Supersymmetry breaking in string theory I

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1 Supergravity limit

String theory is described by a sigma model depending on three external bosonic fields on the target spacet, the metric $g_{\mu\nu}$, Kalb-Ramond 2-form $B_{\mu\nu}$ with field strength $H = \mathrm{d}B$ and the dilaton ϕ (a function on target space). In superstring theory these fields are extended by their superpartners to a $D = 10$, $\mathcal{N} = 1$ or $\mathcal{N} = 2$ gravity multiplet. Besides g, B and ϕ, the $\mathcal{N} = 1$ multiplet contains the gravitino ψ_μ (left-handed Majorana-Weyl spinor) and dilatino λ (right-handed Majorana-Weyl spinor). The additional field content of heterotic and type II string theories is as follows:

- heterotic: an $\mathcal{N} = 1$ super Yang-Mills multiplet (A_μ, χ) in the adjoint of the gauge group $E_8 \times E_8$ or SO(32).

- IIA: a 1-form and 3-form potential $C^{(1)}$, $C^{(3)}$. There are two gravitinos ψ^+_μ, ψ^-_μ of opposite chirality and two dilatinos λ^-, λ^+ of opposite chirality. Massive IIA has an additional parameter (a zero-form) m.

- IIB: 0-, 2- and 4-form potentials $C^{(0)}, C^{(2)}, C^{(4)}$, the latter having self-dual field strength. There are two left-handed gravitinos and two right-handed dilatinos.

The condition for conformal invariance (vanishing of the β-functions) of the sigma model is given by the supergravity field equations:

$$\begin{aligned} \mathrm{Ric}_{\mu\nu} + 2(\nabla \mathrm{d}\phi)_{\mu\nu} - \tfrac{1}{4} H_{\mu\kappa\lambda} H_\nu{}^{\kappa\lambda} + \ldots &= 0 \\ \frac{2}{3\alpha'}\big(\dim M - 10\big) - 2\Delta\phi + 4|\mathrm{d}\phi|^2 - |H|^2 + \ldots &= 0 \\ \mathrm{d} * (e^{-2\phi} H) + \ldots &= 0, \end{aligned} \tag{1.1}$$

where dots indicate fermionic terms, terms involving the additional fields of the specific string theory at hand (which also lead to additional field equations), and contributions of higher order in α'. In the following we assume $\dim M = 10$. The field equations (1.1) can be derived from an action

$$S = \int \mathrm{d}^{10}x \sqrt{|g|} e^{-2\phi} \Big(\mathrm{Scal}^g - 4|\mathrm{d}\phi|^2 + \frac{1}{2}|H|^2 \Big) + \ldots \tag{1.2}$$

The SUSY transformations of the fermions are

$$\begin{aligned}\delta\psi_\mu &= \Big(\nabla_\mu - \frac{1}{8} H_{\mu\nu\lambda}\gamma^{\nu\lambda} + \dots\Big)\epsilon \\ \delta\lambda &= \frac{1}{2}\Big(\partial_\mu\phi\,\gamma^\mu - \tfrac{1}{12} H_{\mu\nu\lambda}\gamma^{\mu\nu\lambda} + \dots\Big)\epsilon\end{aligned} \tag{1.3}$$

Supersymmetry is left unbroken by a background $(M, g, B, \phi, \dots)$ if the variations (1.3) vanish:

$$\delta\psi_\mu = \delta\lambda = 0. \tag{1.4}$$

Hence, one mechanism to break supersymmetry in a string vacuum is to find a solution to (1.1) which has $\delta\psi_\mu, \delta\lambda \neq 0$ for all non-vanishing spinors $\epsilon \to$ SUSY breaking at compactification scale.

1.1 Compactification

To make contact with real world physics we have to reduce the number of dimensions from ten to four. This is usually accomplished by choosing a background of the form $M^{10} = M^4 \times M^6$, with M^6 compact. Dimensional reduction on M^6 then leads to an effective field theory on M^4. We will focus mainly on the case $M^4 = \mathbb{R}^{3,1}$, i.e. 4-dimensional Minkowski space, and use the following index convention: indices $a, b, c = 0, \dots 3$ parametrize M^4, whereas $i, j, k = 4, \dots 9$ belong to M^6.
A 10D Majorana-Weyl spinor ϵ can be decomposed into 4D and 6D spinors

$$\epsilon = \epsilon^4_+ \otimes \epsilon^6_+ + \epsilon^4_- \otimes \epsilon^6_-, \tag{1.5}$$

where the indices $\pm$ denote opposite chiralities, and $\epsilon^{4,6}_-$ is the charge conjugate of $\epsilon^{4,6}_+$. The gravitino equation splits into internal and external equations $\delta\psi_i = 0$ and $\delta\psi_a = 0$ as well. We assume that the fields have no components in M^4; then the external gravitino equation is trivially satisfied by constant spinors, and we are left with two BPS equations

$$\delta\psi_i = \delta\lambda = 0 \tag{1.6}$$

on M^6. The amount of supersymmetry of the effective 4-dimensional theory is equal to the number of independent solutions to (1.6) for the heterotic theory, and twice that much for type II theories.

Ricci-flat solutions. The simplest class of solutions to the field equations (1.1) has $H = \mathrm{d}\phi = 0$ (= RR-field strengths) and $\mathrm{Ric} = 0$. The gravitino equation reduces to $\nabla\epsilon = 0$, so the Levi-Civita connection ∇ of M^6 must admit a parallel spinor for unbroken supersymmetry. According to the general holonomy principle the number of solutions ϵ is equal to the number of spinors invariant under the holonomy group G of ∇. Recall that the generic holonomy group on a spin manifold of dimension n is $\mathrm{SO}(n)$, which doesn't fix any spinor. Wang's theorem gives a classification of possible (irreducible) Riemannian holonomy groups with invariant spinors:

n	G	$\sharp$ spinors		geometry
7	G_2	1		exceptional
8	Spin(7)	(1,0)		exceptional
$2n$	SU(n)	(1,1)	(n odd)	Calabi-Yau
$2n$	SU(n)	(2,0)	(n even)	Calabi-Yau
$4n$	Sp(n)	($n+1$,0)		hyperkähler

Table 1: Invariant spinors for subgroups $G \subset \mathrm{SO}(n)$. In even dimensions even and odd chirality spinors are listed separately. Subgroups of G can fix additional spinors and lead to enhanced supersymmetry.

We are mainly interested in the 6D case, but for instance the holonomy group G_2 in 7D plays a role in compactifications of M theory (i.e. 11D supergravity) down to four dimensions. One can show that the existence of a parallel spinor implies vanishing of the Ricci-curvature, so every BPS solution satisfies the field equations. The converse is an open problem:

Q: does compact Ricci-flat imply existence of a parallel spinor?

This is not true in the non-compact situation. In physic terms, the question is whether there are vacuum string backgrounds that break supersymmetry completely.

If the holonomy group is a proper subgroup of G, then additional SUSYs can be preserved. A flat torus for instance, with $\mathrm{Hol}(\mathbb{T}^6) = \{1\}$, has four parallel Weyl spinors, and hence leads to a 4D theory with $\mathcal{N} = 4$ (heterotic) or $\mathcal{N} = 8$ (type II) supersymmetry.

Flux compactifications. Solutions with non-vanishing form fields ('fluxes') on the internal manifold are called flux compactifications. The simple classification in terms of holonomy groups becomes more complicated then, and one has to work with G-structures. For instance, a 6-manifold with a globally defined non-vanishing Weyl-spinor ϵ automatically carries an SU(3)-structure, defined by a 2-form ω and 3-form Ω

$$\omega_{ij} = \frac{\mathrm{i}}{2}\bar{\epsilon}\gamma_{ij}\epsilon, \qquad \Omega_{ijk} = \frac{1}{6}\epsilon^T\gamma_{ijk}\epsilon. \tag{1.7}$$

SU(3)-structure manifolds are classified by five torsion classes $W_1, \ldots, W_5$:

$$\begin{aligned} \mathrm{d}\omega &= \frac{3}{2}\mathrm{Im}(W_1\overline{\Omega}) + W_4 \wedge \omega + W_3 \\ \mathrm{d}\Omega &= W_1\omega \wedge \omega + W_2 \wedge \omega + \overline{W}_5 \wedge \Omega \end{aligned} \tag{1.8}$$

If all $W_i = 0$, we are back in the Calabi-Yau case, whereas vanishing of W_1 and W_2 is necessary for the manifold to be complex. Supersymmetric string compactifications to 4D require an internal manifold with SU(3)-structure, the details depend on the type of string theory considered:

- heterotic: to obtain $\mathcal{N} = 1$ in 4D Minkowski space the internal space must be a conformally balanced Hermitian manifold (M^6, g, ω, Ω), equipped with a holomorphic vector bundle E with connection. The forms ω, Ω have to satisfy a set of differential equations known as Strominger's equations [4], and the fluxes are determined by

$$H = \frac{\mathrm{i}}{2}(\bar{\partial} - \partial)\omega, \qquad \phi - \phi_0 = \frac{1}{8}\log ||\Omega||. \tag{1.9}$$

In terms of torsion classes the conditions can be rephrased as

$$W_1 = W_2 = 2W_4 + W_5 = 0, \tag{1.10}$$

and W_4, W_5 exact. Few solutions known.

- IIA & B: $\mathcal{N} = 1$ SUSY on 4D Minkowski space requires twisted generalized Calabi-Yaus $\rightarrow$ generalized complex geometry [1].

Typically, it is simpler to obtain AdS-vacua than Minkowski space in flux compactifications. For instance, the D3-brane in IIB has near-horizon geometry $\mathrm{AdS}_5 \times S^5$, with 5-form flux equal to the sum of the volume forms of both factors.

Example: IIA flux compactification to $\mathbf{AdS}_4$. Massive IIA on the other hand admits a large class of $\mathcal{N} = 1$ $\mathrm{AdS}_4 \times M^6$ solutions, which have

$$W_3 = W_4 = W_5 = 0, \tag{1.11}$$

and hence are non-complex [5]. The fluxes are given in terms of the SU(3)-invariant forms as

$$\begin{aligned} H &= \frac{2m}{5} e^{\phi} \mathrm{Re}\,\Omega, \qquad \mathrm{d}\phi = 0, \\ F^{(2)} &= \frac{f}{9}\omega + F_2', \\ F^{(4)} &= f \mathrm{Vol}_{\mathrm{AdS}_4} + \frac{3m}{10}\omega \wedge \omega, \end{aligned} \tag{1.12}$$

where the field strengths $F^{(2)}, F^{(4)}$ are defined as

$$F^{(2)} = \mathrm{d}C^{(1)} + mB, \qquad F^{(4)} = \mathrm{d}C^{(3)} - H \wedge C^{(1)} + \frac{m}{2}B^2. \tag{1.13}$$

The 2-form F' is of type (1,1) and orthogonal to ω, and f is a constant parameter. In general the solution contains smeared O6 planes, which modify one of the Bianchi identities for the field strengths. The torsion classes are

$$W_1 = -\frac{4\mathrm{i}}{9} e^{\phi} f, \qquad W_2 = -\mathrm{i}\, e^{\phi} F_2'. \tag{1.14}$$

Explicit examples are nearly Kähler manifolds ($W_2 = 0$), including the 6-sphere, a flat 6-torus with non-trivial fluxes ($W_1 = W_2 = 0$), and the Iwasawa manifold $\mathbb{T}^6/(\mathbb{Z}_2 \times Z_2)$ (both $W_1, W_2 \neq 0$). The effective 4D field theory has been worked out in [5], where it was found that in many cases all moduli are stabilized.

The Iwasawa manifold is a special nilmanifold/twisted torus. The latter have also been considered in the context of Minkowski space compactifications [6], where existence of solutions has been deduced by performing three T-dualities on a IIB Calabi-Yau orientifold background with fluxes. The idea is as follows. Start from a 3-torus with metric

$$\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2, \tag{1.15}$$

where $x \sim x + 1$ etc., and NS-3-form flux $H = N\mathrm{d}x \wedge \mathrm{d}y \wedge \mathrm{d}z$. One can choose a gauge where the 2-form potential is

$$B = Nx\mathrm{d}y \wedge \mathrm{d}z. \tag{1.16}$$

In the T-dual along z the B-field vanishes and one gets the following metric

$$\mathrm{d}x^2 + \mathrm{d}y^2 + (\mathrm{d}z + Nx\mathrm{d}y)^2, \tag{1.17}$$

where one should identify

$$(x, y, z) \simeq (x, y + 1, z) \simeq (x, y, z + 1) \simeq (x + 1, y, z - Ny). \tag{1.18}$$

This is an example of a nilmanifold or twisted torus; it can be written in the form of a coset space. Consider the Heisenberg group H^N, generated by matrices

$$\begin{pmatrix} 1 & y & -\frac{1}{N}z \\ 0 & 1 & x \\ 0 & 0 & 1 \end{pmatrix}, \tag{1.19}$$

$x, y, z \in \mathbb{R}$, and the subgroup $H^N_{\mathbb{Z}}$:

$$\begin{pmatrix} 1 & b & -\frac{1}{N}c \\ 0 & 1 & a \\ 0 & 0 & 1 \end{pmatrix}, \tag{1.20}$$

with $a, b, c \in \mathbb{Z}$. The quotient $H^N/H^N_{\mathbb{Z}}$ is isomorphic to the twisted torus above. Although it is not itself a solution of the supergravity equations of motion, it illustrates a method to obtain solutions of IIA, by mirror dualizing (equivalent to three T-dualities in this case) certain warped IIB Calabi-Yau orientifolds with NS- and RR-3-form flux [6]. The resulting IIA background contains two copies of the twisted torus.

From a cosmological point of view 4D de Sitter vacua would be most appealing, but these appear to be impossible in pure supergravity. Inclusion of non-perturbative stringy effects may however give rise to de Sitter vacua, and examples based on negatively curved internal nil- and solvmanifolds are constructed in [7].

2 Effective four dimensional theory

2.1 Calabi Yau topology

A Calabi-Yau 3-fold is a complex manifold of $\dim_{\mathbb{C}} = 3$, equipped with a Kähler form $\omega \in \Omega^{1,1}$ and a holomorphic 3-form $\Omega \in \Omega^{3,0}$, which are both closed (even parallel): $\mathrm{d}\omega = \mathrm{d}\Omega = 0$. The Calabi-Yau condition restricts the topology severly; it can be shown that the so-called Hodge diamond, consisting of the Betti numbers $h^{p,q} = \dim H^{p,q}(M, \mathbb{C})$, assumes the following form if the holonomy group is strictly SU(3):

$$\begin{array}{ccccccc}
 & & & 1 & & & \\
 & & 0 & & 0 & & \\
 & 0 & & h^{1,1} & & 0 & \\
1 & & h^{1,2} & & h^{1,2} & & 1 \\
 & 0 & & h^{1,1} & & 0 & \\
 & & 0 & & 0 & & \\
 & & & 1 & & &
\end{array}$$

Due to the correspondence between cohomology and harmonic forms this means e.g. that there is exactly one harmonic 0-form (a constant), no harmonic 1- or 5-form, $h^{1,1}$ harmonic (1,1)-forms, etc.

Many examples of Calabi-Yau 3-folds are known, but their metrics cannot be written down in closed form. A simple example is the blown-up orbifold; define a 6-torus as the quotient of $\mathbb{C}^3$ by relations

$$z^m = z^m + 1, \qquad z^m = z^m + e^{2\pi \mathrm{i}/3}, \tag{2.1}$$

$m = 1, 2, 3$, and quotient the resulting $\mathbb{T}^6$ by the following $\mathbb{Z}_3$-action:

$$z^m \mapsto e^{2\pi \mathrm{i}/3} z^m. \tag{2.2}$$

The quotient is singular at the 27 fixed points of the action, but can be blown-up to a smooth Calabi-Yau. This introduces a 2- and a 4-cycle at each fixed point, so one gets $h^{1,1} = 27$.

Other famous CY^3 examples are quintic hypersurfaces in $\mathbb{C}P^4$ [3, 2].

2.2 Kaluza-Klein reduction

IIA. We consider the dimensional reduction of IIA supergravity on a Calabi-Yau 3-fold M^6. To expand the 10-dimensional fields in terms of harmonics on M^6, denote by ω_A a basis of harmonic (1,1)-forms on M^6, $A = 1, \ldots, h^{1,1}$, and by (α_K, β^K) a symplectic basis of 3-forms, $K = 0, \ldots h^{2,1}$, satisfying

$$\int \alpha_K \wedge \beta^L = \delta_K^L. \tag{2.3}$$

Furthermore, we denote 4D-coordinates by x. The NS-fields are then expanded as

$$\phi = \phi(x), \qquad B = B_2(x) + b^A(x)\omega_A. \tag{2.4}$$

Deformations of the internal metric components $g_{i\bar{\jmath}}$ can be shown to correspond to $H^{1,1}$, and those of g_{ij} to $H^{2,1}$:

$$\delta g_{i\bar{\jmath}} = v^A(x)(\omega_A)_{i\bar{\jmath}}, \qquad \delta g_{ij} = \bar{z}_k(x)(\beta^k)_{i\overline{kl}}\Omega^{\overline{kl}}{}_j, \tag{2.5}$$

where $k = 1, \dots, h^{2,1}$. The (1,1)-deformations are called Kähler deformations, since they modify the Kähler form as $\delta\omega = \delta g_{i\bar{j}} \mathrm{d}z^i \wedge \mathrm{d}\bar{z}^j$. Deformations in g_{ij} on the other hand require a new set of complex coordinates in which the metric is again purely of type (1,1), and they are called complex structure deformations [3]. It turns out that the moduli spaces of complex and Kähler structures for a Calabi-Yau are both themselves Kähler manifolds, whose Kähler potentials are given by

$$K^C = -\log\Big[\mathrm{i}\int \Omega \wedge \overline{\Omega}\Big], \qquad K^K = -\log\Big[\int \omega^3\Big]. \tag{2.6}$$

The type IIA RR-fields are expanded as follows:

$$\begin{aligned} C^{(1)} &= A^0(x), \\ C^{(3)} &= A^A(x) \wedge \omega_A + \xi^K(x)\alpha_K + \tilde{\xi}_K(x)\beta^K \end{aligned} \tag{2.7}$$

We end up with the following 4D fields: metric $g_{\mu\nu}$, scalars $(\phi, v^A, b^A, z^k, \xi^K, \tilde{\xi}_K)$, 1-forms (A^0, A^A), 2-form B_2. They fit into 4D $\mathcal{N} = 2$ supermultiplets (here only bosonic fields):

gravity multiplet	1	$(g_{\mu\nu}, A^0)$
vector multiplets	$h^{1,1}$	(A^A, v^A, b^A)
hypermultiplets	$h^{2,1}$	$(z^k, \xi^k, \tilde{\xi}_k)$
tensor multiplet	1	$(B_2, \phi, \xi^0, \tilde{\xi}_0)$

Table 2: $\mathcal{N} = 2$ multiplets for Type IIA supergravity compactified on a Calabi-Yau manifold. The tensor multiplet can be dualized to an additional hypermultiplet.

The effective 4D field theory is a $\mathcal{N} = 2$ ungauged supergravity, given in terms of a Lagrangian as

$$\begin{aligned} S^{(4)} = \int_{M^4} & -\tfrac{1}{2} * \mathrm{Scal} + \tfrac{1}{2}\mathrm{Re}\mathcal{N}_{\alpha\beta}\, F^\alpha \wedge F^\beta + \tfrac{1}{2}\mathrm{Im}\mathcal{N}_{\alpha\beta}\, F^\alpha \wedge *F^\beta \\ & - G_{A\overline{B}}\, \mathrm{d}t^A \wedge * \mathrm{d}\bar{t}^B - h_{uv}\, \mathrm{d}q^u \wedge * \mathrm{d}q^v, \end{aligned} \tag{2.8}$$

where $\alpha = (0, A)$, and $F^\alpha = \mathrm{d}A^\alpha$. The Kähler and B-field deformations are combined into complexified Kähler moduli

$$t^A = b^A + \mathrm{i}\, v^A. \tag{2.9}$$

They span a special Kähler manifold of complex dimension $h^{1,1}$, whereas the hypermultiplet scalars are collectively denoted by q^u and span a quaternionic Kähler manifold of quaternionic dimension $h^{2,1}+1$. The full scalar moduli space is a product

$$M^v \times M^h \tag{2.10}$$

of the vector and hypermultiplet moduli spaces. The Kähler potential K of M^v is given essentially by the volume of M^6,

$$\begin{aligned} K &= -\log\Big(\frac{4}{3}\int\omega^3\Big) \\ &= -\log\Big(\frac{\mathrm{i}}{6}\mathcal{K}_{ABC}(t-\bar{t})^A(t-\bar{t})^B(t-\bar{t})^C\Big), \end{aligned} \tag{2.11}$$

where the intersection numbers $\mathcal{K}_{ABC}$ are

$$\mathcal{K}_{ABC} = \int \omega_A\wedge\omega_B\wedge\omega_C. \tag{2.12}$$

The Kähler potential can be derived from a holomorphic prepotential

$$e^{-K(t,\bar{t})} = \mathrm{i}\Big(\bar{t}^A\frac{\partial\mathcal{F}}{\partial t^A} - t^A\frac{\partial\overline{\mathcal{F}}}{\partial\bar{t}^A}\Big), \qquad \mathcal{F}(t) = -\frac{1}{6}\mathcal{K}_{ABC}\frac{t^At^Bt^C}{t^0}, \tag{2.13}$$

where t^0 is an auxiliary coordinate to be set to one after differentiation. This is a characteristic property of special Kähler manifolds. An alternative definition of special Kähler is in terms of a flat torsionfree connection ∇ on the tangent bundle of a Kähler manifold such that

- $\nabla\omega = 0$, i.e. ∇ is symplectic
- $\nabla_{[i}{J_{j]}}^k = 0$, where J is the complex structure.

The Kähler metric is

$$G_{A\bar{B}} = \partial_{t^A}\partial_{\bar{t}^B}K = 2e^K\int\omega_A\wedge *\omega_B, \tag{2.14}$$

and the matrix $\mathcal{N}_{\alpha\beta}$ assumes the following form

$$\begin{aligned} \mathrm{Re}\mathcal{N} &= \begin{pmatrix} -\frac{1}{3}\mathcal{K}_{ABC}b^Ab^Bb^C & \frac{1}{2}\mathcal{K}_{ABC}b^Bb^C \\ \frac{1}{2}\mathcal{K}_{ABC}b^Bb^C & -\mathcal{K}_{ABC}b^C \end{pmatrix}, \\ \mathrm{Im}\mathcal{N} &= -\frac{e^{-K}}{8}\begin{pmatrix} 1+4G_{AB}b^Ab^B & -4G_{AB}b^B \\ -4G_{AB}b^B & 4G_{AB} \end{pmatrix} \end{aligned} \tag{2.15}$$

The quaternionic Kähler metric h_{uv} can be found e.g. in [8].

IIB. The effective theory for IIB is very similar to IIA, but the roles of the moduli get interchanged. The RR-form expansion is

$$\begin{aligned} C^{(0)} &= C_0(x) \\ C^{(2)} &= C_2(x) + c^A(x)\omega_A \\ C^{(4)} &= V^K(x)\alpha_K + \rho_A(x)\tilde{\omega}^A + ..., \end{aligned} \tag{2.16}$$

where dots indicate terms required for self-duality of $F^{(5)}$, the field strength of $C^{(4)}$. The 4-forms $\tilde{\omega}^A$ are dual to the 2-forms ω_A, i.e. $\int\omega_A\wedge\tilde{\omega}^B = \delta_A^B$. The complex structure moduli z^k are defined differently from their IIA counterparts. We can expand the (3,0)-form Ω for a deformed metric in terms of harmonic 3-forms:

$$\Omega = Z^K \alpha_K - \mathcal{F}_K \beta^K, \tag{2.17}$$

and set $z^k = Z^K/Z^0$, for $k = 1, \ldots, h^{2,1}$. One obtains the following field content:

gravity multiplet	1	$(g_{\mu\nu}, V^0)$
vector multiplets	$h^{2,1}$	(A^k, z^k)
hypermultiplets	$h^{1,1}$	(v^A, b^A, c^A, ρ_A)
tensor multiplet	1	(B_2, C_2, ϕ, C_0)

Table 3: $\mathcal{N} = 2$ multiplets for Type IIB supergravity compactified on a Calabi-Yau manifold. The tensor multiplet can be dualized to an additional hypermultiplet.

The effective action takes the same form as for IIA (2.8), only the range of indices is switched (see [1]); the complex structure deformations are now in the vector multiplet and span a special Kähler manifold, the Kähler deformations are in the hypermultiplet and give rise to a quaternionic Kähler manifold. The coupling matrix becomes

$$\mathcal{N}_{KL} = \overline{\mathcal{F}}_{KL} + 2\mathrm{i}\,\frac{\mathrm{Im}(\mathcal{F})_{KM} Z^M \mathrm{Im}(\mathcal{F})_{LN} Z^N}{\mathrm{Im}(\mathcal{F})_{MN} Z^M Z^N}, \tag{2.18}$$

where $\mathcal{F}_{KL} = \partial_L \mathcal{F}_K$. The Kähler potential for the vector moduli space is

$$K = -\log\left[\mathrm{i}\int \Omega \wedge \overline{\Omega}\right] = -\log \mathrm{i}\left[\overline{Z}^K \mathcal{F}_K - Z^K \overline{\mathcal{F}}_K\right]. \tag{2.19}$$

The metrics G_{kl} and h_{uv} can be found in [8, 1].

2.3 Orientifolds

IIA. One possibility to break the effective $\mathcal{N} = 2$ down to $\mathcal{N} = 1$ is the introduction of orientifolds. We consider again type IIA, with O6 planes. Suppose that M^6 admits an involutive isometry σ, i.e. σ is a diffeomorphism satisfying $\sigma^2 = 1$ and $\sigma^* g = g$, and assume additionally

$$\sigma^* \omega = -\omega, \qquad \sigma^* \Omega = \overline{\Omega}. \tag{2.20}$$

The fixed-point set $\{x \in M^6 | \sigma(x) = x\}$ of the involution is a so-called special Lagrangian submanifold of M^6, which can be thought of as the world-volume of the orbifold. Define an operator

$$\mathcal{O} = (-1)^{F_L} \Omega_p \sigma, \tag{2.21}$$

where $(-1)^{F_L}$ is the spacetime fermion number in the left-moving sector and Ω_p is worldsheet parity. The transformation behaviour of the various fields under $(-1)^{F_L}$ and Ω_p is given in the following table.

	ϕ	g	B	C_1	C_3
$(-1)^{F_L}$	$+$	$+$	$+$	$-$	$-$
Ω_p	$+$	$+$	$-$	$+$	$-$

Table 4: Parity of the bosonic fields under the actions of $(-1)^{F_L}$ and Ω_p.

The orientifold projection eliminates all states that are odd under $\mathcal{O}$, so the surviving (10D) fields have to satisfy

$$\begin{aligned} \sigma^* g &= g, & \sigma^* B &= -B \\ \sigma^* C^{(1)} &= -C^{(1)}, & \sigma^* C^{(3)} &= C^{(3)} \\ \sigma^* \phi &= \phi. \end{aligned} \tag{2.22}$$

The harmonic forms on M^6 split into even and odd eigenspaces of σ,

$$H^p = H^p_+ \oplus H^p_-, \tag{2.23}$$

and the $\mathcal{O}$-invariant states reside either in H^p_+ or in H^p_-. Since σ is antiholomorphic it interchanges $H^{p,q}$ with $H^{q,p}$; then $H^{1,1}$ splits into two eigenspaces $H^{1,1}_\pm$ of dimensions $h^{1,1}_\pm$, whereas for $p \neq q$ the sum $H^{p,q} \oplus H^{q,p}$ splits into two eigenspaces of dimension $h^{p,q}$ each. We split the indices accordingly

$$\omega_A \to (\omega_a, \omega_r) \in H^{1,1}_- \oplus H^{1,1}_+ \tag{2.24}$$

Now the expansion of the fields contains only those harmonics with the correct σ-parity. For instance, the B-field gets expanded as

$$B = b^a(x)\omega_a, \tag{2.25}$$

where $a = 1, \dots, h^{1,1}_-$. Similarly, the Kähler deformations v^A of the metric reduce to v^a, whereas the 3-form $C^{(3)}$ retains the $H^{1,1}_+$ expansion.

$\mathcal{N}=2$		$\mathcal{N}=1$
1 gravity	$\longrightarrow$	1 gravity
$h^{1,1}$ vector	$\longrightarrow$	$\begin{cases} h^{1,1}_- \text{ vector} \\ h^{1,1}_+ \text{ chiral} \end{cases}$
$(h^{2,1}+1)$ hyper	$\longrightarrow$	$(h^{2,1}+1)$ chiral

Table 5: Splitting of $\mathcal{N}=2$ into $\mathcal{N}=1$ multiplets

The resulting effective Lagrangian assumes the standard $\mathcal{N}=1$ form

$$S = -\int \tfrac{1}{2} * \mathrm{Scal} + K_{I\overline{J}} \mathrm{d}M^I \wedge * \mathrm{d}\overline{M}^J + \frac{1}{2} \mathrm{Re}\, f_{rs} F^r \wedge * F^s + \frac{1}{2} \mathrm{Im}\, f_{rs} F^r \wedge F^s + *V, \tag{2.26}$$

where the M^I encompass all complex scalars in the theories and $K_{I\overline{J}} = \partial_I \overline{\partial}_J K$ is a Kähler metric. The potential V is given in terms of a superpotential W and the D-terms D_r as

$$V = e^K (K^{I\overline{J}} D_I W D_{\overline{J}} \overline{W} - 3|W|^2) + \frac{1}{2} (\mathrm{Re}\, f)^{-1\, rs} D_r D_s, \tag{2.27}$$

where $D_I W = \partial_I W + W \partial_I K$, and

$$f_{rs} = -\mathrm{i} \overline{\mathcal{N}}_{rs} = \mathrm{i}\, \mathcal{K}_{rsa} t^a. \tag{2.28}$$

The full scalar moduli space spanned by the M^I is the product of two Kähler manifolds, the vector and chiral moduli spaces

$$\tilde{M}^v \times \tilde{M}^c. \tag{2.29}$$

Here $\tilde{M}^v \subset M^v$ is a subspace of the $\mathcal{N} = 2$ vector moduli space, which is again special Kähler and of complex dimension $h^{1,1}_-$. The total Kähler potential assumes the form

$$K = -\log \Big[\frac{4}{3} \int_{M^6} \omega^3\Big] - 2 \log \Big[\int_{M^6} \mathrm{Re}(C\Omega) \wedge * \mathrm{Re}(C\Omega)\Big], \tag{2.30}$$

where C is a compensator field, related to the dilaton modulus, and both expressions should be written in terms of appropriate coordinates. For the vector multiplet this simply means that

$$K^V = -\log \Big[\frac{4}{3} \int_{M^6} \omega^3\Big] = -\log \Big(\frac{\mathrm{i}}{6} \mathcal{K}_{abc} (t - \bar{t})^a (t - \bar{t})^b (t - \bar{t})^c\Big). \tag{2.31}$$

Both perturbative and non-perturbative effects like world-sheet instantons and D2-branes give additional contributions to K.

IIB. There are more possibilities in IIB. One can introduce O3, O5, O7 and O9 planes. One needs again an isometric involution, but in contrast to IIA it has to be holomorphic, and satisfy $\sigma^* \Omega = \pm \Omega$. The plus sign leads to O5/O9 planes, minus to O3/O7. Since σ is holomorphic, its eigenspaces are subspaces of $H^{p,q}$, so $H^{p,q} = H^{p,q}_+ \oplus H^{p,q}_-$.

	O3/O7		O5/O9	
gravity multiplet	1	$g_{\mu\nu}$	1	$g_{\mu\nu}$
vector multiplets	$h^{2,1}_+$	V^α	$h^{2,1}_-$	V^κ
chiral multiplets	$h^{2,1}_-$	z^κ	$h^{2,1}_+$	z^α
	$h^{1,1}_+$	(v^r, ρ_r)	$h^{1,1}_+$	(v^r, c^r)
	$h^{1,1}_-$	(b^a, c^a)	$h^{1,1}_-$	(b^a, ρ_a)
	1	(ϕ, C_0)	1	(ϕ, C_2)

Table 6: $\mathcal{N} = 1$ multiplets in IIB orientifolds.

The effective action is again as in IIA. The complex structure moduli z^k remain good coordinates, but the situation in the remaining chiral multiplets is more complicated. For O3/O7 orientifolds we introduce

$$\begin{aligned} \tau &= C_0 + ie^{-\phi} \ , \qquad G^a = c^a - \tau b^a \ , \\ T_r &= \frac{1}{2}\mathcal{K}_{rst}v^s v^t + i\rho_r - \frac{i}{2(\tau-\bar\tau)}\mathcal{K}_{rab}G^a(G-\bar G)^b \ . \end{aligned} \tag{2.32}$$

The Kähler potential on chiral moduli space is then

$$K_{\mathrm{O3/O7}} = -\log\left[-i\int\Omega(z)\wedge\overline{\Omega}(\bar z)\right] - \log\left[-i(\tau-\bar\tau)\right] - 2\log\tfrac{1}{6}\mathcal{K}(\tau,G,T) \tag{2.33}$$

where $\mathcal{K} = \mathcal{K}_{rst}v^r v^s v^t$, with the vs reexpressed in terms of the Kähler coordinates τ, G^a, T_r. This can only be done explicitly when $h^{1,1}_+ = 1$. If additionally $h^{1,1}_- = 0$, one arrives at a simple expression for $\mathcal{K}$:

$$\log\mathcal{K} = \frac{3}{2}\log(T+\overline{T}). \tag{2.34}$$

Analogously for O5/O9 [1].

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