Minimization of Transverse Emittance Growth Due to CSR

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1.

Emittance Growth

- Numerically and experimentally: Emittance growth depends sensitively on initial Twiss parameters
- Numerical method: parameter scan
- Time-consuming: can it be avoided?

- Emittance growth within dispersive regions is partially spurious; subtract dispersion
- Emittance growth is caused by
  - Nonlinear transverse forces
  - Longitudinal forces in dispersive regions

Consider a system of $D \geq d_\varepsilon$ degrees of freedom. We are interested in the correlation matrix C_ε in the first d_ε degrees of freedom. The system evolves according to

$$\frac{\mathrm{d}}{\mathrm{d}s}x = T(s)x + F(s) \quad , \tag{1}$$

where T describes a unimodular (symplecticity is too strong a condition here) behavior.

With

$$\frac{\mathrm{d}}{\mathrm{d}s}M(s) = T(s) \quad , \quad M(0) = 1 \tag{2}$$

and

$$I(s) = \int_0^s M^{-1}(s')F(s')\mathrm{d}s' \tag{3}$$

the system is determined by its initial conditions according to

$$x(s) = M(s)[x(0) + I(s)] \quad . \tag{4}$$

Now we are interested in the system's correlations matrix:

$$
\begin{aligned}
C(s) = \left\langle x(s)x^{\top}(s)\right\rangle &= M(s)\left(C_0(0)\right.\\
&+ \int_0^s\int_0^s M^{-1}(s_1)\left\langle F(s_1)F^{\top}(s_2)\right\rangle {M^{-1}}^{\top}(s_2)\mathrm{d}s_1\mathrm{d}s_2\\
&\left.+ \int_0^s\left\langle x(0)F^{\top}(s_1)\right\rangle \mathrm{d}s_1 + \text{transp.}\right) M^{\top}(s)\\
&= M(s)(C_0 + C_{FF} + C_{Fx} + C_{Fx}^{\top})M^{\top}(s) \quad , \qquad (5)
\end{aligned}
$$

so the system behaves as if it had been transported linearly with an initial correlation given by the middle term in the last line.

Emittance Growth by Dispersion Mismatch

The equations of motion for a single particle in a lattice of focal strength n and curvature ρ without external forces read

$$\frac{\mathrm{d}}{\mathrm{d}s}\begin{pmatrix} x(s) \\ p(s) \\ \delta(s) \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ n(s) & 0 & \rho(s) \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} x(s) \\ p(s) \\ \delta(s) \end{pmatrix} \quad , \tag{6}$$

where we restrict all considerations to x, p, δ-subspace (which is approximately closed under the CSR-beam-interaction, if all dipoles bend in the same plane) of the full phasespace.

We can introduce new coördinates by

$$\begin{pmatrix} \bar{x}(s) \\ \bar{p}(s) \\ \delta(s) \end{pmatrix} = \begin{pmatrix} 1 & 0 & -\eta(s) \\ 0 & 1 & -\eta'(s) \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x(s) \\ p(s) \\ \delta(s) \end{pmatrix} \quad . \tag{7}$$

In these coördinates, the equations of motion are

$$\left.\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}s}\bar{x}(s) &= \bar{p}(s) - \eta(s)\frac{\mathrm{d}}{\mathrm{d}s}\delta(s) \\ \frac{\mathrm{d}}{\mathrm{d}s}\bar{p}(s) &= n(s)\bar{x}(s) - \frac{\mathrm{d}}{\mathrm{d}s}\eta(s)\frac{\mathrm{d}}{\mathrm{d}s}\delta'(s) + \\ &\quad + \delta(s)\underbrace{\left(\rho(s) - n(s)\eta(s) - \eta''(s)\right)} \end{aligned}\right\} \quad . \tag{8}$$

where the underbraced term vanishes if we choose η to be the usual dispersion function. Then, (8) has the same form as (2) with an inhomogeneity of

$$F(s) = \delta'(s)\begin{pmatrix}\eta(s)\\ \eta'(s)\end{pmatrix} \quad . \tag{9}$$

Let's assume the energy kicks are uncorreleated across the bunch. Then, the total emittance is (because $\det(M) = 1$)

$$\epsilon^2 = \det(C_0 + C_{FF}) \quad . \tag{10}$$

For a transport line, we are free to choose the shape of C_0, while its determinant is fixed by the initial emittance. It is easy to see that (10) becomes minimal if we choose $C_0 \propto C_{FF}$, in which case $\epsilon^2_{total} = \epsilon^2_0 + \epsilon^2_{FF}$. In terms of distibutions this means that the final distribution is the convolution of the initial distribution subjected to the linear forces only, and a zero-emittance-distribution subjected to the perturbation and the linear forces.

The problem becomes significantly simpler when the energy noise is white, i. e., if $\langle \delta'(s_1)\delta'(s_2)\rangle = \delta(s1-s2)\left\langle \delta^2(s)\right\rangle'$, where $\left\langle \delta^2(s)\right\rangle'$ is the noise amplitude squared. In this case, we can write down the optimum values for the initial Twiss parameters in terms of the transport matrix and the noise amplitude:

$$\begin{aligned}
\alpha_0(0) &= \lambda \int_0^L \left\langle \delta^2 \right\rangle'(s) \left(M_{12}(s)\eta(s)' - M_{22}(s)\eta(s)\right)\left(M_{11}\eta(s)' - M_{21}\eta(s)\right) \mathrm{d}s \\
\beta_0(0) &= \lambda \int_0^L \left\langle \delta^2 \right\rangle'(s) \left(M_{12}\eta' - M_{22}\eta\right)^2 \mathrm{d}s \\
\gamma_0(0) &= \lambda \int_0^L \left\langle \delta^2 \right\rangle' \left(M_{11}\eta' - M_{21}\eta\right)^2 \mathrm{d}s
\end{aligned} \tag{11}$$

where λ has to be chosen such that $\beta(0)\gamma(0) = 1 + \alpha(0)^2$ and $\beta > 0$.

For purely longitudinal dependence of F: emittance can be minimized by matching C_0 to C_{FF}.

For 'fat' bunches: C_{Fx} matters: scan parameter space for minimal emittance or use a descent method.

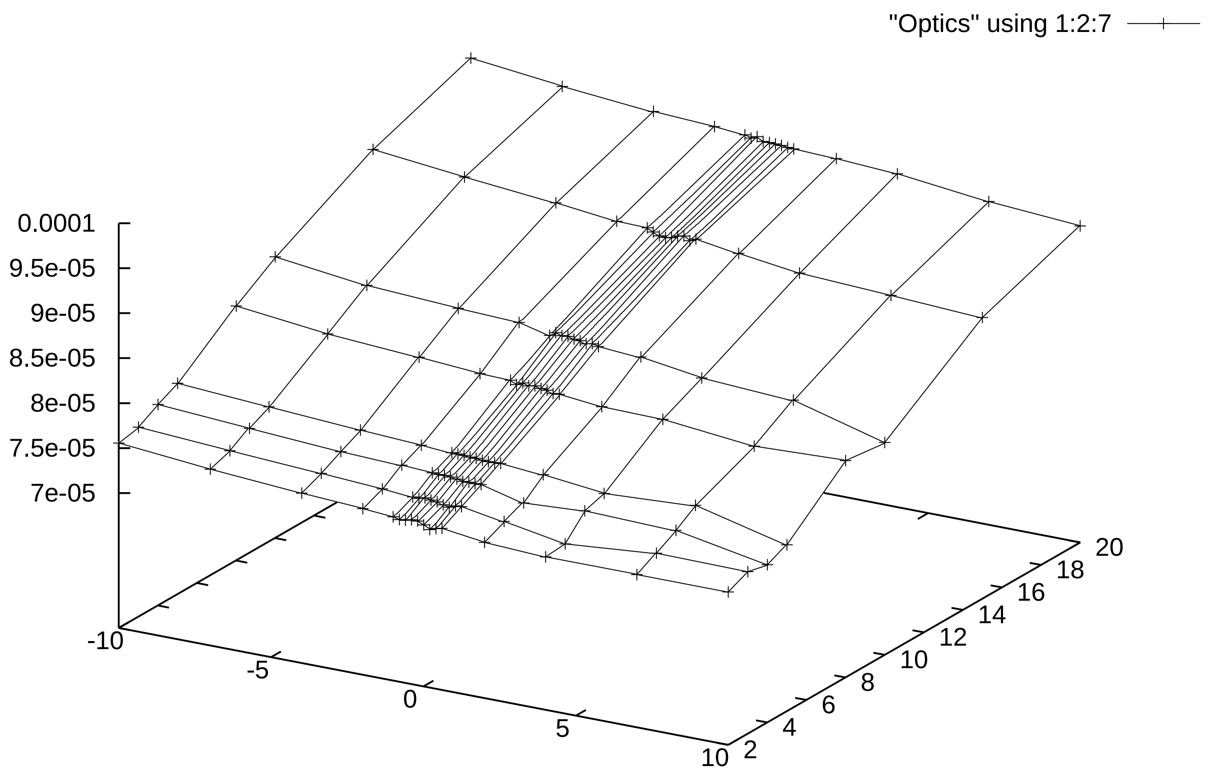


Figure 1: Emittance growth from numerical simulation for a single bend: $R = 2\text{m}, L = 1\text{m}, q = 1\text{nC}, E = 40.7\text{MeV}, \varepsilon_{\text{normalized}} = 70 \cdot 10^{-6}\text{m}$

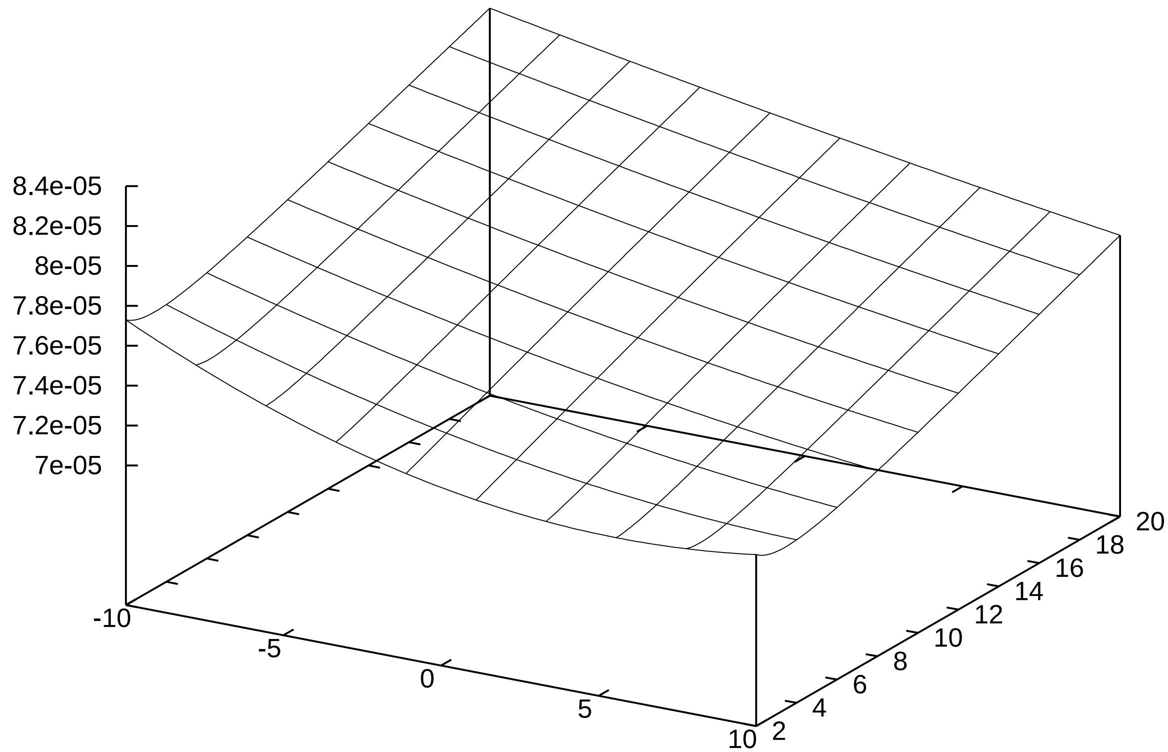
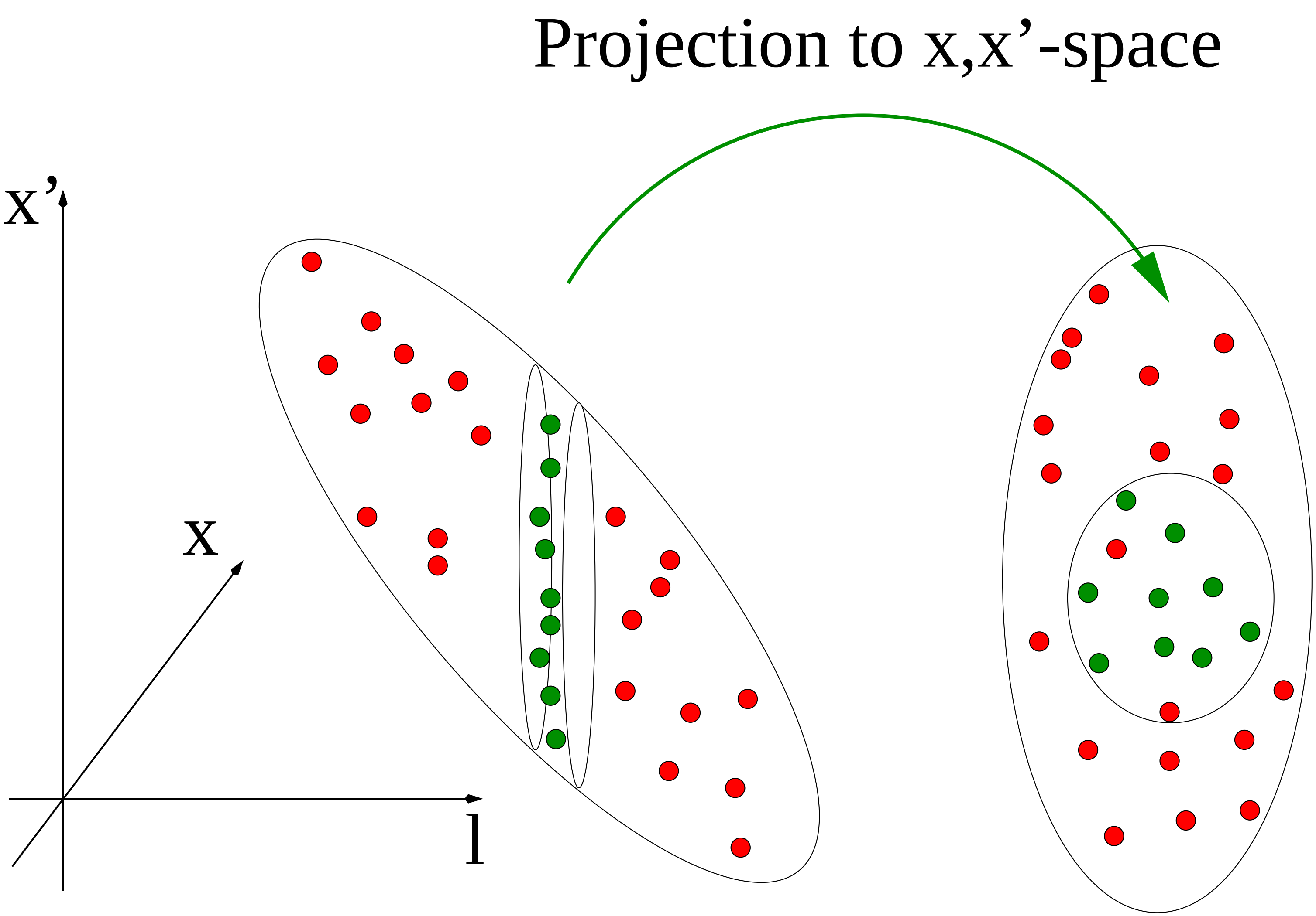


Figure 2: Emittance growth from C_{FF} extrapolation at $\alpha = 0, \beta = 10$m

For FEL operation, beam quality is crucial.

The FEL process involves only particles within a certain longitudinal range ("slippage length").

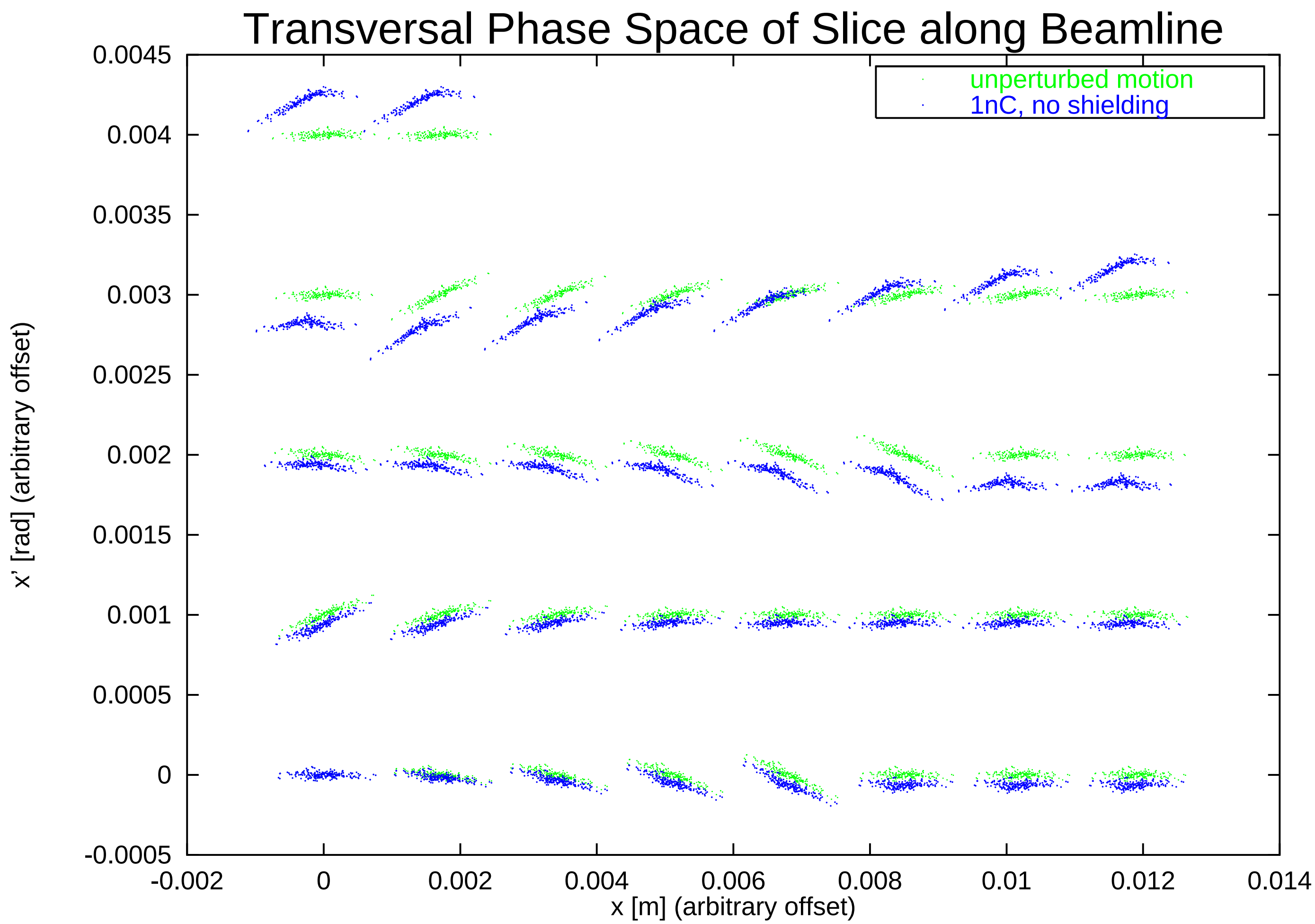
This range may be much smaller than the bunch length:



For Phase 1 of TTF-FEL, the slippage length $\Delta l \approx 10\mu$m $\ll l_{\text{final}} = 250\mu$m

$\rightarrow$ Projective emittance misleading for judging beam quality for FEL operation.

(But important for optics considerations)


Transversal Phase Space of Slice along Beamline
unperturbed motion
1nC, no shielding
x' [rad] (arbitrary offset)
x [m] (arbitrary offset)
0.0045
0.004
0.0035
0.003
0.0025
0.002
0.0015
0.001
0.0005
0
-0.0005
-0.002
0
0.002
0.004
0.006
0.008
0.01
0.012
0.014