu_n} | p' \rangle$$

Off-forward matrix elements

Define generalized form factors

$$\bar{P} = \frac{1}{2}(P' + P), \quad \Delta = P' - P, \quad t = \Delta^2$$

$$\begin{aligned}
\langle p|O^{\mu_1}|p\rangle &\sim \langle\gamma^{\mu_1}\rangle A_{10}(t) \\
&\quad +\langle\sigma^{\mu_1\alpha}\rangle\Delta_\alpha B_{10}(t) \\
\langle p|O^{\mu_1\mu_2}|p\rangle &\sim \langle\gamma^{\{\mu_1}\rangle\bar{P}^{\mu_2\}}A_{20}(t) \\
&\quad +\langle\sigma^{\{\mu_1\alpha}\rangle\Delta_\alpha\bar{P}^{\mu_2\}}B_{20}(t) \\
&\quad +\Delta^{\{\mu_1}\Delta^{\mu_2\}}C_{20}(t) \\
\langle p|O^{\mu_1\mu_2\mu_3}|p\rangle &\sim \langle\gamma^{\{\mu_1}\rangle\bar{P}^{\mu_2}\bar{P}^{\mu_3\}}A_{30}(t) \\
&\quad +\langle\sigma^{\{\mu_1\alpha}\rangle\Delta_\alpha\bar{P}^{\mu_2}\bar{P}^{\mu_3\}}B_{30}(t) \\
&\quad +\langle\gamma^{\{\mu_1}\rangle\Delta^{\mu_2}\Delta^{\mu_3\}}A_{31}(t) \\
&\quad +\langle\sigma^{\{\mu_1\alpha}\rangle\Delta_\alpha\Delta^{\mu_2}\Delta^{\mu_3\}}B_{31}(t)
\end{aligned}$$

Off-forward matrix elements

- **Limits**

  - **Moments of parton distributions** $t \to 0$

    $$A_{n0}(t) = \int dx x^{n-1} q(x)$$

  - **Form factors**

    $$A_{10}(t) = F_1(t) \quad B_{10}(t) = F_2(t)$$

  - **Total quark angular momentum**

    $$J_q = \frac{1}{2}[A_2(0) + B_2(0)]$$

- **t-Dependence**

  $q(x,t)$: Transverse Fourier transform of light cone parton distribution at given x

  x $\to$ 1: Single Fock space component - slope $\to$ 0.

  x $<$ 1: Transverse structure - slope steeper

n=1: Electromagnetic Form Factors

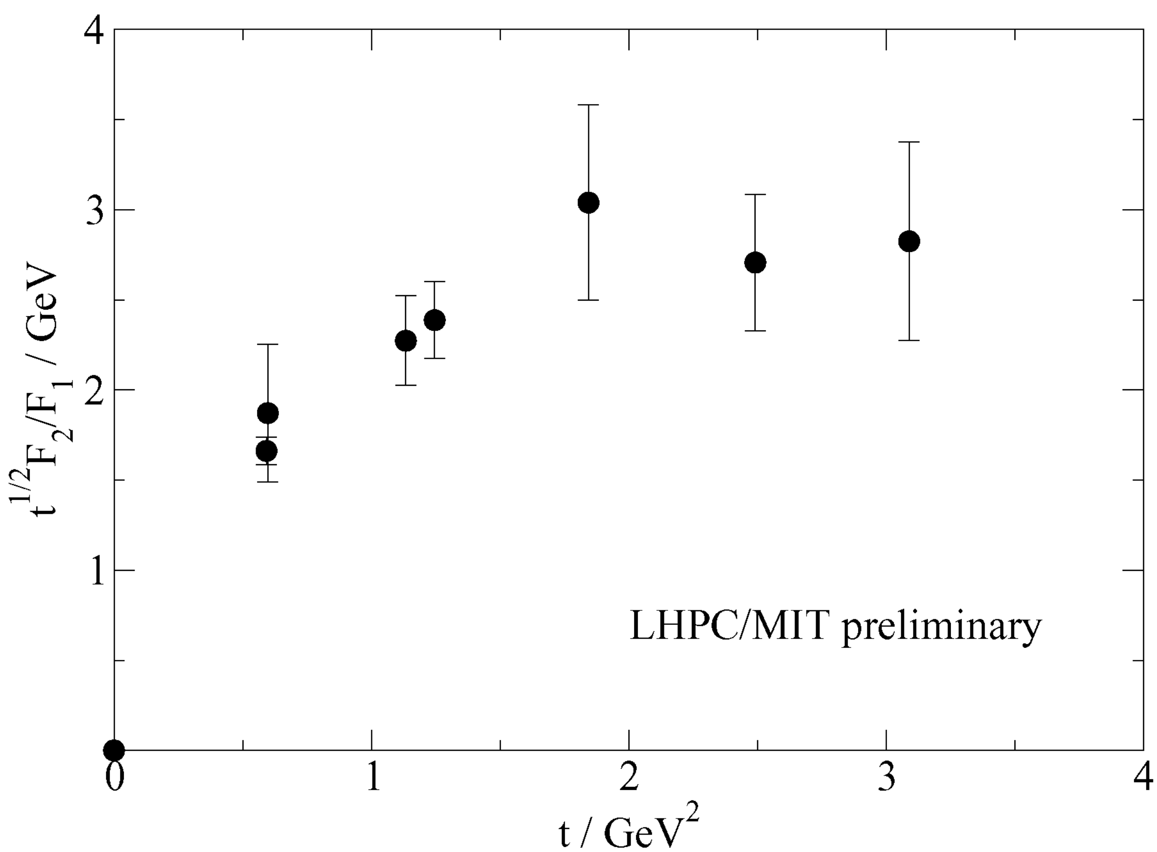
- $Q^2 = t$ **dependence**

$$\frac{Q^2 F_2(Q^2)}{F_1(Q^2)} \sim \textbf{const.} \qquad \textbf{counting rules}$$

$$\frac{Q F_2(Q)}{F_1(Q^2)} \sim \textbf{const.} \qquad \textbf{experiment}$$

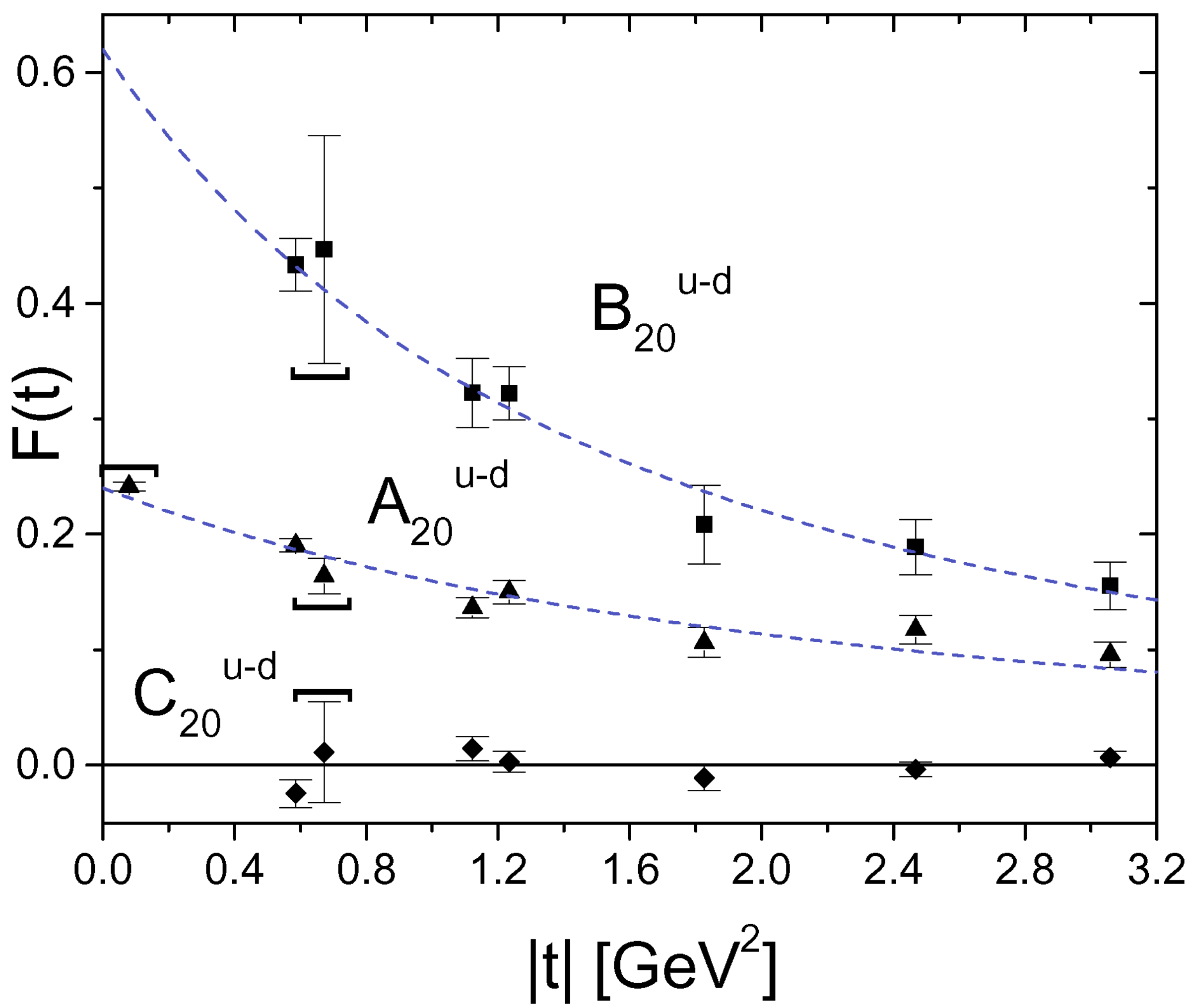
$$\frac{Q^2 F_2(Q^2)}{\log^2(Q^2) F_1(Q^2)} \sim \textbf{const.} \qquad \textbf{hep-ph/0212351}$$

- **Lattice results** $m_\pi \sim 900$ **MeV**



n=2: First x Moments

$A_{20}^{u-d}(t), B_{20}^{u-d}(t), C_{2}^{u-d}(t)$

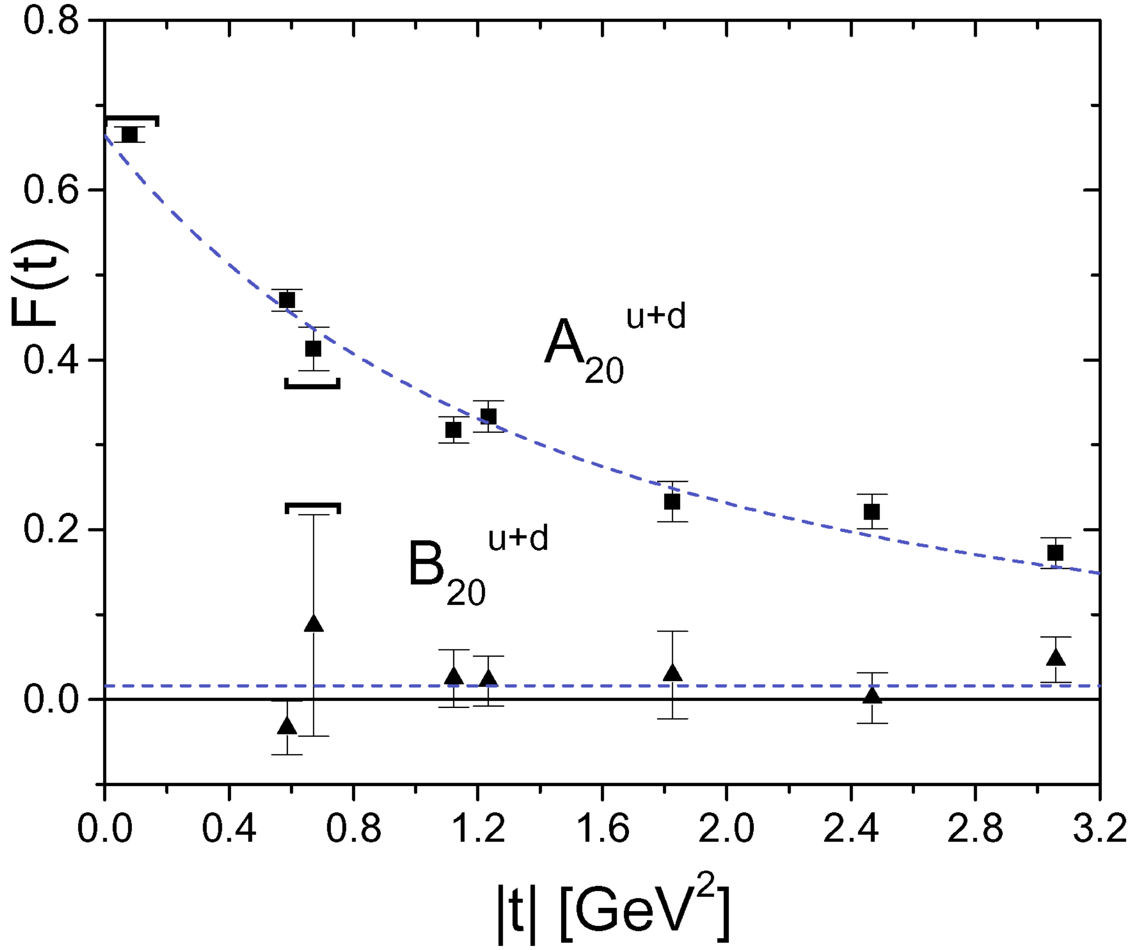


n=2: Quark Angular Momentum

Connected diagrams, $m_\pi = 900$ MeV

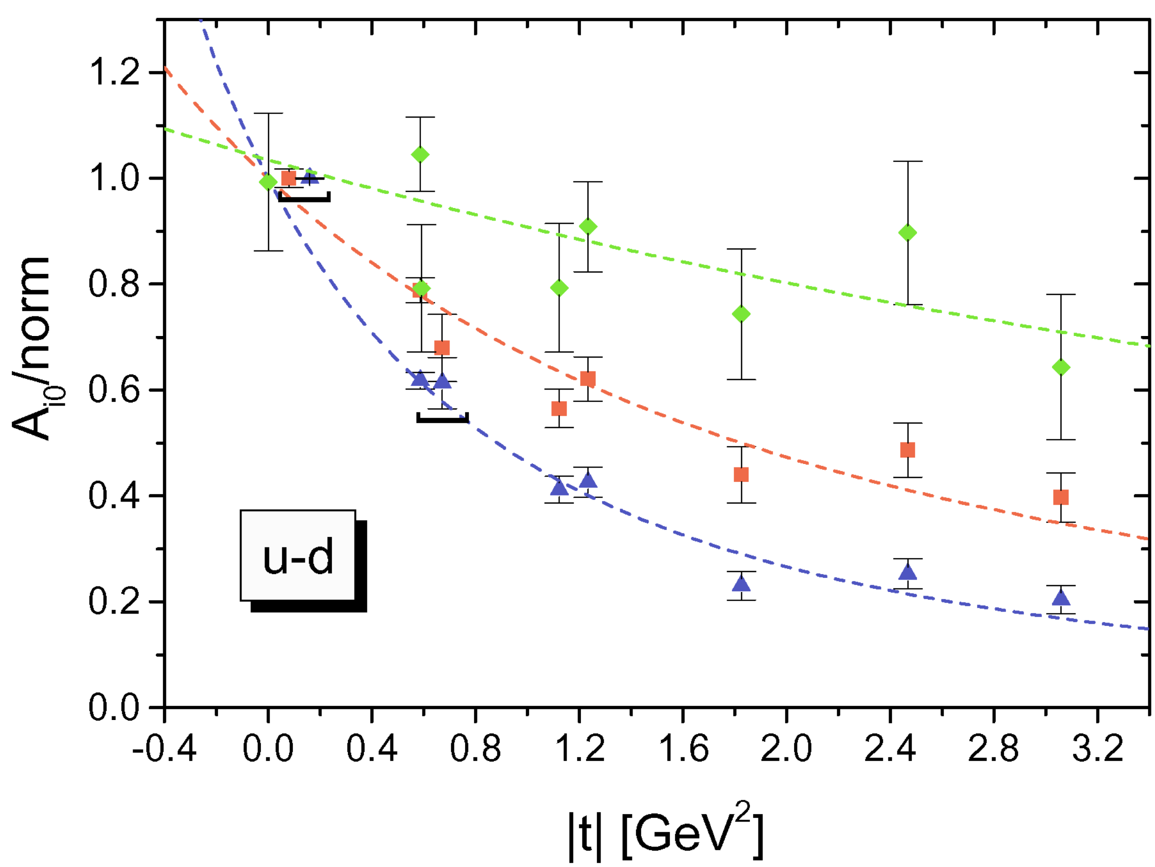
$$\frac{1}{2}\Delta\Sigma = \frac{1}{2}[\langle 1\rangle_{\Delta u} + \langle 1\rangle_{\Delta d}] \sim \frac{1}{2}\,0.682(18)$$

$$J_q = \frac{1}{2}[A_{20}^{\text{u+d}}(0) + B_{20}^{\text{u+d}}(0)] \sim \frac{1}{2}[\langle x\rangle_u + \langle x\rangle_d + 0] \sim \frac{1}{2}0.675(7)$$



n=1, 2, 3: x Dependence of Slope

- **Transverse Fourier transform of light cone parton distribution**
- **Expect slope $\rightarrow$ 0 as $x \rightarrow 1$**
- **Expect higher moments have smaller slope**
- **Lattice results for n = 1, 2, 3 ($m_\pi = 900$MeV)**
- **Factorization Ansatz invalid**



Summary and Prospects

- Striking progress in heavy quark systems

  Full QCD, few percent errors

  Expect decisive impact on weak M.E.

- QCD Thermodynamics

  μ=0 in good shape

  Promising connection with high T pert. theory

  $\mu \neq$0 remains a challenge

- Hadron Structure

  Chiral extrapolation present roadblock - practical, not conceptual

  Expect same success as heavy quark systems with next generation machines

  Hybrid calculations - staggered sea quarks and chiral valence quarks

Shopping List

- Chiral regime
- $\mu \neq 0$, QCD phase diagram
- Disconnected diagrams - flavor singlet sector
- Gluon distributions
- Large N vs instantons
- Confinement mechanism - vortices, merons, $\ldots$
- m_u, m_d strong CP, axion coupling, $m_n - m_p$
- Dependence on m_s