

The 5D composite Higgs Andreas Weiler

- very basic 5D (gauge field + spinor)
- gauge-Higgs unification (the $A_5^{(0)}$ as the Higgs)
- calculation of the scalar ($A_5^{(0)}$) potential
- Remarks on realistic models

Literature: Sundrum - Tasin '04
Contino - Tasin '09
Saroue - 0909.5619
Oda / Weiler '04
Agashe / Contino / Ronan '04

5① gauge-field:

$$S = \text{Tr} \int d^4x dx_5 \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right)$$

$$= \dots \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} F_{\mu\nu} F^{\mu\nu} \right)$$

gauge transf. $A'_\mu = \sum_i \int D_\mu(x) \Delta$

gauge away $A_5 : A'_5 = 0 \quad (\Delta = \text{path-ordered } \exp(i g \int_0^{x^5} dx'_5 A_5(x, x'_5)))$

axial-gauge $\dots \left(-\frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2} (\partial_\mu A_\nu)^2 \right)$

Compactify $x^5 : x^5 = R \cdot \varphi, -\pi \leq \varphi \leq \pi$

$$A_\mu \sim A_\mu^{(0)} + \sum_{n=1}^{\infty} (e^{inx} A_n^{(n)}(x) + h.c.)$$

$(A_n^{(n)})^* = A_n^{(-n)}$

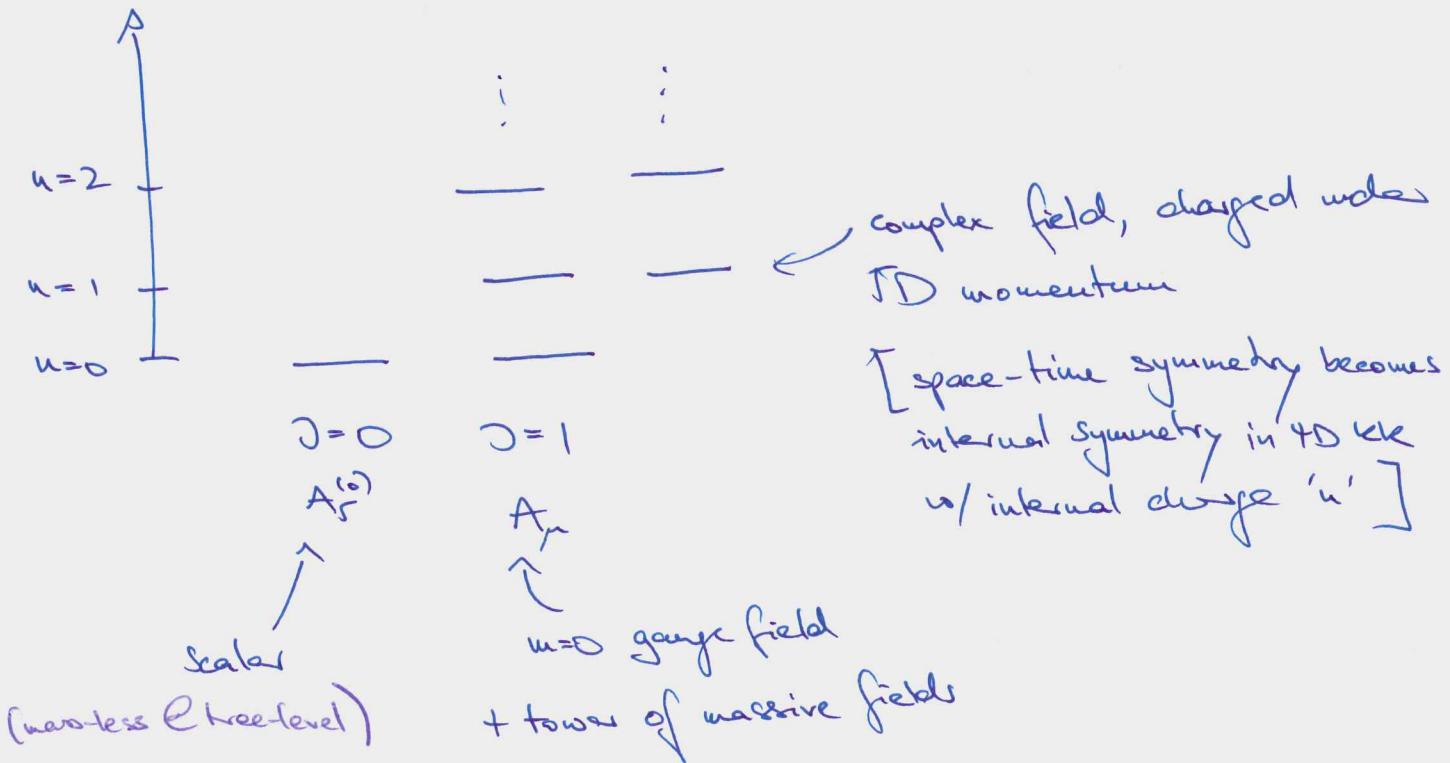
Can not gauge away zero-mode

$$A_5^{(0)} \rightarrow A_5^{(0)} - \partial_5 \alpha^{(0)}(x, x_5)$$

by definition
 $\alpha^{(0)}(x, x^5) = \alpha^{(0)}(x)$

(Best we can do is take $\tilde{\Delta}_{51} = \tilde{\Delta}_2 \cdot e^{-ig A^{(0)}(x)} \phi$)

$$\begin{aligned}
 S = & 2\pi R \cdot \text{Tr} \left(d^4x \left\{ -\frac{1}{4} F_{\mu\nu}^{(0)} \bar{F}_{\mu\nu}^{(0)} + \frac{1}{2} (\partial_\mu A_5^{(0)})^2 \right. \right. \\
 & + \sum_{n=1}^{\infty} \left[-\frac{1}{2} \left| \partial_\mu A_n^{(n)} - \partial_\nu A_n^{(n)} \right|^2 + \left(\frac{k}{R}\right)^2 |A_n^{(n)}|^2 \right] \\
 & \left. \left. + \mathcal{O}(A^3) \right\} \right)
 \end{aligned}$$



5D fermions

Extend Clifford algebra to 5D $\{P_M, P_N\} = 2\gamma_{MN}$

Define $P_\mu = \sigma_\mu$ $P_5 = -i\sigma_5$

Minimal rep. 4D Dirac spinor (σ_5 is part of rep!)

$W_{\mu\nu} \sim [\sigma_\mu, \sigma_\nu]$ commutes w/ σ_5 } chirality not a good
 but $W_{MN}^5 \sim [\sigma_\mu, \sigma_5]$ does not } Quantum number in 5D

$$\psi = \sum_{n=-\infty}^{\infty} \psi^{(n)} e^{inx}$$

$$S_4 = \int d^4x \bar{\psi} (\not{D}^m \not{D}_m - m) \psi$$

$$= \dots \bar{\psi} (\not{i\not{x}} - m) \psi - \bar{\psi} \not{\partial}_5 \not{\partial}_5 \psi + i g \bar{\psi} \gamma_5 \not{\partial}_5 \gamma_5 \psi$$

$$= 2\pi R \int dx \sum_n \bar{\psi}^{(n)} \left(\not{i\not{x}} - m - i \frac{n}{R} \not{\gamma}_5 \right) \psi^{(n)} \dots$$

$$m_{\text{phys}}^2 = m^2 + \frac{n^2}{R^2}$$



Problematic, b/c SM is chiral! Let's postpone this problem for now. Can be taken care of by compactifying on S^1/\mathbb{Z}_2 or interval.

L5

$$\text{EFT} \quad E \ll 1/R \quad S = 2\pi R \int d^4x \left\{ \bar{\psi}^{(0)} (\not{D} - m) \psi^{(0)} + ig \bar{\psi}^{(0)} \gamma_5 A_5^{(0)} \psi^{(0)} \right\}$$

- Yukawa coupling to 4D scalar $A_5^{(0)}$

same strength as gauge coupling

→ gauge-Yukawa-unification

A_5 is a natural candidate for the Higgs

A_5 is massless (on tree-level) because locality and 5D gauge-invariance forbid a potential

Only gauge-inv. operator $F_{\mu 5} = \partial_\mu A_5 - \partial_5 A_\mu - i[A_\mu, A_5]$

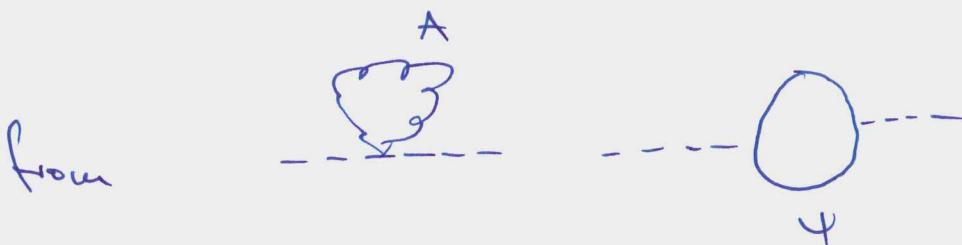
↪ no way to form mass-term

(NB, in 6D can have tree-level mass using A_5, A_6)
see Quiros et.al.

Light scalar loop-potential

$A_5^{(0)}$ is a 4D scalar, what about its mass after quantum corrections?

- pure EFT picture: $S_{\text{scalar}}^2 \approx \frac{g_Y^2}{16\pi^2} A_{\mu\nu}^2$



- 5D viewpoint: $A_5^{(0)}$ is massless b/c it's a 5D gauge field whose mass is protected

What is the answer?

Compute 1-loop effective potential for $A_5^{(0)}$

Can treat $A_5^{(0)}$ as a background field (do not need terms $\partial_\mu A_5$ for now, just the potential)

Set $\alpha \equiv g A_5^{(0)} = \text{const}$ and $t_\mu = 0$

$$S_\psi = \int d^4x \sum_n \bar{\psi}^{(n)}(x) \left[i\cancel{\partial} - m - i\left(\frac{n}{R} - \alpha\right)\gamma_5 \right] \psi^{(n)}(x)$$

chiral rotation

$$\psi \rightarrow e^{i\frac{R\phi_5}{2}}$$

$$m + i\left(\frac{n}{R} - \alpha\right) \rightarrow \sqrt{m^2 + \left(\frac{n}{R} - \alpha\right)^2} = \Gamma(x, R)$$

$$\left[\text{Proof: } \bar{\psi}\psi \rightarrow \bar{\psi} e^{i\beta \gamma_5} \psi = \bar{\psi} (\cos\beta + i\gamma_5 \sin\beta) [A + i\gamma_5 B] \psi \right]$$

$$\Gamma = A + iB\gamma_5 \quad \text{choose: } \tan\beta = \frac{-B}{A} \rightarrow \Gamma \rightarrow \sqrt{A^2 + B^2} \cdot 1$$

$$= \int d^4x \sum_n \bar{\psi}^{(n)}(x) [\not{x} - M(\alpha, R)] \psi^{(n)}(x)$$

$$e^{-iV_{\text{eff}}} = \prod_{\alpha, n} \det(\not{x} - M(\alpha, n, R))$$

$$= \exp \left(\sum_n \int \frac{d^4p}{(2\pi)^4} \text{tr} \ln [\not{x} - M] \right)$$

Only R dependent piece non-trivial ($R \rightarrow \infty$ 5D theory)
 $\quad + u_{\alpha} \delta_{\alpha\beta} = 0$

$$-i \frac{\partial V_{\text{eff}}}{\partial R} = \sum_n \left\{ \int \frac{d^4p}{(2\pi)^4} \text{tr} \left(\frac{\partial M}{\partial R} \cdot \frac{1}{\not{x} - M} \right) \right\}$$

$$\frac{\partial M}{\partial R} = \frac{1}{2} \frac{1}{R} 2 \left(\frac{u}{R} - \alpha \right) \left(-\frac{u}{R^2} \right)$$

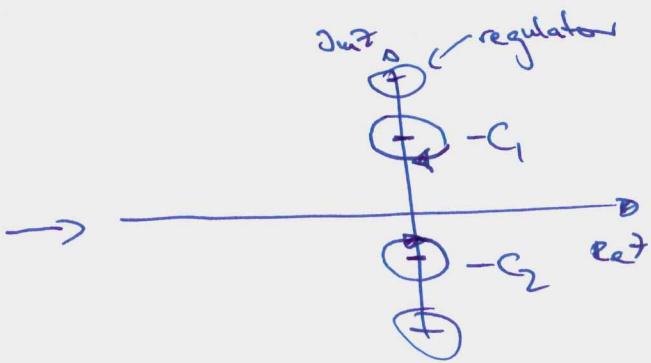
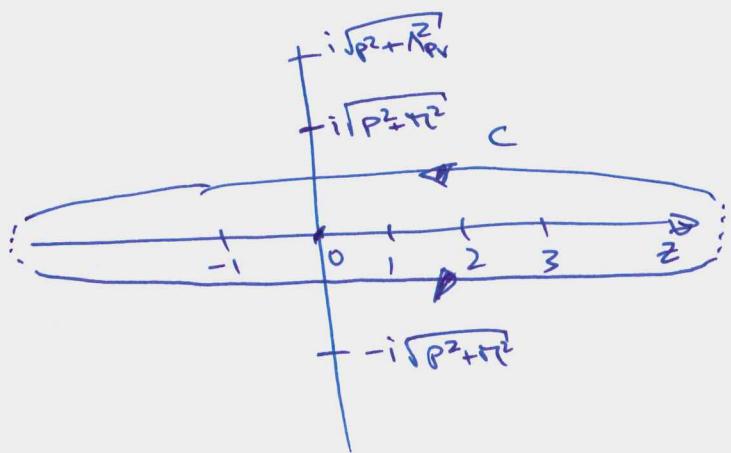
$$= \int \text{tr} \left(\frac{\not{x} + M}{p^2 - M^2} \frac{1}{R} \left(\frac{u}{R} - \alpha \right) \left(-\frac{u}{R^2} \right) \right)$$

$$\frac{\partial V_{\text{eff}}}{\partial R} = i \int \frac{4u(u-\alpha)}{p^2 - M^2} \stackrel{\text{With-rotate}}{\Rightarrow} \int \frac{4u(u-\alpha)}{p_E^2 + M^2} + \text{Regulator}$$

Divergent $\sim \underbrace{\sum_n u^2}_{\Lambda^3} \cdot \underbrace{\int d^4p \frac{1}{p^2}}_{\Lambda^2} \sim \Lambda^5 !$

- two options:
- Zeta function regularization
 - assume Pauli-Villars regulator w/
 α -independent mass

$$\sum_n f(n) \rightarrow \oint_C \frac{dz}{e^{2\pi iz} - 1} f(z) = \sum_{z=-c_1, -c_2}^{\text{res}} f(z) \frac{1}{e^{2\pi iz} - 1}$$



$$p_E^2 + m^2 + (n-\alpha)^2$$

Shift integration $z \rightarrow z+\alpha$, get residue's:

$$-C_1 = -4\pi i \frac{e^{2\pi i \sqrt{m^2 + p^2}} (\alpha + i\sqrt{m^2 + p^2})}{e^{(2\pi i \alpha - 2\pi i \sqrt{m^2 + p^2})R} - 1}$$

$$-C_2 = -4\pi i \frac{\alpha - i\sqrt{m^2 + p^2}}{e^{(2\pi i \alpha - 2\pi i \sqrt{m^2 + p^2})R} - 1}$$

Integrate w.r.t. R

$$V_{\text{eff}} = \int_{\frac{1-p}{2\pi}}^{\frac{1+p}{2\pi}} \left[4\pi i \alpha \cdot R - 2\pi R \sqrt{m^2 + p^2} \right] - 2\pi R \sqrt{m^2 + p^2}$$

$$= 2 \ln \left(1 + e^{-2\pi R \sqrt{m^2 + p^2}} - 2 e^{-2\pi R \sqrt{m^2 + p^2}} \cos(2\pi R g A_5^{(0)}) \right)$$

+ reg.

In the limit of infinite 5D ($R \rightarrow \infty$), the potential terms for $A_5^{(0)}$ must vanish (gauge invariance)

$$V_{\text{eff}} \xrightarrow{R \rightarrow \infty} R X^5 \cdot R$$

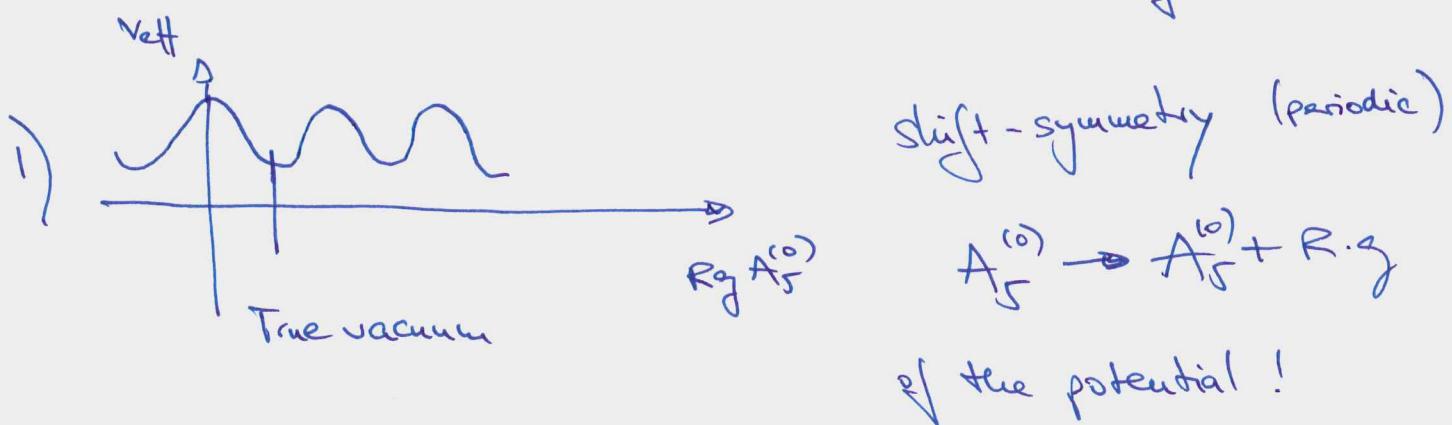
constant, independent of a, R

This means

$$V_{\text{eff}} \xrightarrow{R \rightarrow \infty} \left(\frac{d^4 p}{(2\pi)^4} \left[4\pi i a R - \pi R \sqrt{p^2 + m^2} \right] + \mathcal{O}(e^R) \right) + \text{Reg.}$$

$$\text{cut-off dependent} \quad = \quad X^5 \cdot R \quad \text{conjugate}$$

$$V_{\text{eff}} = X^5 \cdot R - 2 \left(\frac{d^4 p}{(2\pi)^4} \ln \left[1 + e^{-4\pi R \sqrt{p^2 + m^2}} \right] - 2 e^{-2\pi R \sqrt{p^2 + m^2}} \cos(2\pi R g A_5^{(0)}) \right) + \text{Reg.}$$



2) Regulator terms $\sim e^{-2\pi R \lambda_{\text{UV}}}$ \rightarrow can be neglected

3) Potential is UV-insensitive : non-local effect, requires winding around extra-dimension

For small $A_5^{(0)}$ we can write

$$V_{\text{eff}} \approx N R + \int \frac{d^4 p}{(2\pi)^4} \left[\dots + (2\pi R g A_5^{(0)})^2 \left[\frac{e^{-2\pi R \sqrt{p^2 + 1}}}{\dots} \right] + (2\pi R g A_5^{(0)})^4 \left[\text{consequent...} \right] \right]$$

non-vanishing vev: $v = \frac{\mu^2}{\lambda} \approx \frac{1}{R \cdot g}$

Can generalize to $U(1) \rightarrow SU(2)$, add ~~new mass term~~

$$|A_5^{(0)}| \rightarrow \sqrt{\text{tr } A_5^2} \quad (\text{tr is over } SU(2))$$

Self-interactions of gauge field are symmetry

restoring. If fermion contribution dominates, will get

$$m_{A_5^{(0)}} = 0$$

$$m_{W^\pm} \sim 1/Rg$$

$$m_{A_1^{(0)}} \sim \sqrt{m^2 + \frac{1}{Rg^2}} \xrightarrow{m \gg 0} \frac{1}{Rg}$$

$$m_{KK} \sim \frac{1}{R}$$

$$m_{\text{Higgs}}^2 \sim \frac{g^2}{32\pi^3 R^2}$$

Can tune gauge vs. fermion contribution

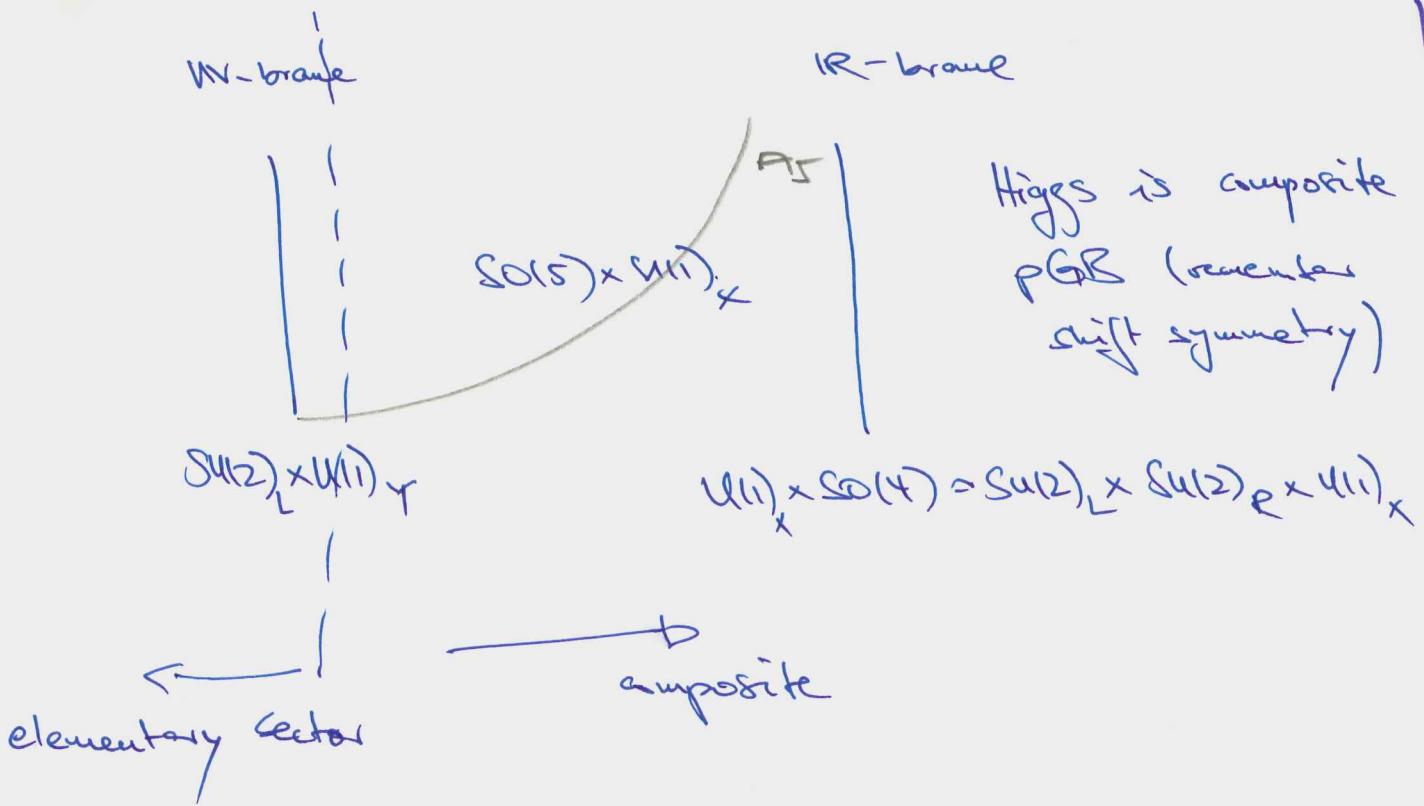
$$A_5^{(i)} \sim \frac{e}{gR} \quad m_{W^{\pm}} \sim \frac{e}{R} \sim e_{\text{max}}$$

Problems Usually : 1) Higgs too light \rightarrow quartic radiatively generated!
2) top Yukawa too small

Realistic models : $ds^2 = g_{MN} dx^M dx^N \rightarrow \left(\frac{R}{z}\right)^2 \left(g_{\mu\nu} dx^\mu dx^\nu - dz^2\right)$

warped Extra-dimensions (for potential calculation
see e.g. Oda/Welke 04)

- valid until Planck scale
- logarithmic gauge coupling running
(unification w/o susy)
- AdS/CFT₄ interpretation as a large-N
strongly coupled theory w/ conformal
sector (à la walking TC)
- elegant solution to the TC-flavor
problem
- $SO(5) \times U(1)_{B-L}$



Fermions are embedded in $SO(5) \times U(n)$, e.g. $4, \bar{1}_3$ (spinorial)

$$\left(\begin{array}{c} q_L^{+ +} \\ Q_L^{--} \end{array} \right)$$

$$\left(\begin{array}{c} q_L^u (+-) \\ u_L^c (-+) \\ d_L^c (++) \end{array} \right)$$

$$\left(\begin{array}{c} q_L^d (+-) \\ \cancel{u_L^c (++)} \\ d_L^c (-+) \end{array} \right)$$

$$(+ -)$$

$\uparrow \uparrow$
uv IR boundary conditions

\circlearrowleft = exotics

$SO(4)$ invariant IR brane mixing ... potential calculation

Effective theory \rightarrow see Kohri's talk

Alternatively use $4 \times \underline{5}$ (fundamentals) or $5, \bar{5}, 10$