

Susy and the MSSM

Whistler - SS12

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1/1: LBN (1981/341), N.Lauten (05/09/024), S.Martin (1970/356)

SUSY: Transformation among fields of different spins

Ex: $A(x)$, $\chi_\alpha(x)$ chiral multiplet
 $\begin{array}{c} \uparrow \\ \text{complex} \\ \text{scalar} \end{array}$
 $\begin{array}{c} \uparrow \\ \text{wspf} \\ \text{fermion} \end{array}$

$$\text{Def: } \delta_s := \bar{\varphi}^a Q_a + \bar{Q}_\alpha \bar{\varphi}^\alpha , \quad \bar{\varphi}_a \bar{\varphi}_\beta = -\delta_{ab} \delta_\alpha^\beta$$

Grassmann parameters

$$\{Q_a, Q_b\} = 2 \frac{\partial \varphi^m}{\partial \varphi^a} \frac{\partial \varphi^m}{\partial \varphi^b} , \quad (\text{F})$$

$$\{Q_a, Q_\beta\} = 0 = \{\bar{Q}_i, \bar{Q}_j\}$$

$$\begin{aligned} \delta_s A &= \sqrt{2} \bar{\varphi}^a X_\alpha \\ \delta_s X_\alpha &= i \sqrt{2} \bar{\varphi}_{\alpha i} \bar{\varphi}^a \partial_a A \\ \delta_s F &= i \sqrt{2} \bar{\varphi}_\alpha \bar{\sigma}^{mn} \partial_m X_n \end{aligned} \quad \left. \begin{array}{l} \left. \begin{array}{l} \text{satisfies (F) if} \\ \bar{\varphi}^a \partial_a X = 0 \text{ is used} \end{array} \right\} \\ \text{no q. sl. motion} \\ \text{(off shell)} \end{array} \right\} \quad \begin{array}{l} \text{F: auxiliary field} \end{array}$$

vector multiplet $(V_m(x), \lambda_\alpha(x))$

$$\delta V_m = -i \bar{\lambda} \bar{\sigma}^{mn} \dot{q} + i \bar{\varphi}^m \lambda$$

$$\delta \lambda = \bar{\sigma}^{mn} \delta F_{mn} \quad (F_{mn} = \partial_m V_n - \partial_n V_m)$$

Non-Abelian gauge group G with generators $\{T^a, T^b\} = i f^{abc} T^c$

$$[T^a, Q] = 0$$

 (V_m^a, λ^a) in adjoint rep of G $a = 1, \dots, \text{dim}(G)$
 (A^i, χ^i) in rep Σ of G $i = 1, \dots, \text{dim}(\Sigma)$

$$\mathcal{L} = -\frac{1}{4} F_{mn}^a F^{amn} - i \bar{\lambda}^a D^a \lambda^a - D_m \bar{A}^i D^m A^i - i \bar{\pi}^i D^i \pi^i$$

$$+ i \sqrt{2} g \bar{A}^i T_{ij}^a \chi^j \lambda^a + \text{c.c.}$$

$$- \sum_i W_{ij}^{(A)} \chi^j \bar{\chi}^i + \text{c.c.} - V(A, \bar{A})$$

$$W(A) = \frac{1}{2} m_{ij} A^i A^j + \frac{1}{3} Y_{ijk} A^i A^j A^k \quad (\text{holomorphic superpot.})$$

$$W_{ij} := \frac{\partial W}{\partial A^i}, \quad W_{ij} = \frac{\partial^2 W}{\partial A^i \partial A^j}$$

$$V = F_i \bar{F}_i + \frac{1}{2} D^a D^a , \quad \bar{F}_i = - \bar{W}_i$$

$$D_a = -g \bar{A}^i T_{ij}^a A^j$$

Remarks

- $V \geq 0$
- V is not general; cf. no $\lambda F A_1^2$ coupling

instead: $\lambda \sim \begin{cases} y^2 \\ g^2 \end{cases}$ - F-term
- D-term

- parameters of \mathcal{L} : θ_i , m_{ij} , y_{ijk}

HSSM: promote each field of SM to supermultiplet

	supermultiplet	F	B	SU(3)	SU(2)	U(1) _Y	U(1) _{em}
quarks	$Q_L = \begin{pmatrix} U_L \\ D_L \end{pmatrix}$	q_L	\bar{q}_L	3	2	$\frac{1}{3}$	$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$
	U_h^L D_h^L	u_h^L d_h^L	\bar{u}_h^L \bar{d}_h^L	3 3	1 1	$-\frac{1}{3}$ $\frac{1}{3}$	$\begin{pmatrix} -1 \\ 1 \end{pmatrix}$
leptons	$L_L = \begin{pmatrix} N_L \\ E_L \end{pmatrix}$	ℓ_L	$\bar{\ell}_L$	1	2	$-\frac{1}{2}$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$
	E_R^L \sqrt{F}_R^L	e_R^L \sqrt{F}_R^L	\bar{e}_R^L \sqrt{F}_R^L	1 1	1 1	1 0	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$
Higgs	$H_d = \begin{pmatrix} h_d^0 \\ h_d^+ \end{pmatrix}$	$\begin{pmatrix} h_d^0 \\ h_d^- \end{pmatrix}$	$\begin{pmatrix} h_d^0 \\ h_d^+ \end{pmatrix}$	1	2	$-\frac{1}{2}$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$
	$H_u = \begin{pmatrix} h_u^0 \\ h_u^+ \end{pmatrix}$	$\begin{pmatrix} h_u^0 \\ h_u^+ \end{pmatrix}$	$\begin{pmatrix} h_u^0 \\ h_u^+ \end{pmatrix}$	1	2	$\frac{1}{2}$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$
gauge bosons	G W B	\tilde{G} \tilde{W} \tilde{B}	G W B	8 1 1	1 3 1	0 0 0	$(0, \pm 1)$

Table 2: Particle content of the supersymmetric Standard Model. The column below 'F' ('B') denotes the fermionic (bosonic) content of the model.

$I = 1, 2, 3$

g_1, g_2, g_3 as in SM

$$h = \mu h_u h_d + (Y_u)_S h_u \tilde{q}_L^T \tilde{u}_R^T + (Y_d)_S h_d \tilde{q}_L^T \tilde{d}_R^T$$

$$+ (Y_e)_S h_d \tilde{\ell}_L^T \tilde{e}_R^T$$

additional couplings possible (respect gauge + Lorentz inv.)

$$h_u \tilde{\ell}_L, \tilde{\ell}_L \tilde{q}_L d_R, \tilde{d}_R \tilde{d}_R \tilde{u}_R, \tilde{\ell}_L \tilde{\ell}_L \tilde{e}_R$$

dim-4 photon decay!

can be forbidden by R-parity: R-parity +1 + SM
-1 \neq SM parity

Motivation for supersymmetry

- gauge couplings unification
- "natural" Higgs-sector
- DM candidate
- (\neq -masses)

- extra terms above have odd # of filled fields
- standard-SM terms have even # of filled fields

Note: 2 Higgs - doublets necessary (sum Higgs-sector)

Spontaneous sunny breaking

thus vacua $\Rightarrow m_B = m_F \Rightarrow$ SSM necessary

$$\text{problematique} \quad \text{sum rule:} \quad \sum_i m_i^2 = 2 \sum_f m_f$$

way out:

- explicit but soft breaking
- \approx of local vars
+ superglobals

} \approx the same

$$\text{soft breaking: } \quad L = L_{\text{SM}} + L_{\text{Soft}}$$

- no quadratic div. introduced
 \Rightarrow hypers. sector stays natural
- class. field $\mathbb{F}_{\text{Galois}}$, $\mathbb{F}_{\text{Galois}}$

$$\mathcal{L}_{\text{Soft}} = -m_{ij}^2 A^i \bar{A}^j - (b_{ij} A^i A^j + c_{ijkl} A^i A^j A^k + h.c.) - \frac{1}{2} \tilde{m} \bar{\lambda} \lambda + h.c.$$

\Rightarrow (max) her parameters $m_{ij}^*, b_{ij}^*, a_{ij}^*, u_i^*$

and constrained by ganz ihrer

choice of soft parameter \Rightarrow specific model
 specific UV-theory
 + medietio medium

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local supersegment = Supergram

Polymer: $\xi_x \rightarrow \xi_x(x)$

\Rightarrow moments of fermionic gauge fields w/ $C = \frac{1}{2}$:

$$\partial_m \delta_s A \approx \partial_m \delta x = \underbrace{\delta \partial_m x}_{s=1} + (\partial_m \delta) x$$

\Rightarrow need operations Ψ_m with transformation

$$\delta\psi_{m_2} = - \partial_m \varphi_2 +$$

Ψ_{rad} is part of gravitational multiplet $(g_{mn}(x), \Psi_{\mu\nu}(x))$
 \Rightarrow gravitational interaction has to be included!
 \Rightarrow complete theory of $(g_{mn}, \Psi_{\mu\nu})$ + allow non-loc. interactions

$$Y(U_i, W_j, f) = -\frac{1}{2} \epsilon^{ijk} R + \sigma^{mnp} \bar{\Psi}_m D_p \Psi_j$$

$$-\mathbb{G}_{ij}(A, \bar{A}) D_m A^i D^n \bar{A}^j = V(A, \bar{A})$$

$$-\frac{1}{4}g_{as}^2 F_{mn}^a F^{lmn} + \frac{\Theta_{as}}{32\pi^2} FF^L + T_{mn}^L$$

$$G_{ij} = \frac{\partial}{\partial A^i} \frac{\partial}{\partial \bar{A}^j} K(A, \bar{A}) \quad , \quad f_{as}(A) = \frac{1}{g_{as}^2(A)} + i \frac{g_{as}^{(A)}}{8\pi^2}$$

$\overset{\uparrow}{\text{K\"ahler metric}}$ $\overset{\uparrow}{\text{K\"ahler pot.}}$

$$V = e^{k^2 k} \left[D_i W \partial^{i,j} D_j \bar{W} - 3k^2 (W \bar{W}) \right], \quad D_i W := \frac{\partial W}{\partial A^i} + k^2 \frac{\partial \bar{W}}{\partial A^i} \cdot W$$

order parameters for susy breaking:

$$\delta \Phi_m = 2 D_m \delta + i e^{k \frac{Y_2}{\lambda}} W T_m \bar{\delta} + \dots$$

$$\delta x^i = - \sqrt{2} F^i \delta + \dots, \quad F^i = e^{k \frac{Y_2}{\lambda}} \theta^{ij} \bar{D}_j \bar{W}$$

$$\delta \lambda^\alpha = - i g D^\alpha \delta + \dots$$

$\Rightarrow \langle F^i \rangle, \langle D^\alpha \rangle$ parameterize susy breaking

$i \Rightarrow$ manufacture potential such that $\langle F^i \rangle \neq 0$
and/or $\langle D^\alpha \rangle \neq 0$

Remarks:

- is possible, e.g. Polonyi model

$$W = W_{\text{obs}} + W_{\text{ind}}(\phi), \quad W_{\text{ind}} = m_\phi (\phi + \delta)$$

$$\beta = \frac{2-\sqrt{5}}{K}, \quad \langle v \rangle \approx$$

- Ψ_{ind} gets a mass $m_{3/2}$, g_m stay massless

- Sam rank modified:

$$\sum_b m_b^2 - 2 \sum_{f,h} m_{fh}^2 \approx m_{3/2}^2$$

- low energy limit: $M_{\text{Pl}} \rightarrow \infty$, $\langle F^i \rangle, \langle D^\alpha \rangle$ fixed

expand L in $\frac{\langle F^i \rangle \langle D^\alpha \rangle}{M_{\text{Pl}}}$: $L = L_{\text{global SUSY}} + L_{\text{SOFT}} + \dots$
 $+ O\left(\frac{m_{3/2}^2}{M_{\text{Pl}}^2}\right)$