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St. Petersburg,
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Small-x Physics -

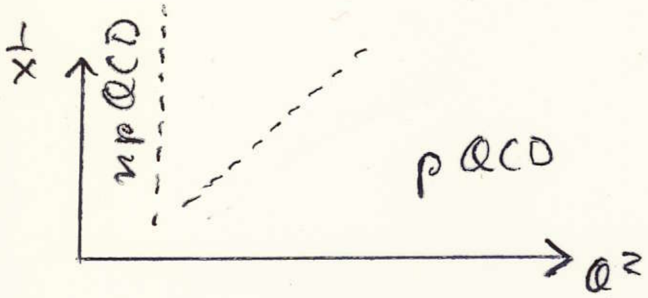
Where are we going?

Small-x Physics:

- novel branch of QCD
- stimulated by HERA measurements
- roots in Gatchina
- created lots of theoretical activities

Introduction

What defines small-x physics:
in DIS



- new kinematic limit: large Q^2 and large $1/x$

  pQCD offers DGLAP and BFKL (CCFM)

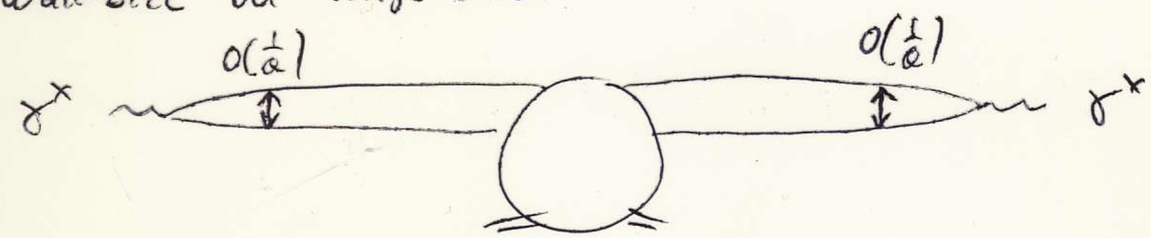
- close to nonperturbative region:

  Regge limit in hadron-hadron scattering

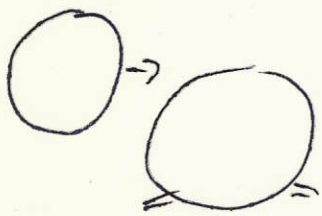
A more physical definition:

- a new type of high energy scattering process

small-size on large-size:



compared to 'old type' hadron-hadron:



- by varying the size of the $q\bar{q}$-pair: continuous transition

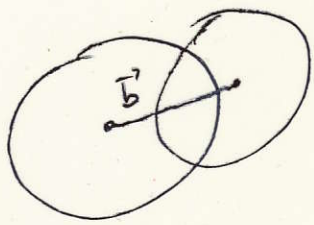
What could we learn from this transition?

- transition from short to long distances in QCD

For comparison: static potential

$$V(r) = -\frac{\alpha_s}{r} + c + \alpha' r$$

High energy scattering process: in transverse plane



Th: $\sigma_{\gamma^*\gamma^*} \sim s^{\lambda > 0}$

$\langle \vec{b}^2 \rangle$ grows exponentially with energy

$T(s, \vec{b}) \sim \frac{1}{(\vec{b}^2)^2}$

"unconfined", Coulomb force

Exp: $\sigma_{pp} \sim s^{\epsilon}$

$\langle b^2 \rangle \sim 2R_0^2 + 2\alpha_P' \ln s$

$(R \sim c \cdot \ln s)$

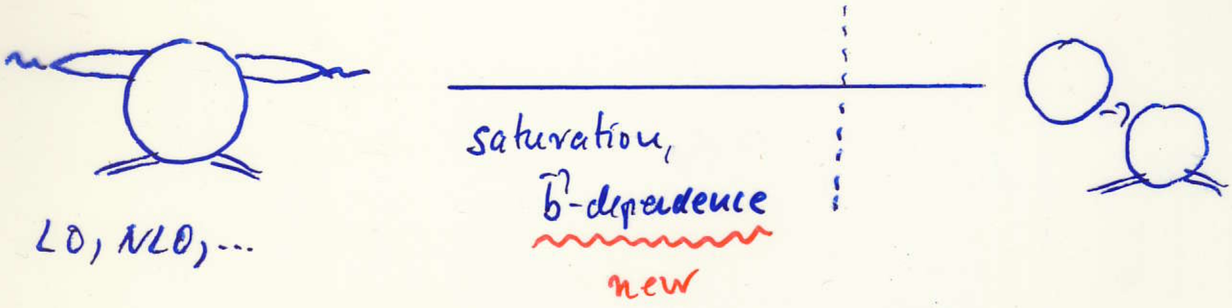
$T(s,b) \sim e^{-b^2/R^2}$

"confined"

unitarity problem

Observation (e.g. this conference):

- move along the route from pQCD to npQCD:



In the following:

- pQCD
- saturation
- $\vec{b}$-dependence

A few theoretical highlights,
wishes for experimental measurements

Short Distance – pQCD

1) DGLAP, collinear factorization

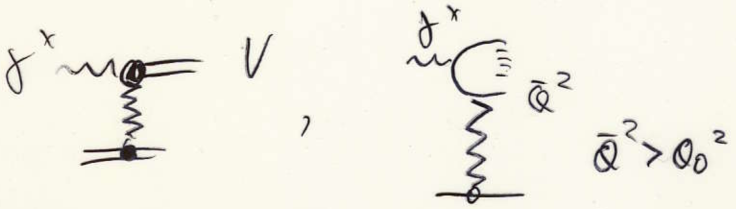
- DGLAP, NNLO — Moch
- DGLAP, resummation + improvement — Altarelli, Thorne
- Observables — Marchesini

Question:

- HERA has measured F_2 in new kinematic region
- How far down in Q^2 and/or x can we describe F_2 by **DGLAP** (leading twist)?

Warnings, doubts:

- **small or even negative gluon at low Q^2** — MRST
- systematic studies of uncertainties — Martin
- curvature of F_2 — Haidt
- Part of diffractive events $\notin$ leading-twist DGLAP — Ryskin



- Higher-twist studies: cancellations inside $F_2 = F_T + F_L$ — JB, Golec-Biernat, Peters

needs to be made quantitative!

Need for F_L:

- interesting per se
- potentially very usefull for testing validity of leading-twist DGLAP

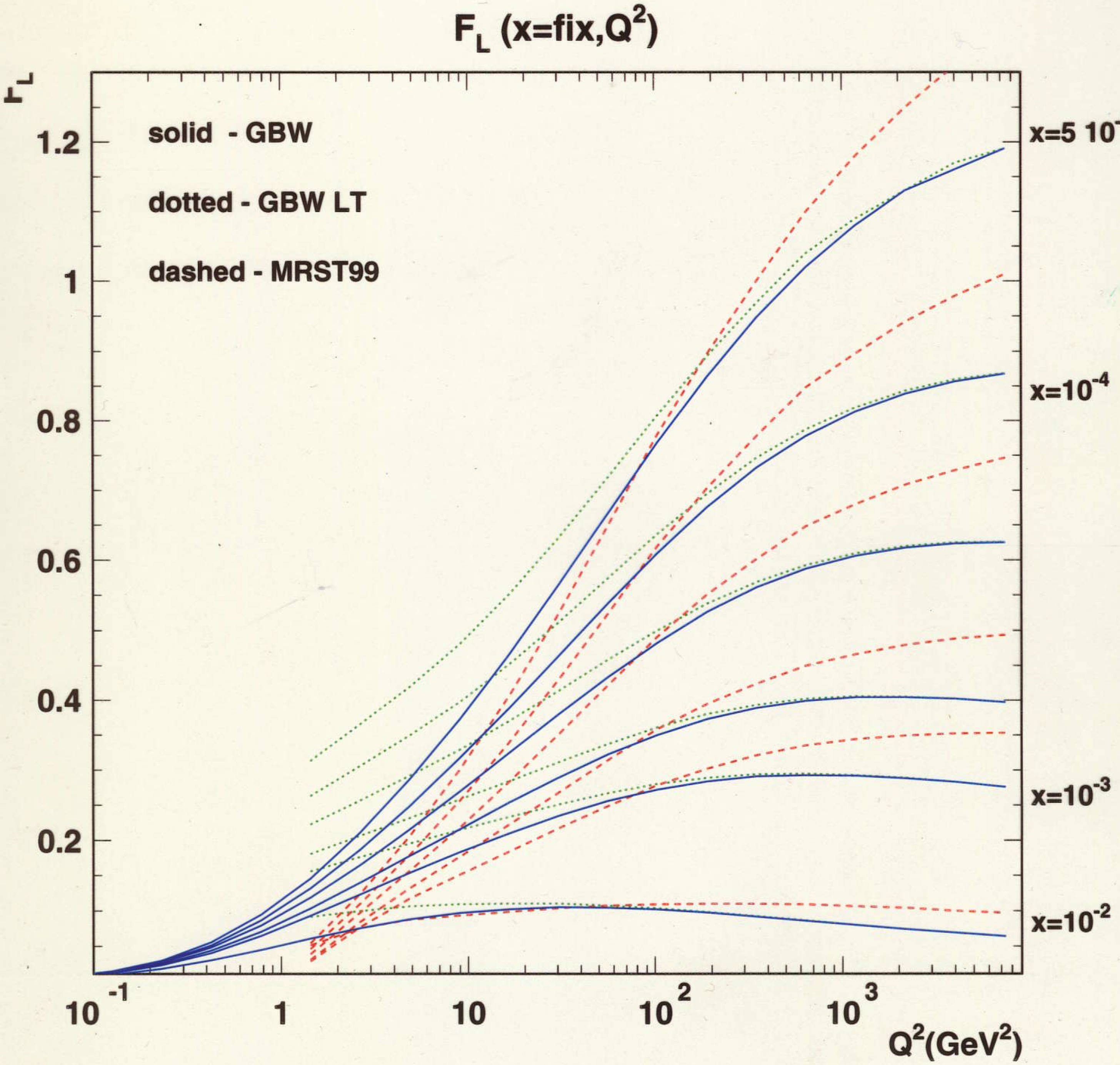
  Thorne

- theoretical argument: F_L could have large twist-4 correction

  → Fig.

→ Please measure F_L !

Golec-Biernat



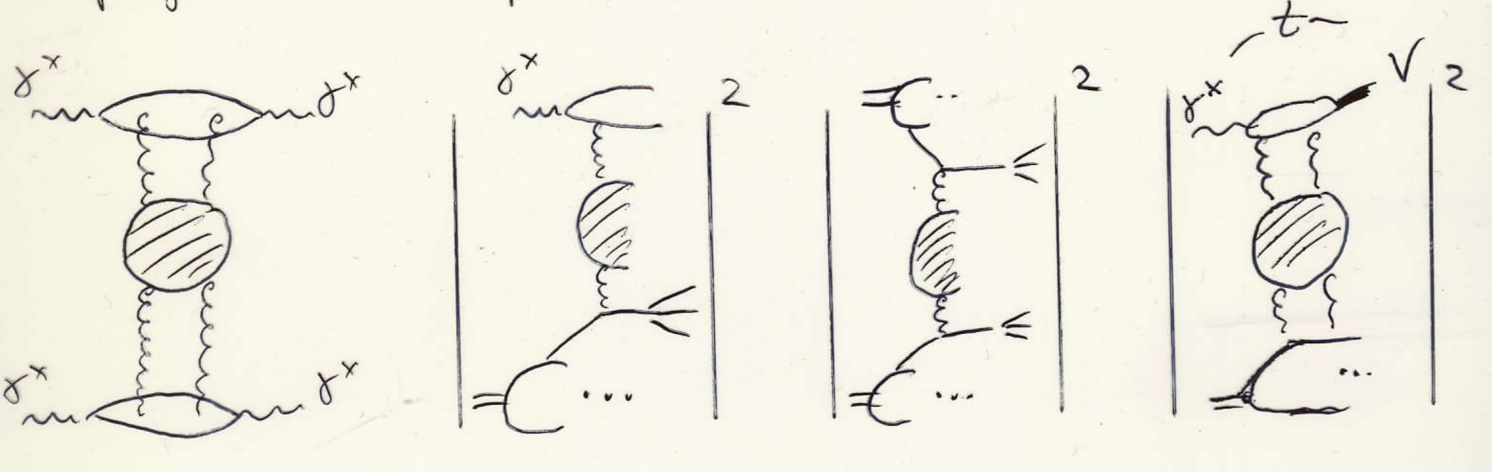
2) BFKL - calculations:

- NLO calculations on the way but not complete

The kernel:

- analytic NLO calculations done for $t=0$ — Fadin, Lipatov; Ciafaloni, Camici
- RG- improvements — Ciafaloni, Salam, Colferai
- numerical algorithms — Ciafaloni, Colferai, Stasto; Sabio-Vera

Couplings to external particles:



$\sigma_{\gamma^*\gamma^*}$ — forward jets — Mueller-Navelet — large-t vector production

Three ingredients:

- photon impact factor ($t=0$)
- jet vertex
- $\gamma^* \to V$ impact factor for $t \neq 0$

Photon impact factor:

- analytic part ✓ — JB, Gieseke, Qiao, Kyrieleis, Colferai
- numerical part on the way — Fadin, Kotsky; Colferai, Ciafaloni

Jet vertex:

- analytic part ✓ — JB, Colferai, Vacca
- numerical part to be done

Kernel for $t \neq 0$:

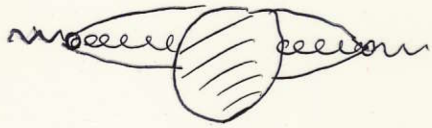
- color octet ✓ — Fadin, Papa, ..
- color singlet: partially done — Fadin, Papa, ..

$\gamma^* \to V$ for $t \neq 0$:

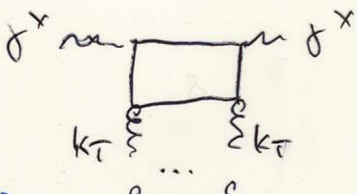
?

Questions to be answered in NLO:

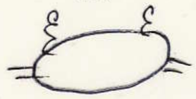
- dipole picture



- k_T-factorization



- unintegrated gluon density



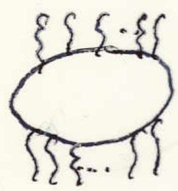
Other BFKL issues:

- bootstrap equations in NLO: fundament of BFKL — BFKL: Braun, Vacca; Fadin, Papa, Fiore

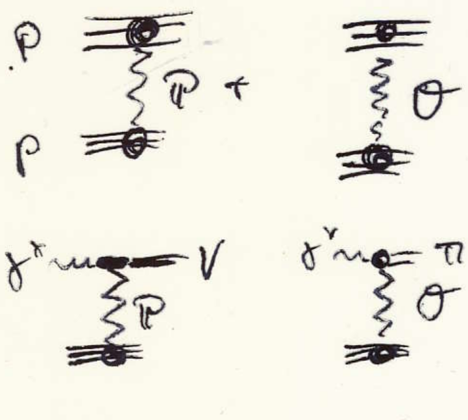
$$T - T^{+} = i T T^{+}$$

set of nonlinear equations
rigorous derivation of BFKL

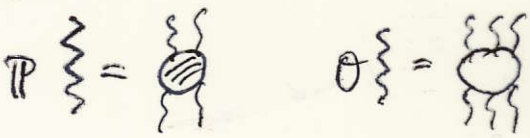
- spectrum of multigluon states — Lipatov



- odderon C-odd partner of Pomeron — ..... Braun



pQCD:



→ Phenomenology

Saturation

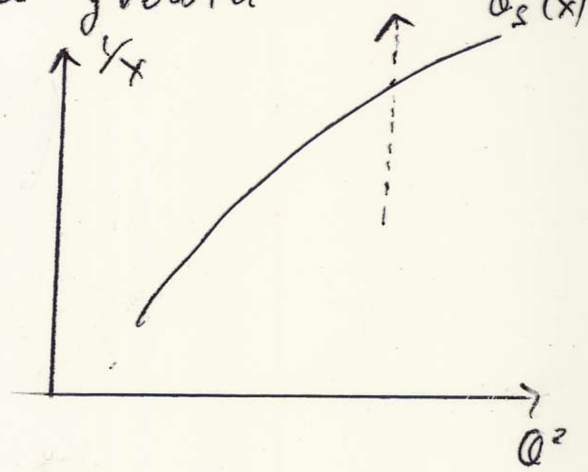
What is it: original idea

Gribov, Levin, Ryskin
Mueller, Qiu

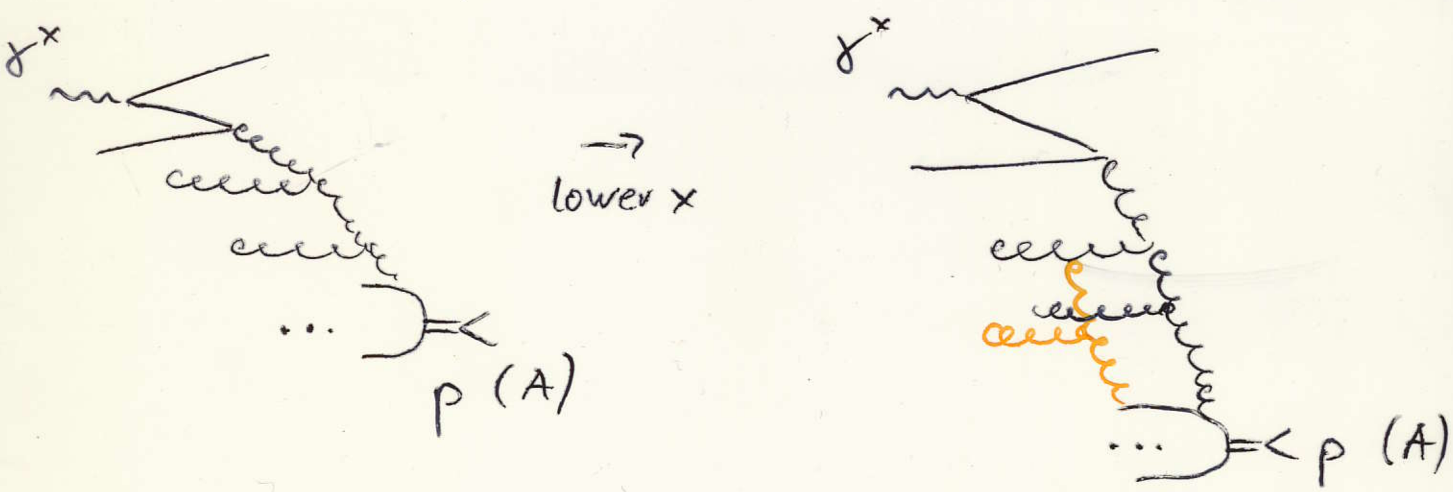
Q^2 not small:

DGLAP predicts rise of $xg(x, Q^2)$
must reach a point in x where gluons are dense
partonic interactions stop the growth

new scale $Q_s^2(x)$



Physical picture: (frame where proton (nucleus) is fast)

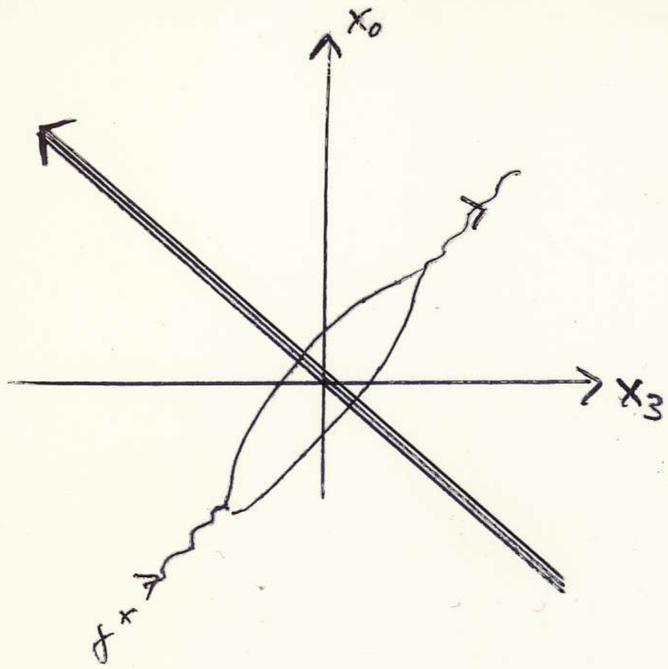


→ still lower x

large number of "cascades" which interact

strong field: universal feature of QCD

Modern formulation: Color Glass Condensate



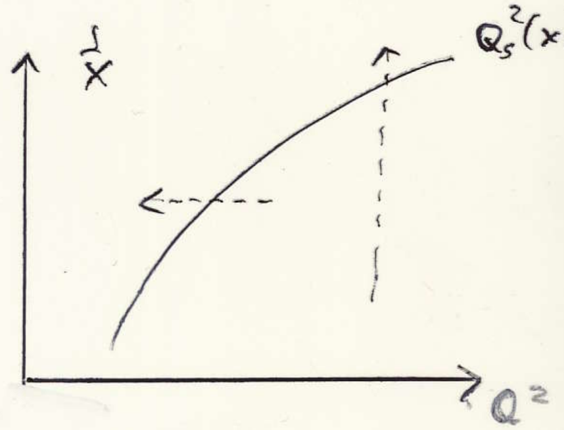
photon "sees" strong color field which remains frozen during interaction

Have we seen saturation in DIS?

Golec-Biernat + Wüsthoff

Evidence from success of models which are based upon saturation idea

(DIS: mainly at low Q^2)

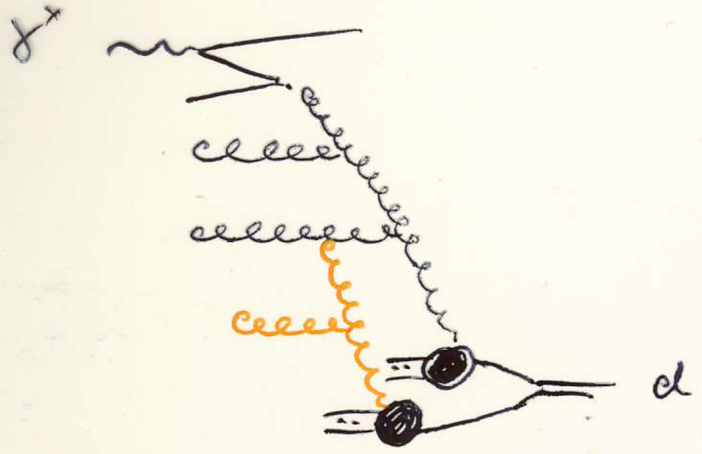


Many modifications, alternative models, new applications

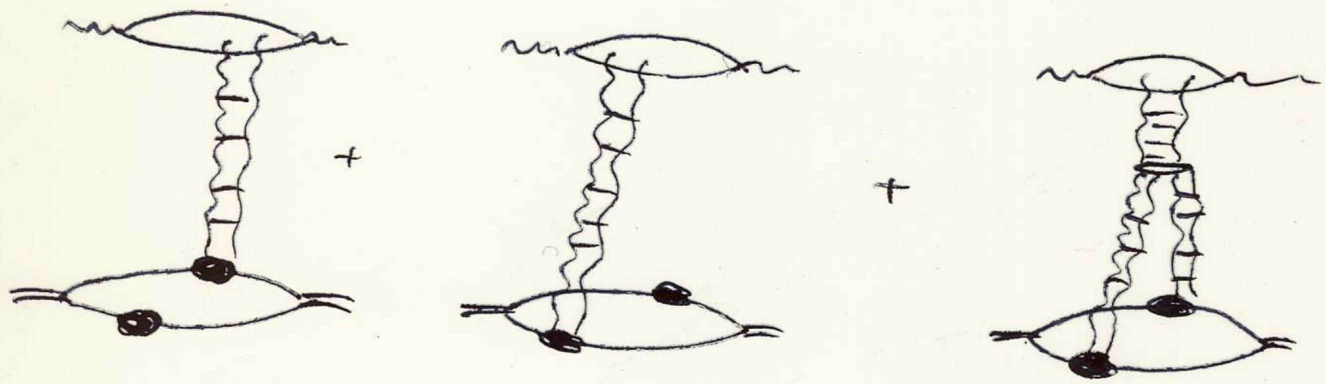
Levin et al.

How could we see saturation more directly:

DIS on nuclei, e.g. deuterium



larger probability to see two gluons in deuterium than in single proton



negative correction

$$\frac{F_2^d}{F_2^p + F_2^n} < 1 \quad \text{at small } x$$

Effect amplifies at larger A.

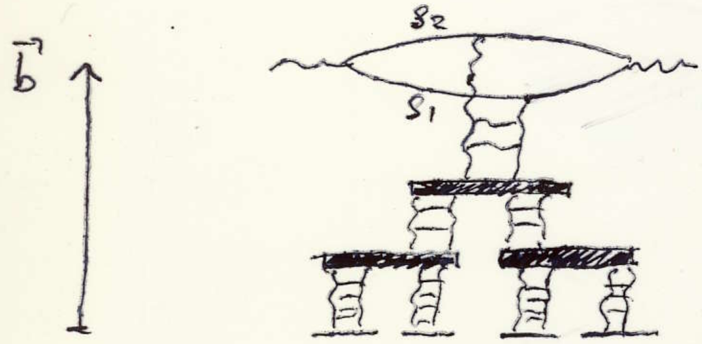
Need DIS on Nuclei, e.g. deuterium

How to investigate Saturation:

different (but: equivalent) approaches

- evolution equations for operators — Balitsky
- fan diagrams on a nucleus — Kovchegov
- functional approaches — Weigert, Kovner, Iancu, McLerran, ...

BK-equation: sums up fan diagrams



$$\frac{\partial N}{\partial Y} = \alpha_s K_{BFKL} \otimes N - \alpha_s K_{PPP} \otimes N N$$

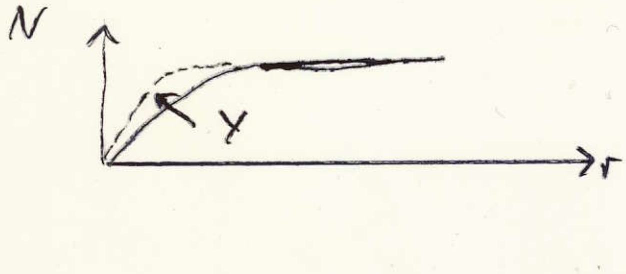
conformal invariance

When applied to single proton: approximations (large N_c)
absence of loops

Solutions:

- semi-analytic — Levin, Tuchin
- numerical, at fixed $\vec{b}$ — Braun; Golec-Biernat, Motyka, Stasto; Levin, Lublinsky, et al.

confirms GBW-model

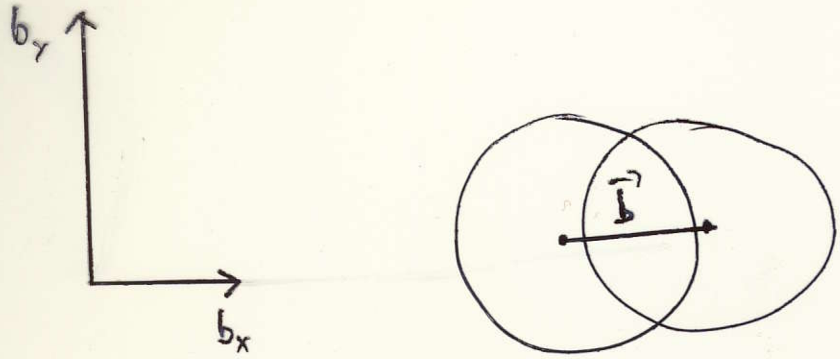


Next Step: $\vec{b}$-dependence

What is missing so far:

transverse dimensions

Diehl

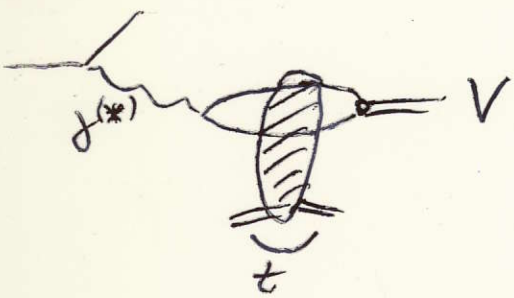


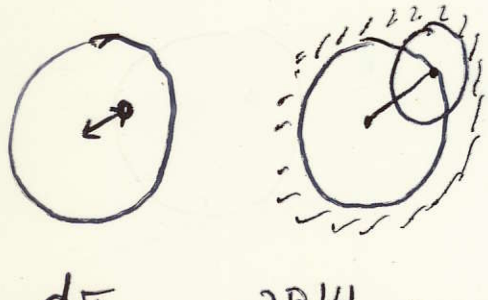
$$T(s, t=-\vec{k}^2) = is\int d^2b\, e^{i\vec{k}\vec{b}} f(s,\vec{b})$$

At $t=0$: only average over all $\vec{b}$

How to reach $t \neq 0$:

DIS diffraction

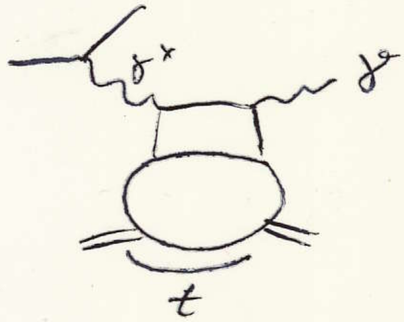


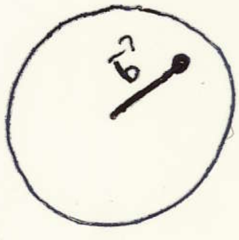


$$\frac{d\sigma}{dt} \sim e^{-2B|t|}, \quad B = B(Q^2, W^2)$$

$$\langle b^2 \rangle \sim B$$

Deeply Virtual Compton Sc.





t-dependence of GPD's

Need data on DVCS and DIS Diffraction:

t- slopes

Need modelling:

- model for b-dependent dipole cross section:

$$\sigma_{q\bar{q}}(\vec{r}, \vec{b}, x)$$

Kowalski

Lipatov

Navelet, Peschanski

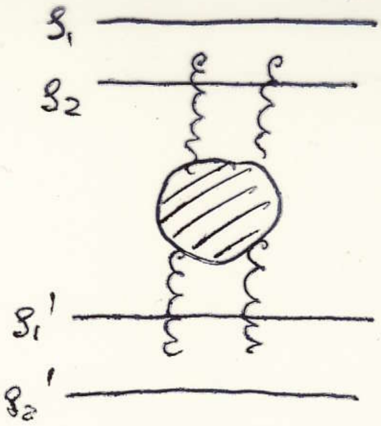
Ryskin

Kozlov

Levin

Theory:

1) BFKL in $\gamma^* \gamma^*$:



$$N(\rho_i, \rho_i'; Y) \sim \sum_n \int \frac{d\nu}{2\pi i}\, e^{Y \chi_{BFKL}(n,\nu)}\, |X|^{2\nu}\, \Phi(\nu)$$

$$X = \frac{\rho_{12}\, \rho_{1'2'}}{\rho_{11'}\, \rho_{22'}}$$

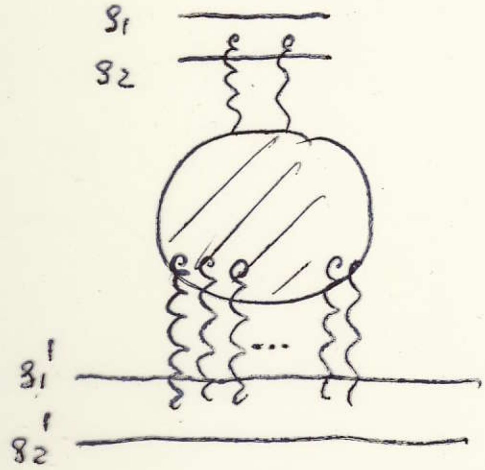
$$\sim s^{\omega_{BFKL}} \frac{|\rho_{12}|\,|\rho_{1'2'}|}{|\vec{b}|^2} \qquad R^2(s) \sim s^{\omega}$$

gluon cloud grows fast
(Coulomb force)
"unconfined system"

2) Analysis of Balitsky-Kovchegov equation:

JB, Ryskin, Vacca

a) conformal invariance:



$$N(\rho_i, \rho_i'; Y) = N\left(\frac{\rho_{12}\,\rho_{1'2'}}{\rho_{11'}\,\rho_{22'}}, y\right)$$

if $Q_s^2(x) \sim \left(\frac{1}{x}\right)^{\lambda}$

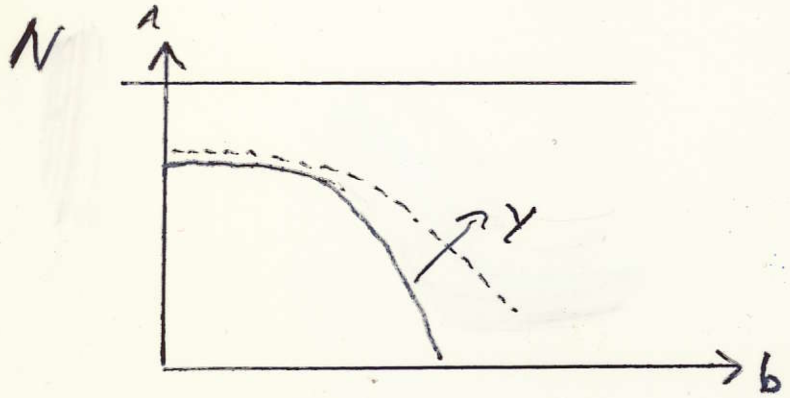
then $R^2 \sim \left(\frac{1}{x}\right)^{\lambda/2}$

3) Numerical analysis:

Stasto, Golec-Biernat

Initial condition: $y = \ln \frac{1}{x} = 0$

$$N \sim 1 - e^{-r^2 S(b)} \quad , \quad S(b) = S_0 e^{-b^2/R^2}$$



from exponential fall-off at $Y=0$
to power-like fall-off at larger Y

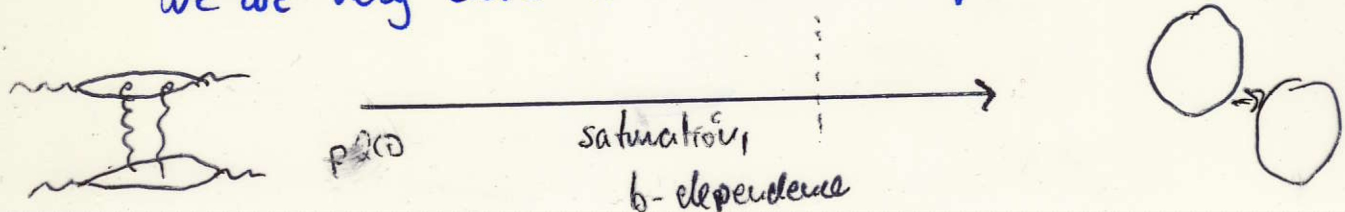
From this:

- "positive": present tools allow to describe "saturation" with suitable initial conditions realistic picture for range of energies

- "conservative": so far, we have not left pQCD need to include npQCD corrections into evolution kernels

In any case:

we are very close to 'transition point'



Conclusions

Past and present:

- small-x story has started at HERA
- has stimulated a lot of activities
- "small-x virus" has spread out, from DIS to Heavy Ion (RHIC)
- move from safe grounds of pQCD (NLO, NNLO) into more adventurous dynamics of QCD (Saturation)

Future:

- Experimental data (F_L, deuteron, DVCS/Diffraction)
- region of applicability of DGLAP
- NLO BFKL-theory: vertices, k_T-factorization
- more on saturation: models, experimental signals
- transition from saturation to confinement: large b-behavior, pion cloud

Great help can come from future e^+e^- Collider:
$\gamma^*\gamma^*$ cross sections are the cleanest probe for small-x physics!