CKM vs. PMNS matrix

- CKM matrix close to unity
  
  $$V_{\text{CKM}} = \begin{pmatrix} \ast & \ast & \ast \\ \ast & \ast & \ast \\ \ast & \ast & \ast \end{pmatrix}$$

- small off-diagonal: generate mixings radiatively?

- different pattern for the leptonic mixing matrix:
  
  $$U_{\text{PMNS}} = \begin{pmatrix} \ast & \ast & \ast \\ \ast & \ast & \ast \\ \ast & \ast & \ast \end{pmatrix}$$

- large mixings
- non-vanishing $\theta_{13}$: possible CP violation in $\nu$ oscillations

[Weinberg 1972]

[T2K, DoubleChooz, Reno, DayaBay]

- try to model quark and lepton mixing using the same mechanism?
Radiative Flavour Violation in the MSSM

Theories with Additional Sources of Flavour Violation

- non-minimal flavour violating MSSM
- arbitrary flavour structure in the soft breaking terms?

\[ \mathcal{M}_{\tilde{Q}}^2, \mathcal{M}_{\tilde{u}}^2, \mathcal{M}_{\tilde{d}}^2, \mathcal{M}_{\tilde{\ell}}^2, \mathcal{M}_{\tilde{e}}^2, \quad A^u, A^d, A^e \]

- additional flavour mixing in fermion–sfermion–gaugino interaction
- especially non-CKM-like: e.g. quark–squark–gluino and lepton–slepton–neutralino

[Crivellin, Nierste 2009]
radiative lepton flavour violation

PMNS matrix renormalization

\[ i \frac{g}{\sqrt{2}} \gamma^\mu P_L U_{\text{PMNS}}^\dagger \rightarrow i \frac{g}{\sqrt{2}} \gamma^\mu P_L \left( U^{(0)}_i \rightarrow \Delta U^{e}_i U^{(0)}_i \right) + \Delta U^{\nu}_i U^{(0)}_i \]

flavour changing self energies and sensitivity to neutrino mass

\[ \Delta U^{\nu}_{fi} \sim \frac{m_{\nu_f} \Sigma_{fi}}{\Delta m^2_{\nu}} \]
Neutrino masses and seesaw

Standard Model + righthanded Neutrinos = Seesaw Type I

\[-\mathcal{L}_{\nu,\text{mass}} = \bar{\nu}_L m_D \nu_R + \frac{1}{2} \bar{\nu}_L^c m_R \nu_R + \text{h. c.}\]

Dirac mass

Majorana mass
Neutrino masses and seesaw

Standard Model + righthanded Neutrinos = Seesaw Type I

\[ -\mathcal{L}_{\nu,\text{mass}} = \bar{\nu}_L m_D \nu_R + \frac{1}{2} \bar{\nu}_L^c m_R \nu_R + \text{h. c.} \]

Dirac mass \hspace{1cm} Majorana mass

Neutrino mass matrix:

\[ \mathcal{M}_\nu = \begin{pmatrix} 0 & m_D \\ m_D^T & m_R \end{pmatrix}. \]

What about \( m_R \)?

- righthanded neutrinos are SM singlets \( \rightarrow \) no constraint for mass
- seesaw: \( m_\nu = -m_D m_R^{-1} m_D \approx \mathcal{O}(0.1 \text{ eV}) \)
- assumption: Dirac mass of order EW scale (\( \mathcal{O}(10 \ldots 100 \text{ GeV}) \)): \( m_R \sim \mathcal{O}(10^{13\ldots14} \text{ GeV}) \)
The MSSM with righthanded neutrinos

Superpotential of the $\nu$MSSM

$$\mathcal{W}^\ell = \mu H_d \cdot H_u - Y_H^{ij} H_d \cdot L^I_L E^J_R + Y_{\nu}^{ij} H_u \cdot L^I_L N^J_R + \frac{1}{2} m_R^{ij} N^I_R N^J_R,$$

with $L_L = (\ell_L, \tilde{\ell}_L) \in SU(2)_L$ and $E_R = (e^c_L, \tilde{e}^*_R), N_R = (\nu^c_L, \tilde{\nu}^*_R)$.

Soft-breaking terms

$$\mathcal{V}_{\text{soft}} = (M_\ell^{2})^{ij} \tilde{L}^I_L \tilde{L}^J_L + (M_{\tilde{e}}^{2})^{ij} \tilde{e}^I_R \tilde{e}^J_R + (M_{\tilde{\nu}}^{2})^{ij} \tilde{\nu}^I_R \tilde{\nu}^J_R$$

$$- \left[ (B_\nu)^{ij} \tilde{\nu}^I_R \tilde{\nu}^J_R + A_{\nu}^{ij} H_1 \cdot \tilde{L}^I_L \tilde{e}^J_R - A_{\nu}^{ij} H_2 \cdot \tilde{L}^I_L \tilde{\nu}^J_R + \text{h.c.} \right],$$
One Numerical Example

- try to generate PMNS mixing completely radiatively
- "νMSSM": \( Y_ν \) in general arbitrary (for simplicity taken diagonal)
- all soft breaking masses assumed flavour blind
- only source of flavour mixing: soft trilinear couplings
- large off-diagonal values of \( A^e \) ruled out by \( \ell_j \rightarrow \ell_i \gamma \)
- toy numbers for \( m_0, M_1, M_2, \mu, \tan \beta, \ldots \)

Some remarks...

- corrections to \( U_{ij}^{PMNS} \) more or less linear in \( A_{ij}^V \) (for \( i \neq j \))
- no decoupling, if all SUSY parameters shifted uniformly
- size of corrections correlated to \( \Delta m^2_{ij} \) and, of course, \( U_{ij}^{phys} \)
One Numerical Example

- scan over uniform soft mass
  (assuming $M_{\tilde{Q}}^2, M_{\tilde{u}}^2, M_{\tilde{d}}^2, M_{\tilde{\ell}}^2, M_{\tilde{e}}^2 = m_{\text{soft}}^2$)
- display values of off-diagonal $A$ parameters needed to generate corresponding mixing matrix element fully radiatively
- since corrections are linear in $A$, those grow linearly with $m_{\text{soft}}$
One Numerical Example

The graph shows the relationship between the soft mass $m_{\text{soft}}$ and the neutrino mixing parameters $A_{ij}^{\nu}$, where $i$ and $j$ are the indices of the neutrino states. The parameters $A_{12}$, $A_{13}$, and $A_{23}$ are plotted against $m_{\text{soft}}$.

The graph indicates that as the $m_{\text{soft}}$ increases, the values of $A_{ij}^{\nu}$ also increase linearly for each parameter.

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dividing by \( m_{\text{soft}} \ldots \)

\[
c_{ij} = \frac{A_{\nu ij}(m_{\text{soft}})}{m_{\text{soft}}}
\]
corrections crucially depend on the neutrino mass spectrum:

\[ \Delta U_{fi}^\nu \sim \frac{m_{\nu_f} \Sigma_{fi}}{\Delta m_{\nu}^2} \]

rough estimate: self energy \( \Sigma_{fi} \sim m_{\nu_{i,f}} \)

divide by that:

\[ f_{ij} = \frac{m_{\nu_f} m_{\nu_i}}{\Delta m_{fi}^2} \]

\[ c_{ij} = \frac{A_{ij}(m_{\text{soft}})}{m_{\text{soft}}} \rightarrow \frac{c_{ij}}{f_{ij}} \]
One Numerical Example

\[ c_{ij} / f_{ij} \]

\[ m_{\text{soft}} \text{ [GeV]} \]

\[ c_{12} / f_{12}, c_{13} / f_{13}, c_{23} / f_{23} \]
One Numerical Example

- generate large PMNS elements
  - size of correction larger for large $U_{ij}$
  - correct for that

$$\tilde{c}_{ij} = \frac{c_{ij}}{f_{ij}} / U_{ij}^{\text{phys}}$$
One Numerical Example

\begin{align*}
  c_{12} / f_{12} / U_{12} \\
  c_{13} / f_{13} / U_{13} \\
  c_{23} / f_{23} / U_{23}
\end{align*}

\[ m_{\text{soft}} \ \text{[GeV]} \]
One Numerical Example

For completeness: numerical input values used for those plots

- $M_R = 10^{12}$ GeV
- SUSY scale: 2 TeV, $m_{\text{soft}} = 200, \ldots, 4000$ GeV
- $\tan \beta = 10$, $\mu = -3600$ GeV
- $m^{(0)}_\nu = 0.35$ eV (potential \textsc{Katrin} discovery)
- $\Delta m^2_{12} = 7.54 \times 10^{-23}$ GeV$^2$, $|\Delta m^2_{13}| = 2.47 \times 10^{-21}$ GeV$^2$
- $|U_{12}| = 0.53$, $|U_{13}| = 0.15$, $|U_{23}| = 0.58$
- $M_1 = M_2 = m_{\text{soft}}$
- all other $A$ values set to zero
- neutrino $B$ term set to zero
Conclusion

- described corrections very general to theories with new flavour structures
- can completely spoil tree-level mixing patterns
- simple extension of MSSM to incorporate $\nu$ masses can lead to lepton mixing from SUSY breaking in sneutrino sector
- numer(olog)ical example: at least for quasi-degenerate neutrino masses potential size of corrections for $(1, 3)$ and $(2, 3)$ mixing rather the same
Backup

Slides
Splitting of the neutrino mass spectrum

degeneracy of neutrino mass spectrum

neutrino mass [eV]
neutrino mass scale $m_0$ [eV]
degeneracy of neutrino mass spectrum
$m_1$
$m_2$
$m_3$

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enhanced corrections to PMNS mixing

flavour changing self energies and sensitivity to neutrino mass

\[ 
\Delta U_{\nu fi} \sim \frac{m_{\nu f} \Sigma_{fi}}{\Delta m_{\nu}^2} \sim \frac{m_{\nu f} m_{\nu i}}{\Delta m_{\nu fi}^2} \leq 5 \times 10^3 \text{ for } m_\nu^0 \sim 0.35 \text{ eV and } f, i = 1, 2
\]
Superpotential of the $\nu$MSSM

\[ \mathcal{W}^\ell = \mu H_d \cdot H_u - Y^{ij}_\ell H_d \cdot L_L E_R + Y^{ij}_\nu H_u \cdot L_L N_R^J + \frac{1}{2} m_{IJ}^R N_R^I N_R^J, \]

where the chiral superfields are $L_L = (\ell_L, \tilde{\ell}_L) \in SU(2)_L$ and $E_R = (e_R^c \equiv (e_R)^c, \tilde{e}^*_R)$, $N_R = (\nu_R^c, \tilde{\nu}_R^*) \in SU(2)_R$, but leftchiral.

Soft-breaking terms

\[ \mathcal{V}_{\text{soft}} = (\mathcal{M}^2_\ell)^{IJ} \tilde{L}^I_L \tilde{L}^J_L + (\mathcal{M}^2_{\tilde{e}})^{IJ} \tilde{\tilde{e}}^I_R \tilde{e}^J_R + (\mathcal{M}^2_{\tilde{\nu}})^{IJ} \tilde{\tilde{\nu}}^I_R \tilde{\nu}^J_R \]

\[ - \left[ (B_\nu)^{IJ} \tilde{\tilde{\nu}}^I_R \tilde{\nu}^J_R + A^{IJ}_e H_1 \cdot \tilde{L}^I_L \tilde{e}^J_R - A^{IJ}_\nu H_2 \cdot \tilde{L}^I_L \tilde{\nu}^J_R + \text{h.c.} \right], \]
effects on sneutrino mass matrix

- charged slepton mass matrix as in the MSSM
- sneutrino mass matrix in the MSSM: simple

\[
\mathcal{M}^2_{\tilde{\nu}} = \left( \begin{array}{cc}
\mathcal{M}^2_{\tilde{\ell}} + M^2_Z T_{3L} \cos 2\beta & 1 \\
1 & 0
\end{array} \right)
\]

- Majorana mass term $\nu^T_R m_R \nu_R$ inflates sneutrino mass matrix: additional terms $\sim \tilde{\nu}_R \tilde{\nu}_R, \tilde{\nu}^*_R \tilde{\nu}^*_R$

\[
\mathcal{M}^2_{\tilde{\nu}} = \left( \begin{array}{cccc}
\mathcal{M}^2_{L^* L} & \mathcal{M}^2_{L^* L^*} & \mathcal{M}^2_{L^* R} & \mathcal{M}^2_{L^* R^*} \\
\mathcal{M}^2_{L L} & \mathcal{M}^2_{L L^*} & \mathcal{M}^2_{L R} & \mathcal{M}^2_{L R^*} \\
\mathcal{M}^2_{R L} & \mathcal{M}^2_{R L^*} & \mathcal{M}^2_{R R} & \mathcal{M}^2_{R R^*} \\
\mathcal{M}^2_{R^* L} & \mathcal{M}^2_{R^* L^*} & \mathcal{M}^2_{R^* R} & \mathcal{M}^2_{R^* R^*}
\end{array} \right)
\]

12 × 12-Matrix
effects on sneutrino mass matrix

- charged slepton mass matrix as in the MSSM
- sneutrino mass matrix in the MSSM: simple

$$\mathcal{M}_{\tilde{\nu}}^2 = \begin{pmatrix} M_{\tilde{\nu}}^2 + M_Z^2 T_{3L} \cos 2\beta & 1 \\ 1 & 0 \end{pmatrix}$$

- Majorana mass term $\nu_R^T m_R \nu_R$ inflates sneutrino mass matrix: additional terms $\sim \tilde{\nu}_R \tilde{\nu}_R, \tilde{\nu}_R^* \tilde{\nu}_R^*$

$$\mathcal{M}_{\tilde{\nu}}^2 = \begin{pmatrix} M_{LL}^2 & M_{LR}^2 \\ (M_{LR}^2)^\dagger & M_{RR}^2 \end{pmatrix}$$

12 $\times$ 12-Matrix
full sneutrino squared mass matrix in the $\nu$MSSM

$$\mathcal{M}_{\tilde{\nu}}^2 = \frac{1}{2} \begin{pmatrix} \mathcal{M}_{LL}^2 & \mathcal{M}_{LR}^2 \\ (\mathcal{M}_{LR}^2)^\dagger & \mathcal{M}_{RR}^2 \end{pmatrix}$$

$$\mathcal{M}_{LL}^2 = \begin{pmatrix} \mathcal{M}_{\ell}^2 + \frac{1}{2} M_Z^2 \cos 2\beta & 0 \\ 0 & (\mathcal{M}_{\ell}^2)^* \end{pmatrix},$$

$$\mathcal{M}_{RL}^2 = \begin{pmatrix} \frac{1}{2} m_\nu m_R & -\mu \cot \beta m_\nu - v_2 A_\nu^* \\ -\mu^* \cot \beta m_\nu^* - v_2 A_\nu & \frac{1}{2} m_\nu^* m_R^* \end{pmatrix},$$

$$\mathcal{M}_{RR}^2 = \begin{pmatrix} (\mathcal{M}_{\tilde{\nu}}^2)^T + m_\nu^T m_\nu^* + \frac{1}{2} m_R^* m_R & -2B^* \\ -2B & \mathcal{M}_{\tilde{\nu}}^2 + m_\nu^\dagger m_\nu + \frac{1}{2} m_R m_R^* \end{pmatrix}.$$
effective sneutrino mass matrix

\[ \mathcal{M}_{\tilde{\nu}\ell}^2 = \begin{pmatrix} m_{\Delta L=0}^2 & (m_{\Delta L=2}^2)^* \\ m_{\Delta L=2}^2 & (m_{\Delta L=0}^2)^* \end{pmatrix} + \mathcal{O} \left( M_{SUSY}^2 m_R^{-2} \right), \]

\[ m_{\Delta L=0}^2 = \text{MSSM} + m_\nu^D m_\nu^{D\dagger} - m_\nu^D m_R \left( m_R^2 + \mathcal{M}_{\tilde{\nu}}^2 \right)^{-1} m_R m_\nu^D, \]

\[ m_{\Delta L=2}^2 = X_\nu m_\nu^D \left( m_R^2 + \mathcal{M}_{\tilde{\nu}}^2 \right)^{-1} m_R m_\nu^{D T} + (\rightarrow)^T - 2m_\nu^{D*} m_R \left[ m_R^2 + (\mathcal{M}_{\tilde{\nu}}^2)^T \right]^{-1} \mathcal{B} \left( m_R^2 + \mathcal{M}_{\tilde{\nu}}^2 \right)^{-1} m_R m_\nu^{D\dagger}. \]

\[ X_\nu m_\nu^D = -\mu^* \cot \beta m_\nu^{D*} - v_2 A_\nu \]
radiative flavour violation in the lepton sector

\[ W_\mu W_\mu W_\mu \Sigma(e) \]

\[ \Sigma_{ji}^{(e)} U_{fi}^{(0)\dagger} + \Sigma_{ji}^{(e)} U_{ji} \]

\[ \sum_{j\neq i} U_{fi}^{(0)\dagger} \Delta U_{ji}^{e} \]

\[ \sum_{j\neq f} \Delta U_{fj}^{\nu} U_{ji}^{(0)\dagger} \]

\[ \nu_f \]

\[ \nu_f \]

\[ e_i \]

\[ e_i \]

\[ n_f \]

\[ n_f \]

\[ \nu_f \]

\[ \nu_f \]

\[ \nu_f \]

\[ \nu_f \]

\[ e_i \]

\[ e_i \]

\[ e_i \]

\[ e_i \]

\[ W_\mu \]

\[ W_\mu \]

\[ W_\mu \]

\[ W_\mu \]

\[ \Sigma_{ji}^{(e)} \]

\[ \Sigma_{ji}^{(e)} \]

\[ \Sigma_{ji}^{(e)} \]

\[ \Sigma_{ji}^{(e)} \]
Radiative flavour violation in the lepton sector

Flavour changing self energies

\[ \Sigma^\ell_{fi}(p) = \Sigma^\ell_{fi} (p^2) P_L + \Sigma^\ell_{fi} (p^2) P_R + \phi \left[ \Sigma^\ell_{fi} (p^2) P_L + \Sigma^\ell_{fi} (p^2) P_R \right] \]

PMNS matrix renormalization

\[ i \frac{g}{\sqrt{2}} \gamma^\mu P_L U^\dagger \rightarrow i \frac{g}{\sqrt{2}} \gamma^\mu P_L \left( 1 + D_{L,fi} + D_{R,fi} \right), \]
radiative flavour violation in the lepton sector

\[
D_{\nu f, i} = \sum_{j \neq f} m_{\nu_f} \left( \sum_{f_j}^{(\nu)LR} + m_{\nu_f} \sum_{f_j}^{(\nu)RR} \right) + m_{\nu_j} \left( \sum_{f_j}^{(\nu)RL} + m_{\nu_f} \sum_{f_i}^{(\nu)LL} \right) \frac{m_{\nu_j}^2 - m_{\nu_f}^2}{U_{ji}^{(0)\dagger}}
\]

\[
\equiv \sum_{j=1}^{3} \left[ \Delta U_{\nu L}^f \right]_{f_j} U_{ji}^{(0)\dagger}
\]
Majorana mass renormalization

effects of righthanded Neutrinos

- trilinear couplings $A_\nu$
- see-saw-like terms in sneutrino mass matrix

\[
\delta_{LL^*} \sim X_\nu m^D_\nu \left( m_R^2 + M^2_\nu \right)^{-1} m_R m^{DT}_\nu
\]

\[
\sim \frac{v_u A_\nu}{v_R^2} \quad \text{with} \quad m_R = v_R h_R
\]