Avoiding Death by Vacuum Decay
Cutting on the MSSM parameter space

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The end is nigh...
“I think, we have it!”
Rolf Heuer
“Eureka!”
Archimedes
What do we have?
Having pinned down the missing piece...

Back again


\[ M_H \text{ (GeV)} \]

\[ \Lambda \text{ (GeV)} \]

Theoretical considerations

- $\lambda < 4\pi \rightarrow$ perturbativity
- $\lambda > 0 \rightarrow$ unbounded from below, aka vacuum stability
Constraining the SM Higgs sector

Theoretical considerations
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trivial at the classical (i.e. tree) level

$$V(\Phi) = m^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2$$

Quantum level
- dominant contribution: top quark ($y_t \sim 1$)
- dependence on the energy scale: $\beta$ function!

[courtesy of Max Zoller]
Energy scale dependence

Scale-independent loop-corrected effective potential

$$Q \frac{d}{dQ} V_{\text{loop}}(\lambda_i, \phi, Q) = 0$$

Approximation for large field values

$$V_{\text{loop}}(\phi) = \lambda(\phi)\phi^4,$$
evaluated at $$Q \sim \phi$$

$$\beta_i(\lambda_i) = Q \frac{d \lambda_i(Q)}{dQ}$$

- running of $$\lambda$$ determines stability of the loop potential
- upper bound: Landau pole; lower bound: $$\lambda > 0$$
Precise analysis: up to three loops!

\[ \lambda(\mu) \]

\[ \text{Log}_{10}[\mu/\text{GeV}] \]

3 loop
2 loop
1 loop

[Zoller 2014]
Precise analysis: up to three loops!

\[ \lambda(\mu) \]

\[ \text{Log}_{10}[\mu/\text{GeV}] \]

- 3 loop
- 2 loop
- \( \alpha_s = 0.1185 + 0.0006 \)
- \( \alpha_s = 0.1185 - 0.0006 \)
- \( M_H = 125.7 + 0.4 \text{ GeV} \)
- \( M_H = 125.7 - 0.4 \text{ GeV} \)
- \( M_t = 173.34 - 0.76 \text{ GeV} \)
- \( M_t = 173.34 + 0.76 \text{ GeV} \)

[Zoller 2014]
Stability, instability or metastability?

[Courtesy of Max Zoller]
Stability, instability or metastability?

Avoiding Death by Vacuum Decay

[Courtesy of Max Zoller]
$m_h = 125 \text{ GeV}$: metastable electroweak vacuum
- metastability: decay time of false vacuum large
- instability scale around $10^{10...12} \text{ GeV}$
- SM sufficiently stable
- neutrino masses missing
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**Why New Physics?**
Metastability of the Standard Model

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Why New Physics?

Why Supersymmetry?

- $m_h = 125$ GeV: very suitable for light MSSM Higgs
- Metastability $\rightarrow$ absolute stability
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Task: Do not introduce further instabilities!
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**Why New Physics?**

**Why Supersymmetry?**
- $m_h = 125$ GeV: very suitable for light MSSM Higgs
- metastability $\rightarrow$ absolute stability

Task: Do not introduce further instabilities!
(generically difficult)
Supersymmetry

- relates bosonic and fermionic degrees of freedom: absolute zero
- the only non-trivial extension of Poincaré symmetry
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\[ W_{\text{MSSM}} = \mu H_d \cdot H_u - Y_{ij}^e H_d \cdot L_L, i \bar{E}_{R,j} + Y_{ij}^u H_u \cdot Q_L, i \bar{U}_{R,j} - Y_{ij}^d H_d \cdot Q_L, i \bar{D}_{R,j} \]
Higgs sector of the MSSM and its stability

\[ \mathcal{W}_{\text{MSSM}} = \mu H_d \cdot H_u - Y_{ij}^e H_d \cdot L_{L,i} \bar{E}_{R,j} + Y_{ij}^u H_u \cdot Q_{L,i} \bar{U}_{R,j} - Y_{ij}^d H_d \cdot Q_{L,i} \bar{D}_{R,j} \]

**Higgs potential of 2HDM type II**

\[
V = m_{11}^2 H_d^\dagger H_d + m_{22}^2 H_u^\dagger H_u + (m_{12}^2 H_u \cdot H_d + \text{h.c.}) \\
+ \frac{\lambda_1}{2} (H_d^\dagger H_d)^2 + \frac{\lambda_2}{2} (H_u^\dagger H_u)^2 \\
+ \lambda_3 (H_u^\dagger H_u) (H_d^\dagger H_d) + \lambda_4 (H_u^\dagger H_d) (H_d^\dagger H_u) + \{\lambda_5, \lambda_6, \lambda_7\}
\]

In the MSSM: tree potential calculated from \(D\)-terms and \(\mathcal{L}_{\text{soft}}\)

\[
\begin{align*}
m^2_{11}^{\text{tree}} &= |\mu|^2 + m_{H_d}^2, \\
m^2_{22}^{\text{tree}} &= |\mu|^2 + m_{H_u}^2, \\
m^2_{12}^{\text{tree}} &= B_\mu, \\
\lambda^1_{\text{tree}} &= \lambda^2_{\text{tree}} = -\lambda^3_{\text{tree}} = \frac{g^2 + g'^2}{4}, \\
\lambda^4_{\text{tree}} &= \frac{g^2}{2}, \\
\lambda^5_{\text{tree}} &= \lambda^6_{\text{tree}} = \lambda^7_{\text{tree}} = 0.
\end{align*}
\]
The Higgs sector of the MSSM and its stability

\[ \mathcal{W}_{\text{MSSM}} = \mu H_d \cdot H_u - Y^e_{ij} H_d \cdot L_L, i \tilde{E}_{R,j} + Y^u_{ij} H_u \cdot Q_L, i \tilde{U}_{R,j} - Y^d_{ij} H_d \cdot Q_L, i \tilde{D}_{R,j} \]

Higgs potential of 2HDM type II

\[ V = m^2_{11} H_d^\dagger H_d + m^2_{22} H_u^\dagger H_u + (m^2_{12} H_u \cdot H_d + \text{h.c.}) \]
\[ + \frac{\lambda_1}{2} (H_d^\dagger H_d)^2 + \frac{\lambda_2}{2} (H_u^\dagger H_u)^2 \]
\[ + \lambda_3 (H_u^\dagger H_u) (H_d^\dagger H_d) + \lambda_4 (H_u^\dagger H_d) (H_d^\dagger H_u) + \{\lambda_5, \lambda_6, \lambda_7\} \]

In the MSSM: tree potential calculated from \( D \)-terms and \( \mathcal{L}_{\text{soft}} \)

\[ m^2_{11}^{\text{tree}} = |\mu|^2 + m^2_{H_d}, \quad \lambda_1^{\text{tree}} = \lambda_2^{\text{tree}} = -\lambda_3^{\text{tree}} = \frac{g^2 + g'^2}{4}, \]
\[ m^2_{22}^{\text{tree}} = |\mu|^2 + m^2_{H_u}, \quad \lambda_4^{\text{tree}} = \frac{g^2}{2}, \]
\[ m^2_{12}^{\text{tree}} = B_\mu, \quad \lambda_5^{\text{tree}} = \lambda_6^{\text{tree}} = \lambda_7^{\text{tree}} = 0. \]
The Higgs sector of the MSSM and its stability

\[ W_{\text{MSSM}} = \mu H_d \cdot H_u - Y_{ij}^e H_d \cdot L_i \tilde{E}_{R,j} + Y_{ij}^u H_u \cdot Q_L_i \tilde{U}_{R,j} - Y_{ij}^d H_d \cdot Q_L_i \tilde{D}_{R,j} \]

Higgs potential of 2HDM type II

\[ V = m_{11}^2 H_d^\dagger H_d + m_{22}^2 H_u^\dagger H_u + (m_{12}^2 H_u \cdot H_d + \text{h.c.}) + \frac{\lambda_1}{2} (H_d^\dagger H_d)^2 + \frac{\lambda_2}{2} (H_u^\dagger H_u)^2 + \lambda_3 (H_u^\dagger H_u) (H_d^\dagger H_d) + \lambda_4 (H_u^\dagger H_d) (H_d^\dagger H_u) + \{\lambda_5, \lambda_6, \lambda_7\} \]

Unbounded from below requirements

\[ \lambda_1 > 0, \quad \lambda_2 > 0, \quad \lambda_3 > -\sqrt{\lambda_1 \lambda_2} \]

and others...

always fulfilled in the MSSM @ tree

[Gunion, Haber 2003]

Extending the tree

- loop corrections?

[Gorbahn, Jäger, Nierste, Trine 2011]
integrating out heavy SUSY particles
requirement of large SUSY scale $M_{\text{SUSY}} \gg M_A \sim v_{\text{ew}}$
effective theory: generic 2HDM, $\lambda_i$ calculated from SUSY loops
integrating out heavy SUSY particles

requirement of large SUSY scale $M_{\text{SUSY}} \gg M_A \sim v_{\text{ew}}$

effective theory: generic 2HDM, $\lambda_i$ calculated from SUSY loops

collecting all SUSY contributions:

$$\lambda_i = \lambda_i(\tan \beta, \mu, M_1, M_2, \tilde{m}_Q^2, \tilde{m}_u^2, \tilde{m}_d^2, \tilde{m}_L^2, \tilde{m}_e^2, A_u, A_d, A_e).$$
simple check:

\[ \lambda_1 > 0, \quad \lambda_2 > 0, \quad \lambda_3 > -\sqrt{\lambda_1 \lambda_2}, \]

where now

\[ \lambda_i = \lambda_i^{\text{tree}} + \frac{\lambda_i^{\text{ino}} + \lambda_i^{\text{sferm}}}{16\pi^2}. \]

Severe UFB limits

Bounds on \( \lambda_{1,2,3} \) transfer into bounds on \( m_0, A_t, \mu, \ldots \)
Recovery from unbounded from below???

\[ V(\phi) = -\mu^2 \phi^2 + \lambda \phi^4 \]
Recovery from unbounded from below???

\[ V(\phi) = -\mu^2 \phi^2 - \lambda \phi^4 \]
Recovery from unbounded from below???

\[ V(\phi) = -\mu^2 \phi^2 - \lambda \phi^4 + \lambda^{(6)} \phi^6 \]
Calculating the 1-loop effective potential

- dominant contribution from third generation squarks
- quadrilinear couplings ($\sim |Y_t|^2$)
- trilinear coupling to a linear combination ($\mu^* Y_t h_d^\dagger - A_t h_u^0$)
- series summable to an infinite number of external legs
Calculating the 1-loop effective potential

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- series summable to an infinite number of external legs
- Do not stop after renormalizable / dim 4 terms!
All roads lead to Rome...

- 1-loop effective potential
  \[ V_1(h_u, h_d) = \frac{1}{64\pi^2} \text{STr} M^4(h_u, h_d) \left[ \ln \left( \frac{M^2(h_u, h_d)}{Q^2} \right) - \frac{3}{2} \right] \]

- **field dependent mass** \( M(h_u, h_d) \)
- STr accounts for spin degrees of freedom

- same result can be obtained by the tadpole method
  \[ T \sim \frac{\partial}{\partial h} V_1(h) \quad \leftrightarrow \quad V_1(h) \sim \int dh \, T(h) \]

- functional methods: effective potential for arbitrary number of scalars: \( V_1(\phi_1, \phi_2, \ldots, \phi_n) \)
The effective potential

\[ V_{\text{eff}}(\phi) : \text{average energy density} \]
The effective potential

\( V_{\text{eff}}(\phi) \): average energy density

The ground state of the theory

\( V_{\text{eff}} \) minimized:

\[
\frac{d V_{\text{eff}}}{d \phi} \bigg|_{\phi=v} = 0
\]
The reasoning behind

The effective potential

\( V_{\text{eff}}(\phi) \): average energy density

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\( V_{\text{eff}} \) minimized:

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\frac{d V_{\text{eff}}}{d \phi} \bigg|_{\phi=v} = 0
\]

Technically:

generating function for 1PI \( n \)-point Green’s functions:

\[
V_{\text{eff}}(\phi) = - \sum_{n=2}^{\infty} \tilde{G}^{(n)}(p_i = 0) \phi^n
\]
The reasoning behind

The effective potential

\[ V_{\text{eff}}(\phi): \text{average energy density} \]

The ground state of the theory

\[ V_{\text{eff}} \text{ minimized:} \]

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\[ V_{\text{eff}}(\phi) = - \sum_{n=2}^{\infty} \tilde{G}^{(n)}(p_i = 0) \phi^n \]

Conversely: calculate all 1PI \( n \)-point functions \( \rightarrow V_{\text{eff}} \)
The reasoning behind

The effective potential

$V_{\text{eff}}(\phi)$: average energy density

The ground state of the theory

$V_{\text{eff}}$ minimized:

$$\left. \frac{d V_{\text{eff}}}{d \phi} \right|_{\phi=v} = 0$$

Technically:

generating function for 1PI $n$-point Green's functions:

$$V_{\text{eff}}(\phi) = -\sum_{n=2}^{\infty} \tilde{G}^{(n)}(p_i = 0) \phi^n$$

conversely: calculate all 1PI $n$-point functions $\rightarrow V_{\text{eff}}$

(with subleties)
most dominant contribution from top Yukawa $y_t$ and $A_t$

can be easily summed for $m_{\tilde{t}_R} = m_{\tilde{t}_L} \equiv M$

1-PI potential as generating function for 1-PI Green’s functions

$$-V_{1-\text{PI}}(\phi) = \Gamma_{1-\text{PI}}(\phi) = \sum_n \frac{1}{n!} G_n(p_{\text{ext}} = 0) \phi^n$$

“classical” field value $\phi \to \langle 0|\phi|0 \rangle$

$\frac{dV(\phi)}{d\phi} = 0$ determines ground state of the theory
Summing up external legs

\[ h = h^0_d - \frac{A_t}{\mu^* Y_t} h^0_u \]
Summing up external legs

\[ h = h_0^0 + A_t + h_0^0 \]

\[ V_1 \sim \sum_n \frac{a_n}{n!^2} \left( h^\dagger h \right)^n , \quad \frac{a_n}{n!^2} = \frac{1}{n(n-1)(n-2)} \]
Summing up external legs

\[ h = h_d^{0\dagger} - \frac{A_t}{\mu^* Y_t} h_u^0 \]

\[ V_1 \sim \sum_n \frac{a_n}{n!^2} \left( h^{\dagger} h \right)^n, \quad \frac{a_n}{n!^2} = \frac{1}{n(n-1)(n-2)} \]

\[ V_1 = \frac{N_c M^4}{32\pi^2} \left[ (1 + x)^2 \log(1 + x) + (1 - x)^2 \log(1 - x) - 3x^2 \right] \]

\[ Q^2 = M^2 \]

\[ x^2 = |\mu Y_t|^2 h^{\dagger} h / M^4, \quad m_{\tilde{t}_L}^2 = m_{\tilde{t}_R}^2 = M^2 \]
Field dependent stop mass

\[ M_{\tilde{t}}^2(h_u^0, h_d^0) = \begin{pmatrix}
   m_{\tilde{t}_L}^2 + |Y_{t}h_u^0|^2 & A_t h_u^0 - \mu^* Y_t h_d^0^* \\
   A_t^* h_u^0^* - \mu Y_t^* h_d^0 & m_{\tilde{t}_R}^2 + |Y_{t}h_u^0|^2
\end{pmatrix} \]

- trilinear \( \sim h(h_d^0, h_u^0) \), quadrilinear \( \sim |h_u^0|^2 \)
- diagrams with mixed contributions
Summing up (ctd.)

Field dependent stop mass

\[ M^2_{\tilde{t}}(h^0_u, h^0_d) = \begin{pmatrix} m^2_{\tilde{t}_L} + |Y_t h^0_u|^2 & A_t h^0_u - \mu^* Y_t h^0_d^* \\ A^* t h^0_u^* - \mu Y^* t h^0_d & m^2_{\tilde{t}_R} + |Y_t h^0_u|^2 \end{pmatrix} \]

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Field dependent stop mass

\[ M_{\tilde{t}}^2(h_u^0, h_d^0) = \left( \begin{array}{ccc} m_{\tilde{t}_L}^2 + |Y_t h_u^0|^2 & A_t h_u^0 - \mu^* Y_t h_d^0^* \\ A_t^* h_u^0^* - \mu^* Y_t^* h_d^0 & m_{\tilde{t}_R}^2 + |Y_t h_u^0|^2 \end{array} \right) \]

- trilinear \( \sim h(h_d^0, h_u^0) \), quadrilinear \( \sim |h_u^0|^2 \)
- diagrams with mixed contributions

- gummi bear factor

\[ \frac{(2n + k - 1)!}{k!(2n - 1)!} \]

www.idn.uni-bremen.de/biologiedidaktik
Field dependent stop mass

\[ M^2_{\tilde{t}}(h^0_u, h^0_d) = \begin{pmatrix} m^2_{\tilde{t}_L} + |Y_t h^0_u|^2 & A_t h^0_u - \mu^* Y_t h^0_d^* \\ A^*_t h^0_u^* - \mu Y^*_t h^0_d & m^2_{\tilde{t}_R} + |Y_t h^0_u|^2 \end{pmatrix} \]
Summing up (ctd.)

Field dependent stop mass

\[ \mathcal{M}_{t}^{2}(h_{u}, h_{d}) = \left( \begin{array}{cc}
    m_{tL}^{2} + |Y_{t}h_{u}|^{2} & A_{t}h_{u}^{0} - \mu^{*}Y_{t}h_{d}^{0*} \\
    A_{t}^{*}h_{u}^{0*} - \mu^{*}Y_{t}^{*}h_{d}^{0} & m_{tR}^{2} + |Y_{t}h_{u}^{0}|^{2}
\end{array} \right) \]

\[ V_{1} \sim \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} a_{kn} x^{2n} y^{k}, \quad x^{2} = \frac{|\mu Y_{t}|^{2} h^{\dagger} h}{M^{4}}, \quad y = \frac{|Y_{t}h_{u}^{0}|^{2}}{M^{2}} \]

\[ = \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} \frac{1}{n(2n + k - 1)(2n + k - 2)} \frac{(2n + k - 1)!}{k!(2n - 1)!} x^{2n} y^{k} \]

\[ = \left[ (1 + y + x)^{2} \log(1 + y + x) + (1 + y - x)^{2} \log(1 + y - x) - 3(x^{2} + y^{2} + 2y) \right] \]
Features of the resummed series

\[ V_1(h_u^0, h_d^0) = \frac{N_c M^4}{32\pi^2} \left[ (1 + y + x)^2 \log(1 + y + x) \\
+ (1 + y - x)^2 \log(1 + y - x) \\
- 3(x^2 + y^2 + 2y) \right] \]

\[ x^2 = \frac{|\mu Y_t|^2 h^\dagger h}{M^4}, \quad h = h_d^0 - \frac{A_t}{\mu^* Y_t} h_u^0, \quad y = \frac{|Y_t h_u^0|^2}{M^2} \]

- branch cut at \( x - y = \pm 1 \): take real part (analytic cont.)
- ignore imaginary part: \( \log(1 + y - x) = \frac{1}{2} \log \left( (1 + y - x)^2 \right) \)
- always bounded from below
- minimum independent of Higgs parameters from tree potential
- minimum determined by SUSY scale parameters
Tree, loop and tree + loop

(a) $V(x)$

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Minimum at the electroweak scale $v = 246$ GeV

\[ m_{11}^{\text{tree}} = m_{12}^{\text{tree}} \tan \beta - \frac{v^2}{2} \cos(2\beta) \lambda_1^{\text{tree}} - \frac{1}{v \cos \beta} \frac{\delta}{\delta \phi_d} V_1 \bigg|_{\phi_{u,d} \rightarrow 0, \chi_{u,d} \rightarrow 0}, \]

\[ m_{22}^{\text{tree}} = m_{12}^{\text{tree}} \cot \beta + \frac{v^2}{2} \cos(2\beta) \lambda_1^{\text{tree}} - \frac{1}{v \sin \beta} \frac{\delta}{\delta \phi_u} V_1 \bigg|_{\phi_{u,d} \rightarrow 0, \chi_{u,d} \rightarrow 0}. \]

$m_h = 125$ GeV

- using FeynHiggs 2.10.0 to determine light Higgs mass by adjusting $A_t$ (several solutions: $\text{sign } A_t = - \text{sign } \mu$)
- connection to potential: $m_A$
- pseudoscalar mass $m_A$ less dependent on higher loops
- decoupling limit: $m_A, m_{H^\pm}, m_H \gg m_h$

- include sbottom (drives minimum), take $A_b = 0$
Yukawa coupling not given directly by the mass

\[ y_b = \frac{m_b}{v_d(1 + \Delta_b)} \]

\[ \Delta_{b}^{\text{gluino}} = \frac{2\alpha_s}{3\pi} \mu M_{\tilde{G}} \tan \beta C_0(\tilde{m}_{\tilde{b}_1}, \tilde{m}_{\tilde{b}_2}, M_{\tilde{G}}), \]

\[ \Delta_{b}^{\text{higgsino}} = \frac{Y_t^2}{16\pi^2} \mu A_t \tan \beta C_0(\tilde{m}_{\tilde{t}_1}, \tilde{m}_{\tilde{t}_2}, \mu). \]
\[ \tan \beta = 40 \]
\[ m_A = 800 \text{ GeV} \]
\[ M = 1 \text{ TeV} \]
\[ \mu = 3.75 \text{ TeV} \]
\[ A_t \simeq -1.5 \text{ TeV} \]
Discussion of stability

\[ V_{\text{eff}} / \text{GeV}^4 \]

\[ \tan \beta = 40 \]
\[ m_A = 800 \text{ GeV} \]
\[ M = 1 \text{ TeV} \]

\[ \mu = 3.83 \text{ TeV} \]
\[ A_t \simeq -1.5 \text{ TeV} \]
Avoiding Death by Vacuum Decay
Naive exclusions: Constraint in $\mu$-$\tan\beta$

$M_{\text{SUSY}} = 2\text{TeV}$
$M_{\text{SUSY}} = 1\text{TeV}$

$\mu/\text{TeV}$
$\tan\beta$

Avoiding Death by Vacuum Decay
Access to Charge and Color breaking minima

\[ \mu = 3910 \text{ GeV} \]
\[ \mu = 3860 \text{ GeV} \]
\[ \mu = 3810 \text{ GeV} \]

\[ v_u + \varphi_u / \text{GeV} \]
\[ V_{\text{eff}} / \text{GeV}^4 \]

\[ 6 \times 10^8 \]
\[ 5 \times 10^8 \]
\[ 4 \times 10^8 \]
\[ 3 \times 10^8 \]
\[ 2 \times 10^8 \]
\[ 1 \times 10^8 \]
\[ 0 \times 1 \]
\[ -1 \times 10^8 \]
\[ -2 \times 10^8 \]
\[ -3 \times 10^8 \]
\[ -4 \times 10^8 \]
Access to Charge and Color breaking minima

\[ M_t^2(h_u^0, h_d^0) = \begin{pmatrix} \tilde{m}_Q^2 + |Y_t h_u^0|^2 & A_t h_u^0 - \mu^* Y_t h_d^0 \\ A_t^* h_u^0 - \mu Y_t^* h_d^0 & \tilde{m}_t^2 + |Y_t h_u^0|^2 \end{pmatrix} \]

\[ M_b^2(h_u^0, h_d^0) = \begin{pmatrix} \tilde{m}_Q^2 + |Y_b h_d^0|^2 & A_b h_d^0 - \mu^* Y_b h_u^0 \\ A_b^* h_d^0 - \mu Y_b^* h_u^0 & \tilde{m}_b^2 + |Y_b h_d^0|^2 \end{pmatrix} \]

- non-trivial behaviour of sfermions masses with Higgs vev
New interpretation

Access to Charge and Color breaking minima

\[
M^2_t(h_u^0, h_d^0) = \left( \begin{array}{cc} \tilde{m}_Q^2 + |Y_t h_u^0|^2 & A_t h_u^0 - \mu^* Y_t h_d^0^* \\ A_t^* h_u^0^* - \mu Y_t^* h_d^0 & \tilde{m}_t^2 + |Y_t h_u^0|^2 \end{array} \right)
\]

\[
M^2_b(h_u^0, h_d^0) = \left( \begin{array}{cc} \tilde{m}_Q^2 + |Y_b h_d^0|^2 & A_b h_d^0 - \mu^* Y_b h_u^0^* \\ A_b^* h_d^0^* - \mu Y_b^* h_u^0 & \tilde{m}_b^2 + |Y_b h_d^0|^2 \end{array} \right)
\]

- non-trivial behaviour of sfermions masses with Higgs vev:

\[
m^2_{b_{1,2}}(h_u^0, h_d^0) = \frac{\tilde{m}_Q^2 + \tilde{m}_b^2}{2} + |Y_b h_d^0|^2 \\
\pm \frac{1}{2} \sqrt{ (\tilde{m}_Q^2 - \tilde{m}_b^2)^2 + 4 |A_b h_d^0 - \mu^* Y_b h_u^0|^2 }
\]

- expand theory around new minimum: \( m^2_{b_2} < 0 \)
Access to Charge and Color breaking minima

$$\mathcal{M}^2_t(h^0_u, h^0_d) = \begin{pmatrix} \tilde{m}_Q^2 + |Y_t h_u^0|^2 & A_t h_u^0 - \mu^* Y_t h_d^0* \\ A_t^* h_u^0* - \mu Y_t^* h_d^0 & \tilde{m}_t^2 + |Y_t h_u^0|^2 \end{pmatrix}$$

$$\mathcal{M}^2_b(h^0_u, h^0_d) = \begin{pmatrix} \tilde{m}_Q^2 + |Y_b h_d^0|^2 & A_b h_d^0 - \mu^* Y_b h_u^0* \\ A_b^* h_d^0* - \mu Y_b^* h_u^0 & \tilde{m}_b^2 + |Y_b h_d^0|^2 \end{pmatrix}$$

- non-trivial behaviour of sfermions masses with Higgs vev:
  
  $$m^2_{b_{1,2}}(h_u^0, h_d^0) = \frac{\tilde{m}_Q^2 + \tilde{m}_b^2}{2} + |Y_b h_d^0|^2$$
  
  $$\pm \frac{1}{2} \sqrt{(\tilde{m}_Q^2 - \tilde{m}_b^2)^2 + 4 |A_b h_d^0 - \mu^* Y_b h_u^0*|^2}$$

- expand theory around new minimum: $$m^2_{b_2} < 0$$

- tachyonic squark mass!

New interpretation

Avoiding Death by Vacuum Decay
What does a tachyonic mass mean?

- Mass $\leftrightarrow$ second derivative: $m_{\phi}^2 = \partial^2 V / \partial \phi^2$
- $m_{\phi}^2 < 0 \iff$ negative curvature
- Non-convex potential: imaginary part
- $\log(1 + y - x) \sim \log(m_{\phi}^2)$
**What does a tachyonic mass mean?**

- mass \( \Leftrightarrow \) second derivative: \( m_\phi^2 = \partial^2 V / \partial \phi^2 \)
- \( m_\phi^2 < 0 \Leftrightarrow \) negative curvature
- non-convex potential: imaginary part
- \( \log(1 + y - x) \sim \log(m_\phi^2) \)
Including colored directions

\[ V_{\tilde{b}}^{\text{tree}} = \tilde{b}_L^*(M_Q^2 + |Y_b v_d|^2)\tilde{b}_L + \tilde{b}_R^*(M_b^2 + |Y_b v_d|^2)\tilde{b}_R \]

\[ - \left[ \tilde{b}_L^*(\mu^* Y_b h^0_u - A_b v_d)\tilde{b}_R + \text{h. c.} \right] + |Y_b|^2|\tilde{b}_L|^2|\tilde{b}_R|^2 \]

+ \textit{D}-terms.
Including colored directions

\[ V_{\tilde{b}}^{\text{tree}} = \tilde{b}_L^* (M_{\tilde{Q}}^2 + |Y_b v_d|^2) \tilde{b}_L + \tilde{b}_R^* (M_{\tilde{B}}^2 + |Y_b v_d|^2) \tilde{b}_R \]
\[ - \left[ \tilde{b}_L^* (\mu^* Y_b h_u^0 - A_b v_d) \tilde{b}_R + \text{h. c.} \right] + |Y_b|^2 |\tilde{b}_L|^2 |\tilde{b}_R|^2 \]
\[ + D\text{-terms}. \]

**D-flat direction: minimizing D-term contribution**

- **D-terms**: \( g^2 \phi^4 \)

\[ V_D = \frac{g_1^2}{8} \left( |h_u^0|^2 - |h_d^0|^2 + \frac{1}{3} |\tilde{b}_L|^2 + \frac{2}{3} |\tilde{b}_R|^2 \right)^2 \]
\[ + \frac{g_2^2}{8} \left( |h_u^0|^2 - |h_d^0|^2 + |\tilde{b}_L|^2 \right)^2 + \frac{g_3^2}{6} (|\tilde{b}_L|^2 - |\tilde{b}_R|^2)^2. \]

- will always take over for large field values
- take e.g. \( \tilde{b}_L = \tilde{b}_R = \tilde{b} \) and \( |h_d^0|^2 = |h_u^0|^2 + |\tilde{b}|^2 \)
Including colored directions

\[ V_{\tilde{b}}^{\text{tree}} = \tilde{b}_L^* (M_\tilde{Q}^2 + |Y_b v_d|^2) \tilde{b}_L + \tilde{b}_R^* (M_\tilde{b}^2 + |Y_b v_d|^2) \tilde{b}_R 
- \left[ \tilde{b}_L^* (\mu^* Y_b h_u^0 - A_b v_d) \tilde{b}_R + \text{h. c.} \right] + |Y_b|^2 |\tilde{b}_L|^2 |\tilde{b}_R|^2 + D\text{-terms.} \]

\[ D\text{-flat direction: minimizing } D\text{-term contribution} \]

- \( D\)-terms: \( g^2 \phi^4 \)

\[ V_D = \frac{g_1^2}{8} \left( |h_u^0|^2 - |h_d^0|^2 + \frac{1}{3} |\tilde{b}_L|^2 + \frac{2}{3} |\tilde{b}_R|^2 \right)^2 + \frac{g_2^2}{8} \left( |h_u^0|^2 - |h_d^0|^2 + |\tilde{b}_L|^2 \right)^2 + \frac{g_3^2}{6} (|\tilde{b}_L|^2 - |\tilde{b}_R|^2)^2. \]

- will always take over for large field values
- \( h_d^0 = 0 \) and \( \tilde{b} = h_u^0 \) [large \( D\)-term]
CCB minima and the one-loop Higgs potential

From CCC to CCB

- previously “safe” false but CC conserving minima turn into deep global minima with $\langle \tilde{b}_L \rangle = \langle \tilde{b}_R \rangle \neq 0$ and $\langle h^0_u \rangle \neq v_u$
From CCC to CCB

- previously “safe” false but CC conserving minima turn into deep global minima with $\langle \tilde{b}_L \rangle = \langle \tilde{b}_R \rangle \neq 0$ and $\langle h^0_u \rangle \neq v_u$
Analytic bounds at the tree-level

1. choose appropriate direction $\rightarrow$ one-field problem

$$V_{\phi}^{\text{tree}} = \tilde{m}^2 \phi^2 - A\phi^3 + \lambda\phi^4$$

$$h^0_u = \tilde{b}, \quad h^0_d = 0$$

2. identify parameters, e.g.

$$\tilde{m}^2 = \tilde{m}_Q^2 + \tilde{m}_b^2 + m_{H_u}^2 + \mu^2,$$

$$\lambda = Y_b^2 + \frac{g_1^2 + g_2^2}{2}, \quad A = 2\mu Y_b$$

3. necessary condition:

$$\tilde{m}^2 > \frac{A^2}{4\lambda}$$ $\rightarrow$ second minimum not below first (trivial) one
choose appropriate direction $\leftrightarrow$ one-field problem

\[ V_{\phi}^{\text{tree}} = \bar{m}^2 \phi^2 - A \phi^3 + \lambda \phi^4 \]

\[ h_u^0 = \tilde{b}, \; h_d^0 = 0 \]

identify parameters, e.g. \( \bar{m}^2 = \tilde{m}_Q^2 + \tilde{m}_b^2 + m_{H_u}^2 + \mu^2 \),
\[ \lambda = Y_b^2 + \frac{g_1^2 + g_2^2}{2}, \; A = 2\mu Y_b \]

necessary condition: \( \bar{m}^2 > \frac{A^2}{4\lambda} \leftrightarrow \) second minimum not below first (trivial) one

\[ h_u^0 = \tilde{b}, \; h_d^0 = 0 \]

\[ m_{H_u}^2 + \mu^2 + \tilde{m}_Q^2 + \tilde{m}_b^2 > \frac{(\mu Y_b)^2}{Y_b^2 + (g_1^2 + g_2^2)/2} \]
**Analytic bounds at the tree-level**

1. choose appropriate direction $\leftrightarrow$ one-field problem

$$V_{\phi}^{\text{tree}} = \bar{m}^2 \phi^2 - A\phi^3 + \lambda\phi^4$$

$$h_u^0 = \tilde{b}, \quad h_d^0 = 0$$

2. identify parameters, e.g. $\bar{m}^2 = \tilde{m}_Q^2 + \tilde{m}_b^2 + m_{H_u}^2 + \mu^2$, $\lambda = Y_b^2 + \frac{g_1^2 + g_2^2}{2}$, $A = 2\mu Y_b$

3. necessary condition: $\bar{m}^2 > \frac{A^2}{4\lambda}$ $\leftrightarrow$ second minimum not below first (trivial) one

$$|h_d^0|^2 = |h_u^0|^2 + |\tilde{b}|^2, \quad \tilde{b} = \alpha h_u^0$$

$$m_{11}^2(1 + \alpha^2) + m_{22}^2 \pm 2m_{12}^2\sqrt{1 + \alpha^2} + \alpha^2(\tilde{m}_Q^2 + \tilde{m}_b^2) > \frac{\mu^2\alpha^4}{2 + 3\alpha^2}$$
Avoiding Death by Vacuum Decay
Conclusions

- the Higgs potential in the SM is meta/un/stable
- MSSM: multi-scalar theory, has several unwanted minima
- formation of new CCB *conserving* minima at the 1-loop level
- stability of the electroweak vacuum: bounds on $\mu$-$\tan \beta$
- instability of electroweak vacuum by second minimum in “standard model direction” $\sim v_u$: global CCB minimum
- more severe bounds [new CCB constraints]
- CCB constraints for non-vanishing $D$-Terms
- quantum tunneling: either very long- or very short-lived
Finally...  

Greetings from Señor Higgs  
(courtesy of Jens Hoff)
Backup

Slides
W. G. H. Avoiding Death by Vacuum Decay

References

- Wolfgang Gregor Hollik: “Charge and color breaking constraints in the Minimal Supersymmetric Standard Model associated with the bottom Yukawa coupling” Physics Letters B 752 (2016) 7 – 12