New constraints from vacuum stability on MSSM parameters

PASCOS 2014

Markus Bobrowski†, Guillaume Chalons*, Wolfgang Gregor Hollik†, Ulrich Nierste†

†Institut für Theoretische Teilchenphysik (TTP)
Karlsruher Institut für Technologie (KIT)
* LPSC Grenoble

June 24, 2014
Motivation and outline

- discovery of Higgs boson [ATLAS, CMS: 4th July 2012]
- $m_h = 126$ GeV: light SUSY (MSSM) Higgs

Stability of the electroweak vacuum

- ground state of the theory (global minimum)
- unbounded from below (UFB) limits
- further minima $\rightarrow$ vacuum decay or stable vacuum

This talk

- UFB bounds for effective 2HDM
- UFB direction $\rightarrow$ deeper minimum
- new class of constraints from vacuum stability on SUSY (Higgs) parameters using 1-loop effective potential
What has been done and what will be new

Already done

- effective potential for lightest SM-like Higgs mass [Okada, Yamaguchi, Yanagida 1991; Ellis, Ridolfi, Zwirner 1991; Casas, Espinosa, Quiros 1995]
- charge and color breaking minima [Frère, Jones, Raby 1983; Casas, Lleyda, Muños 1995, Casas, Dimopoulos 1996]
- ...  
- computer code: finding all tree-level minima and perturbe them by one loop [Vevacious: Camargo-Molina, O’Leary, Porod, Staub]

Not yet

- stability conditions for the effective 2HDM
- charge and color conserving deeper minima by one loop
- lightest Higgs mass in the effective 2HDM (not covered)
The Higgs sector of the MSSM and its stability

Higgs potential of 2HDM type II

\[ V = m_{11}^2 H_d^\dagger H_d + m_{22}^2 H_u^\dagger H_u + (m_{12}^2 H_u \cdot H_d + \text{h.c.}) \]
\[ + \frac{\lambda_1}{2} (H_d^\dagger H_d)^2 + \frac{\lambda_2}{2} (H_u^\dagger H_u)^2 \]
\[ + \lambda_3 (H_u^\dagger H_u)(H_d^\dagger H_d) + \lambda_4 (H_u^\dagger H_d)(H_d^\dagger H_u) + \{ \lambda_5, \lambda_6, \lambda_7 \} \]

In the MSSM: tree potential calculated from \( D \)-terms and \( L \)\text{soft}
### Higgs potential of 2HDM type II

\[
V = m_{11}^2 H_d^\dagger H_d + m_{22}^2 H_u^\dagger H_u + (m_{12}^2 H_u \cdot H_d + \text{h.c.}) \\
+ \frac{\lambda_1}{2} (H_d^\dagger H_d)^2 + \frac{\lambda_2}{2} (H_u^\dagger H_u)^2 \\
+ \lambda_3 (H_u^\dagger H_u) (H_d^\dagger H_d) + \lambda_4 (H_u^\dagger H_d) (H_d^\dagger H_u) + \{\lambda_5, \lambda_6, \lambda_7\}
\]

### Unbounded from below requirements

\[
\lambda_1 > 0, \quad \lambda_2 > 0, \quad \lambda_3 > -\sqrt{\lambda_1 \lambda_2}
\]

and others…

- always fulfilled in the MSSM @ tree

### Extending the tree

- loop corrections?
integrating out heavy SUSY particles
requirement of large SUSY scale $M_{\text{SUSY}} \gg M_A \sim v_{\text{ew}}$
effective theory: generic 2HDM, $\lambda_i$ calculated from SUSY loops
integrating out heavy SUSY particles
requirement of large SUSY scale $M_{\text{SUSY}} \gg M_A \sim v_{\text{ew}}$
effective theory: generic 2HDM, $\lambda_i$ calculated from SUSY loops

collecting all SUSY contributions:

$$\lambda_i = \lambda_i(\tan \beta, \mu, M_1, M_2, M_{\tilde{Q}}^2, M_{\tilde{u}}^2, M_{\tilde{d}}^2, M_{L}^2, M_{e}^2, A_u, A_d, A_e).$$
simple check:

\[ \lambda_1 > 0, \quad \lambda_2 > 0, \quad \lambda_3 > -\sqrt{\lambda_1 \lambda_2}, \]

where now

\[ \lambda_i = \lambda_i^{\text{tree}} + \frac{\lambda_i^{\text{ino}} + \lambda_i^{\text{sferm}}}{16\pi^2}. \]

Severe UFB limits

Bounds on \( \lambda_{1,2,3} \) transfer into bounds on \( m_0, A_t, \mu, \ldots \)
Recovery from unbounded from below???

\[ V(\phi) = -\mu^2 \phi^2 + \lambda \phi^4 \]
Recovery from unbounded from below???

\[ V(\phi) = -\mu^2 \phi^2 - \lambda \phi^4 \]
Recovery from unbounded from below???

\[ V(\phi) = -\mu^2 \phi^2 - \lambda \phi^4 + \lambda^{(6)} \phi^6 \]
Summing up external legs

- most dominant contribution from top Yukawa $y_t$ and $A_t$
- can be easily summed for $m_{\tilde{t}_R} = m_{\tilde{t}_L} \equiv M$
- 1-PI potential as generating function for 1-PI Green’s functions

$$-V_{1-\text{PI}}(\phi) = \sum_n \frac{1}{n!} G_n(p_{\text{ext}} = 0) \phi^n$$

- “classical” field value $\phi \to \langle 0|\phi|0 \rangle$
- $\frac{dV(\phi)}{d\phi} = 0$ determines ground state of the theory
Summing up external legs

\[ V_1 \sim \sum_n \frac{a_n}{n!^2} (h^\dagger h)^n, \quad \frac{a_n}{n!^2} = \frac{1}{n(n-1)(n-2)} \]

\[ V_1 = \frac{N_c M^4}{32 \pi^2} \left[ (1 + x)^2 \log(1 + x) + (1 - x)^2 \log(1 - x) - 3x^2 \right] \]

\[ x^2 = |\mu Y_t|^2 h^\dagger h/M^4, \quad m_{\tilde{t}_L}^2 = m_{\tilde{t}_R}^2 = M^2 \]
Field dependent stop mass

\[ M_t^2(h_u^0, h_d^0) = \begin{pmatrix} m_{\tilde{t}_L}^2 + |Y_t h_u^0|^2 & A_t h_u^0 - \mu^* Y_t h_d^0 \\ A^*_t h_u^0 - \mu Y_t^* h_d^0 & m_{\tilde{t}_R}^2 + |Y_t h_d^0|^2 \end{pmatrix} \]

- trilinear \( \sim h(h_d^0, h_u^0) \), quadrilinear \( \sim |h_u^0|^2 \)
- diagrams with mixed contributions
Field dependent stop mass

\[
\mathcal{M}^2_t(h^0_u, h^0_d) = \begin{pmatrix}
    m^2_{\tilde{t}_L} + |Y_t h^0_u|^2 & A_t h^0_u - \mu^* Y_t h^0_d^* \\
    A^* h^0_u^* - \mu Y^*_t h^0_d & m^2_{\tilde{t}_R} + |Y_t h^0_u|^2
\end{pmatrix}
\]

- trilinear \( \sim h(h^0_d, h^0_u) \), quadrilinear \( \sim |h^0_u|^2 \)
- diagrams with mixed contributions
Field dependent stop mass

\[ \mathcal{M}_{\tilde{t}}^2(h_u^0, h_d^0) = \begin{pmatrix} m_{\tilde{t}_L}^2 + |Y_t h_u^0|^2 & A_t h_u^0 - \mu^* Y_t h_d^0^* \\ A_t^* h_u^0^* - \mu Y_t^* h_d^0 & m_{\tilde{t}_R}^2 + |Y_t h_u^0|^2 \end{pmatrix} \]

- trilinear \( \sim h(h_d^0, h_u^0) \), quadrilinear \( \sim |h_u^0|^2 \)
- diagrams with mixed contributions
Summing up (ctd.)

Field dependent stop mass

\[ M^2_t(h_0^u, h_0^d) = \begin{pmatrix} m_{\tilde{t}_L}^2 + |Y_t h_0^u|^2 & A_t h_0^u - \mu^* Y_t h_0^d^* \\ A_t^* h_0^d^* - \mu Y_t^* h_0^u & m_{\tilde{t}_R}^2 + |Y_t h_0^u|^2 \end{pmatrix} \]

- trilinear \( \sim h(h_0^d, h_0^u) \), quadrilinear \( \sim |h_0^u|^2 \)
- diagrams with mixed contributions
- gummi bear factor

\[ \frac{(2n + k - 1)!}{k!(2n - 1)!} \]

W. G. H.  
MSSM vacuum  
www.idn.uni-bremen.de/biologiedidaktik
Summing up (ctd.)

Field dependent stop mass

\[ M^2_{\tilde{t}}(h^0_u, h^0_d) = \begin{pmatrix} m^2_{\tilde{t}_L} + |Y_t h^0_u|^2 & A_t h^0_u - \mu Y^*_t h^0_d^* \\ A^*_t h^0_u - \mu^* Y^*_t h^0_d & m^2_{\tilde{t}_R} + |Y_t h^0_u|^2 \end{pmatrix} \]
Field dependent stop mass

\[ M_{\tilde{t}}^2(h_u^0, h_d^0) = \begin{pmatrix}
    m_{\tilde{t}_L}^2 + |Y_t h_u^0|^2 & A_t h_u^0 - \mu^* Y_t h_d^0^* \\
    A_t^* h_u^0^* - \mu Y_t^* h_d^0 & m_{\tilde{t}_R}^2 + |Y_t h_u^0|^2
\end{pmatrix} \]

\[ V_1 \sim \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} a_{kn} x^{2n} y^k, \quad x^2 = \frac{|\mu Y_t|^2 h^\dagger h}{M^4}, y = \frac{|Y_t h_u^0|^2}{M^2} \]

\[ = \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} \frac{1}{n(2n + k - 1)(2n + k - 2)} \frac{(2n + k - 1)!}{k!(2n - 1)!} x^{2n} y^k \]

\[ = \left[ (1 + y + x)^2 \log(1 + y + x) \right. \]
\[ + (1 + y - x)^2 \log(1 + y - x) - 3(x^2 + y^2 + 2y) \]
Features of the resummed series

\[ V_1(h_u^0, h_d^0) = \frac{N_c M^4}{32\pi^2} \left[ (1 + y + x)^2 \log(1 + y + x) \\
+ (1 + y - x)^2 \log(1 + y - x) \\
- 3(x^2 + y^2 + 2y) \right] \]

\[ x^2 = \frac{|\mu Y_t|^2 h_d^\dagger h}{M^4}, \quad h = h_d^{0*} - \frac{A_t}{\mu^* Y_t} h_u^0, \quad y = \frac{|Y_t h_u^0|^2}{M^2} \]

- always bounded from below
- minimum independent of Higgs parameters from tree potential
- minimum determined by SUSY scale parameters
Features of the resummed series

$M_{\text{SUSY}} = 1\,\text{TeV}, A_t = 1.5\,\text{TeV}, \mu = 3\,\text{TeV}, \tan \beta = 10.$
Radius of convergence, analytic continuation and imaginary part

- Series of 1-PI Green’s functions converges for \( y, x < 1 \)
- Imaginary part for \( y - x > 1 \) from logarithm
- Take real part for stability discussion

**Constraint by radius of convergence**

- Stop discussion at large field values
- Viability of the (effective) theory under consideration
- No physical interpretation possible

**Analytic continuation: discussion of stability**

- Effective potential viable beyond “radius of convergence”
- 1-loop part drives deeper minimum
- Interpretation via stability of the electroweak vacuum

- Ignore imaginary part and interpretation

[Weinberg, Wu 1987]
Discussion of stability

$V_{\text{eff}} / \text{GeV}^4$

- $\tan \beta = 40$
- $m_A = 800 \text{ GeV}$
- $M = 1 \text{ TeV}$
- $A_t \approx 1.5 \text{ TeV}$
- $\mu = 2.55 \text{ TeV}$

$\mu = 2.55 \text{ TeV}$
Discussion of stability

\[ V_{\text{eff}} / \text{GeV}^4 \]

- \( \tan \beta = 40 \)
- \( m_A = 800 \text{ GeV} \)
- \( M = 1 \text{ TeV} \)
- \( A_t \simeq 1.5 \text{ TeV} \)

- \( \mu = 2.51 \text{ TeV} \)

W. G. H. MSSM vacuum
Conclusions

- UFB limits for effective 2HDM turn into bounds from the formation of a deeper minimum
- vacuum stability bounds on MSSM parameters
- constrained and simplified model discussed:
  - gluino and electroweak gauginos heavy
  - only third generation squarks light
  - $A_t$ fixed by $m_h$, $A_b \equiv 0$ (for simplicity)
  - free parameters: $M_{\text{SUSY}} = m_{\tilde{t}} = m_{\tilde{b}}$, $\tan \beta$, $\mu$
  - no new insights if $m_{\tilde{t}_L, \tilde{b}_L} \neq m_{\tilde{t}_R, \tilde{b}_R}$
- instability of electroweak vacuum by deeper (global?) minimum in “standard model direction” $\sim v_u$
Backup

Slides
Cooking up a phenomenologically viable potential

Minimum at the electroweak scale $v = 246$ GeV

\[
\begin{align*}
 m_{11}^{2 \text{tree}} &= m_{12}^{2 \text{tree}} \tan \beta - \frac{v^2}{2} \cos(2\beta) \lambda_1^{\text{tree}} - \frac{1}{v \cos \beta} \frac{\delta}{\delta \phi_d} V_1 \bigg|_{\phi_u,d \to 0}, \\
 m_{22}^{2 \text{tree}} &= m_{12}^{2 \text{tree}} \cot \beta + \frac{v^2}{2} \cos(2\beta) \lambda_1^{\text{tree}} - \frac{1}{v \sin \beta} \frac{\delta}{\delta \phi_u} V_1 \bigg|_{\phi_u,d \to 0}.
\end{align*}
\]

$m_h = 126$ GeV

- pseudoscalar mass $m_A$ less dependent on higher loops
- using FeynHiggs 2.10.0 to determine light Higgs mass by adjusting $A_t$
- connection to potential: $m_A$
- decoupling limit: $m_A, m_{H^\pm}, m_H \gg m_h$

- include sbottom loop as well, taking $A_b = 0$
Check versus known results

1-loop effective potential

\[ V_1(h_u, h_d) = \frac{1}{64\pi^2} \text{STr} \mathcal{M}^4(h_u, h_d) \left[ \ln \left( \frac{\mathcal{M}^2(h_u, h_d)}{Q^2} \right) - \frac{3}{2} \right] \]

- field dependent mass \( \mathcal{M}(h_u, h_d) \)
- STr accounts for spin degrees of freedom
- same result can be obtained by the tadpole method

\[ T \sim \frac{\partial}{\partial h} V_1(h) \quad \Rightarrow \quad V_1(h) \sim \int dh \ T(h) \]

Functional methods: effective potential for arbitrary number of scalars:

\[ V_1(\phi_1, \phi_2, \ldots \phi_n) \]

[Jackiw 1973]