Anomaly mediation

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References

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Contents

1 Motivation

In Planck-scale mediated supersymmetry breaking secarios one imagines that dynamical supersymmetry breaking in a hidden sector is communicated to the MSSM purely through M_{Pl} supressed operators.

Let us assume that X is a chiral superfield whose F-term breaks SUSY in the hidden sector and consider the supersymmetric effective Lagrangian including Planck-scale suppressed operators that communicate between the visible (MSSM) sector and the hidden sector.

$$W = W_{MSSM} - \frac{1}{M_{Pl}}\left(\frac{1}{6}y_X^{ijk}X\Phi_i\Phi_j\Phi_k + \frac{1}{2}\mu^{ij}X\Phi_i\Phi_j\right) + ... \quad (1)$$

$$K = \Phi^{*i}\Phi_i + \frac{1}{M_{Pl}}\left(n_i^j + n^{*j}_i\right)X\Phi^{*i}\Phi_j - \frac{1}{M_{Pl}^2}k_j^i XX^*\Phi^{*i}\Phi_i + ... \quad (2)$$

$$f_{ab} = \frac{\delta_{ab}}{g_a^2}\left(1 - \frac{2}{M_{Pl}f_a X + ...}\right) \quad (3)$$

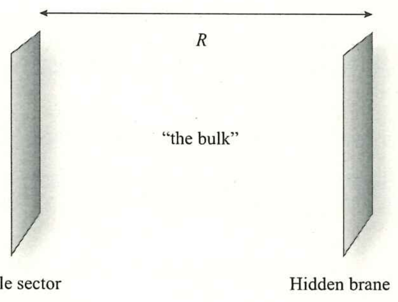
Here the Φ_i's denote the MSSM matter fields. The source of these higher dimensional operators are Planck scale (string states) that couple to the visible as well as hidden sector. After integrating out those states one obtains Planck-scale suppressed couplings, which are not expected to preserve flavor symmetry!

Thus even though gravity couples flavor blind to the visible sector, Plnack-scale mediated supersymmetry breaking models lead generically dangerous flavor violating terms.

Is there a way to further suppress those Planck-scale suppressed operators?

Anomaly mediation setup

Assuming an extra-dimensional theory where the visible (MSSM) sector is spatially separated by a distance r from the hidden sector. Such a setup can be realized within string theory by having two D3-branes separated spatially in the remaining extra dimensions.



Then the potentially dangerous operators are exponentially suppressed by $e^{-M_{Pl}R}$, and if $M_{Pl}R \gg 1$ one can ignore those couplings. Thus there are no allowed couplings on tree level between the visible and hidden sector, however there will be couplings generated on the radiative level between these tow sectors.

2 Weyl compensator formalism

Before we start with the supergravity Lagrangian let us briefly take a look at ordinary Einstein gravity whose action takes the form

$$S = \int d^4x\sqrt{-g}R \tag{4}$$

where $g = det(g_{\mu\nu})$ and R is the Ricci scalar.

One easily sees that the action is not invariant under the scale transformations

$$g_{\mu\nu}(x) \to \Omega^2(x)g_{\mu\nu}(x) \qquad R = g_{\mu\nu}R_{\mu\nu} \to \Omega^{-2}(x)R \qquad \sqrt{-g} \to \Omega^4(x)\sqrt{-g} \tag{5}$$

However one can introduce an additional real scalar η that transforms under scale transformations

$$\eta(x) \to \Omega^{-2}(x)\eta(x) \tag{6}$$

and with the replacement $\tilde{g}_{\mu\nu} = \eta g_{\mu\nu}$ one obtains a scale invariant

$$\int d^4x\sqrt{\tilde{g}}\tilde{R} = \int d^4x\sqrt{-g}\left[\eta R - 6(\partial\eta)^2\right] \tag{7}$$

Note the kinetic term for the scalar has a wrong sign, however this is not a problem since eta is not a physical degree of freedom, it can be gauged away. A non-zero vev of *eta* will break the scale invariance and Einstein gravity can be recovered with the choice

$$\eta(x) = M_{Pl} \tag{8}$$

Supergravity

In the standard superfield formalism on has the following action

$$S = \frac{-3}{\kappa}\int d^4x d^2\theta d^2\bar{\theta}\, \mathbf{E}\, e^{-\frac{1}{3}\kappa^2 K} + \int d^4x d^2\theta d^2\bar{\theta}\, \mathcal{E}\left[W + \frac{1}{4}f_{(a)(b)}\mathcal{W}^{(a)\alpha}\mathcal{W}^{(b)}{}_{\alpha}\right] + h.c. + \ldots \tag{9}$$

where the dots denote higher derivative terms. Here $\mathbf{E} = det(E_\mu^A)$ is the determinant of the supervielbein, $\mathcal{E}$ is the chiral density and $\mathcal{W}^{(a)\alpha}$ is the gauge superfield strength.

- W is the superpotential, a holomorphic function of the superfields, that governs the Yukawa couplings
- K denotes the Kähler potential that is a real function of the superfields and determines in particular the kinetic terms of the chiral superfields.

- f is the gauge kinetic function that is holomorphic and determines the kinetic terms of the gauge fields.

The idea is to extend the local supersymmetric action to a local superconformal action. Under a scale transformation one has the following transformations:

$$
\begin{aligned}
&E_M^{\alpha} \to e^{2\overline{\tau}-\tau}\left(E_M^{\alpha} - \frac{i}{2}\overline{\sigma}^{\dot{\beta}_a\alpha}\left(\mathcal{D}_{\dot{\beta}}\overline{\tau}\right)E_M^{a}\right) \qquad E_M^{\dot{\alpha}} \to e^{2\tau-\overline{\tau}}\left(E_M^{\dot{\alpha}} - \frac{i}{2}\overline{\sigma}^{\dot{\alpha}_a\alpha}\left(\mathcal{D}_{\beta}\tau\right)E_M^{a}\right)\\
&E_M^{a} \to e^{\tau+\overline{\tau}}E_M^{a} \qquad \mathbf{E}^{2(\tau+\overline{\tau})}\mathbf{E} \qquad \mathcal{E} \to e^{6\tau}\mathcal{E} + ... \qquad \overline{\mathcal{E}} \to e^{6\overline{\tau}}\overline{\mathcal{E}} + ...\\
&\mathcal{D}_{\alpha} \to e^{\tau-2\overline{\tau}}\left(\mathcal{D}_{\alpha} - 2\left(\mathcal{D}^{\beta}\tau\right)L_{\alpha\beta}\right) \qquad \mathcal{D}_{\dot{\alpha}} \to e^{\overline{\tau}-2\tau}\left(\mathcal{D}_{\dot{\alpha}} - 2\left(\mathcal{D}^{\dot{\beta}}\overline{\tau}\right)L_{\dot{\alpha}\dot{\beta}}\right)\\
&\Phi^i \to \Phi^i \qquad Q^I \to Q^I \qquad \overline{\Phi}^i \to \overline{\Phi}^i \qquad \overline{Q}^I \to \overline{Q}^I\\
&V^{(a)} \to V^{(a)} \qquad \mathcal{W}_{\alpha}^{a} \to e^{-3\tau}\mathcal{W}_{\alpha}^{a} \qquad \overline{\mathcal{W}}_{\dot{\alpha}}^{(a)} \to e^{-3\overline{\tau}}\mathcal{W}_{\dot{\alpha}}^{(a)}
\end{aligned}
\tag{10}
$$

As it stands the action (9) is not invariant, however this can be compensated by introducing a field ϕ known as the Weyl compensator that transforms as

$$
\phi \to e^{-2\tau}\phi \tag{11}
$$

and ensures superconformal invariance of the action

$$
S = \frac{-3}{\kappa}\int d^4x d^2\theta d^2\overline{\theta}\ \mathbf{E}\, e^{-\frac{1}{3}\kappa^2 K - 6\kappa^{-2} Re\log(\phi)} \tag{12}
$$

$$
+\int d^4x d^2\theta d^2\overline{\theta}\ \mathcal{E}\left[\phi^3 W + \frac{1}{4}f_{(a)(b)}\mathcal{W}^{(a)\alpha}\mathcal{W}^{(b)}{}_{\alpha}\right] + h.c. + ... \tag{13}
$$

In the low energy limit one gets

$$
S = \int d^4x d^2\theta d^2\overline{\theta}\ \phi\overline{\phi}\,\overline{Q}e^{2V}Q \tag{14}
$$

$$
+\int d^4x d^2\theta\left[\phi^3 W + \frac{1}{4g_a^2}\mathcal{W}^{(a)\alpha}\mathcal{W}^{(a)}{}_{\alpha}\right] + h.c. + ... \tag{15}
$$

A vev for ϕ will break superconformal invariance (one uses $\langle\phi\rangle = 1$ as gauge fixing), but moreover in the presence of supersymmetry breaking in a hidden sector $\langle F\rangle \neq 0$ the auxiliary field component of ϕ acquires a non-zero vev

$$
\langle F_{\phi}\rangle = \frac{\langle F\rangle}{M_{Pl}} \sim m_{3/2} \tag{16}
$$

Thus the auxiliary component of ϕ, F_{ϕ}, parametrizes the SUSY breaking in (15).

In case the superpotential takes the form $W \sim Q^3$ with a field redefinition $Q' = \phi Q$ the Weyl compensator seems to disappear from the action. Thus there is no direct coupling between the hidden and visible sector and the action seems to be superconformal invariant. However the scale invariance is broken on loop level by the running of the couplings. Thus it is expected that the breaking of supersymmetry in the visible sector is related to the conformal anomaly. Thus the name *Anomaly mediated supersymmetry breaking.*

3 Couplings as superfields

It turns out to be a very useful idea to promote coupling constants to superfields χ where the vev of the lowest component is identified with the coupling constant. Note that a nonzero constant vev does not break supersymmetry as long as

$$
Q_{\alpha}\langle\chi\rangle = \overline{Q}_{\dot{\alpha}}\langle\chi\rangle = 0 \tag{17}
$$

with $Q_{\alpha} = \frac{\partial}{\partial\theta^{\alpha}} - i\sigma^{\mu}_{\alpha\dot{\alpha}}\overline{\theta}^{\dot{\alpha}}\partial_{\mu}$.

Let us assume we have the following "fairly generic"action

$$
\int d^4x d^4\theta\ Z\overline{Q}e^{2V}Q + \int d^4x d^2\theta\left[y_0\,Q^3 - \frac{i}{16\pi}\tau\,\mathcal{W}^{\alpha}\mathcal{W}_{\alpha}\right] \tag{18}
$$

where Z denotes the wave function renormalization, y_0 the Yukawa coupling and $\tau = \frac{\theta_{YM}}{2\pi} + \frac{4\pi i}{g^2}$. Promoting those constants to superfields (here Z is a real superfield and y_0, τ are chiral superfields)

$$
\int d^4x d^4\theta\ Z(X,\overline{X},\mu)\overline{Q}e^{2V}Q + \int d^4x d^2\theta\left[y_0(X,\mu)\,Q^3 - \frac{i}{16\pi}\tau(X,\mu)\,\mathcal{W}^{\alpha}\mathcal{W}_{\alpha}\right] \tag{19}
$$

one can elegantly proof various non-renormalization theorems.

For instance the holomorphic Yukawa coupling are not renormalized. However the Yukawa coupling of the canonical normalized fields run due to wavefunction renormalization

$$
y^{phys} = \frac{y_0}{Z^{\frac{3}{2}}}\ . \tag{20}
$$

One can also argue that the holomorphic gauge coupling receives on perturbative level only one-loop corrections.

However the introduction of such superfield couplings also allows for an elegant treatment of soft supersymmetry breaking terms, by allowing the superfield couplings to have non-zero higher components, non-zero F-terms.

$$Z \to 1 + (\theta^2 B + \overline{\theta}^2 \overline{B}) + \theta^2 \overline{\theta}^2 C \tag{21}$$

$$y_0 \to y_0 + \theta^2 F_{y_0} \tag{22}$$

$$\tau \to \tau + \theta^2 F_\tau \tag{23}$$

In terms of those auxiliary field vacuum expectation values one obtains the following masses:

- **Gaugino masses**

  The gaugino mass term arises from the Lagrangian

$$\mathcal{L} = -\frac{i}{16\pi}\int d^2\theta\, \tau(X,\mu) W^\alpha W_\alpha \tag{24}$$

  With

$$W^\alpha = -i\lambda^\alpha + \theta D + ... \tag{25}$$

  the gaugino masses compute to

$$M_\lambda = -\frac{i}{16\pi} F_\tau \lambda^\alpha \lambda_\alpha \tag{26}$$

  However, note that the in order to have canonically normalized gauge fields we have to rescale

$$(A_\mu, \lambda_\alpha, D) \to g\,(A_\mu, \lambda_\alpha, D) \longrightarrow W^\alpha \to \frac{4\pi i}{\tau} W^\alpha \tag{27}$$

  which gives

$$M_\lambda = -\frac{1}{4\tau} F_\tau \lambda^\alpha \lambda_\alpha = -\frac{1}{4\langle\tau\rangle} \tau\big|_{\theta^2} \lambda^\alpha \lambda_\alpha \tag{28}$$

- **Scalar masses**

  The scalar masses arise from the D-term

$$\mathcal{L} = \int d^4\theta Z Q \overline{Q} \tag{29}$$

  that computes with

$$Q = q + F_Q \qquad \overline{Q} = \overline{q} + \overline{F}_Q \qquad Z = 1 + (\theta^2 B + \overline{\theta}^2 \overline{B}) + \theta^2 \overline{\theta}^2 C \tag{30}$$

  after integration over the Grassmann variables to

$$C q\overline{q} + F_Q \overline{F}_Q + B \overline{F}_Q q + \overline{B} F_Q \overline{q} + ... \tag{31}$$

  Integrating out the auxiliary fields one obtains $\mathcal{L} \sim (C - |B|^2)\, q\overline{q} + ...$ Thus the mass term for the scalars is

$$m_q^2 = \left(|B|^2 - C\right) q\overline{q} = -\ln(Z)\big|_{\theta^2\overline{\theta}^2} q\overline{q} \tag{32}$$

- **A-terms**

  The A terms arise from the Lagrangian

$$\int d^4x d^4\theta\; Z(X, \overline{X}, \mu) \overline{Q} e^{2V} Q + \int d^4x d^2\theta\, y_0(X,\mu)\, Q^3 \tag{33}$$

  which gives after integration over the Grassmann variables

$$F_{y_0} q^3 + F_Q \overline{F}_Q + B \overline{F}_Q q + 3 y_0 F_Q q^2 + ... \tag{34}$$

  which after integrating out the auxiliary field $\overline{F}_Q$ gives (not that in contrast to the mass term for the scalars here the contributions from the superpotential enter in $\overline{F}_Q$)

$$A_{qqq} \sim (3B y_0 - 2F_{y_0})\, q^3 = -2\hat{y}_0 q^3 = -2\frac{y_0}{Z^{\frac{3}{2}}}\bigg|_{\theta^2} q^3 \tag{35}$$

4 Soft SUSY-breaking masses

Let us apply the procedure laid out above to the case of Anomaly mediation, where as we discussed above the whole supersymmetry breaking arises from the Weyl compensator

$$\langle\phi\rangle = 1 + \theta^2 F_\phi \tag{36}$$

From dimensional analysis it is clear that Z, as well as τ will depend on μ/Λ_{UV}, however as can be shown by Pauli-Villars regularization the cutoff scale is always accompanied by Φ. An illustrative argument is the following: any mass term induced by regularization will scale with Λ_{UV}, but also has to scale with ϕ, since otherwise we break scale invariance of the Lagrangian.

Thus we have

$$\mathcal{L} = \int d^4\theta Z\left(\frac{\mu}{\Lambda_{UV}\phi}, \frac{\mu}{\Lambda_{UV}\overline{\phi}}\right)\overline{Q}e^{2V}Q + \int d^2\theta y_0 Q^3 + \int d^2\theta \tau\left(\frac{\mu}{\Lambda_{UV}\phi}\right) + h.c. \tag{37}$$

and now we will use the formalism discussed above.

- **Gaugino masses**

  Applying (28) and expanding τ around $\phi = 1 + F_\phi$ one gets

$$M_\lambda = -\frac{1}{4\langle\tau\rangle}\frac{\partial\tau\left(\frac{\mu}{\Lambda_{UV}\phi}\right)}{\partial\phi}|_{\phi=1}F_\phi \tag{38}$$

  which after using

$$\frac{\partial\tau\left(\frac{\mu}{\Lambda_{UV}\phi}\right)}{\partial\phi} = -\frac{\mu}{\phi}\frac{\partial\tau\left(\frac{\mu}{\Lambda_{UV}\phi}\right)}{\partial\mu} \tag{39}$$

  gives

$$M_\lambda \sim \frac{\partial\ln\tau}{\partial\ln\mu}F_\phi = -\frac{\beta_g}{g} = \frac{g^2 b}{16\pi^2}F_\phi \tag{40}$$

  where $b = 3N - N_F$ for a non-abelian supersymmetric $SU(N)$ gauge symmetry.

  Summarizing the gaugino masses are induced at one-loop and are suppressed compared to the gravitino mass. Moreover the ratio of the gaugino masses are given by their β function.

- **Scalar masses**

  From (32) one obtains

$$m_q^2 = -\ln(Z)\left(\frac{\mu}{\Lambda_{UV}\phi}\right)|_{\theta^2\overline{\theta}^2} = -\frac{1}{2}\frac{\partial^2\ln Z}{\partial|\phi|^2}|_{\phi=1}|F_\phi|^2 \tag{41}$$

$T(x) = T(x_0) + \frac{\partial T}{\partial x}\Big|_{x_0}(x - x_0)$

$X = x_0 + \theta^2 \cdot F_X$

$\theta^2 \cdot F_X$

Analogously as above we can trade the derivative with respect to $|\phi|$ with he derivative with respect to μ. Then we get

$$m_q^2 = -\frac{1}{2}\left(\frac{\partial}{\partial g}\frac{\partial\ln Z}{\partial\ln\mu}\frac{\partial g}{\partial\ln\mu} + \frac{\partial}{\partial\hat{y}_0}\frac{\partial\ln Z}{\partial\ln\mu}\frac{\partial\hat{y}_0}{\partial\ln\mu}\right)|F_\phi|^2 = -\frac{1}{2}\left(\frac{\partial\gamma}{\partial g}\beta_g + \frac{\partial\gamma}{\partial\hat{y}_0}\beta_{\hat{y}_0}\right)|F_\phi|^2 \tag{42}$$

Here we used the definition of the anomalous dimension and the β functions

$$\gamma(g,\hat{y}_0) = \frac{\partial\ln Z}{\partial\ln\mu} \qquad \beta_g(g,\hat{y}_0) = \frac{\partial g}{\partial\ln\mu} \qquad \beta_{\hat{y}_0}(g,\hat{y}_0) = \frac{\partial\hat{y}_0}{\partial\ln\mu} \tag{43}$$

Scalar masses only appear at 2 loop (similar to gauge mediation). In case we consider scalar masses only appearing from purely gauge field loops, asymptotically free theories give rise to positive masses while infrared free theories give rise to tachyonic states. This is a serious problem for the slepton masses, additional contributions to their masses may be required.

- **A-terms**

  From equation (36) we get

$$A_{qqq} \sim \frac{y_0}{Z^{\frac{3}{2}}}\Big|_{\theta^2} \sim 3\frac{y_0}{Z^{\frac{5}{2}}}\frac{\partial Z}{\partial\phi}|_{\phi=1}F_\phi \sim 3\hat{y}_0\frac{\partial\ln Z}{\partial\ln\mu}F_\phi \sim \gamma\hat{y}_0F_\phi \tag{44}$$

  One can easily generalize that result to a yukawa term $y_{ijk}Q^iQ^jQ^k$ which gives

$$A_{ijk} \sim (\gamma_i + \gamma_j + \gamma_k)\,\hat{y}_{ijk}F_\phi \tag{45}$$

Insensitivity to UV physics

Consider some new chiral superfields P and $\widetilde{P}$ transforming as vector-like representations under the SM gauge symmetry, that have a large supersymmetric mass term

$$\Delta\mathcal{L} = \int d^2\theta M\phi P\widetilde{P} + h.c. \tag{46}$$

Note that the mass term contains the Weyl compensator ϕ. Due to the presence of these additional fields the gauge β functions will have different values above and below the scale M.

$$\tau(\mu) = \tau_0 + \frac{b'}{16\pi^2} \ln\left(\frac{M\phi}{\Lambda\phi}\right) + \frac{b}{16\pi^2} \ln\left(\frac{\mu}{M\phi}\right) \tag{47}$$

where b and b' denote the β function coefficients below and above the scale M, respectively. Note that the second term in (47) is independent of ϕ and thus does not give any contribution to the gaugino mass. Similar arguments apply for the wavefunction normalization Z. Thus the masses are independent of the UV physics.

Another way to think about this is that the gauge mediated mass term arising from the fields P and $\widetilde{P}$ at the scale M is cancelled by the difference in the β function above the scale M. However that opens the door for mass shifts in the gaugino and scalar masses arising from soft supersymmetry breaking mass thresholds. That might help to make sleptons non-tachyonic

5 Mass spectrum and Phenomenology

Now let us apply the above formula to the MSSM spectrum. We make the following simplifying assumptions $\beta_{y_i} = 0\ \forall i$, thus all masses are basically given by the β function of the gauge couplings. At the scale $\mu = 1\ TeV$ we have

$$\alpha_Y = 0.01 \qquad \alpha_2 = 0.032 \qquad \alpha_3 = 0.1 \tag{48}$$

and then get for the gaugino masses

$$M^2_{gluino} = 6.1 \times 10^{-4}|F_\phi|^2 \qquad M^2_{wino/zino} = 6.4 \times 10^{-6}|F_\phi|^2 \qquad M^2_{bino} = 7.0 \times 10^{-5}|F_\phi|^2 \tag{49}$$

and for the scalar masses

$$M^2_{squarks} = 5.5 \times 10^{-5}|F_\phi|^2 \qquad M^2_{sleptons} = -1.3 \times 10^{-5}|F_\phi|^2 \qquad M^2_{Higgs} = -1.3 \times 10^{-5}|F_\phi|^2 \tag{50}$$

One sees that squarks and gluinos are the heaviest particles. Sleptons are tachyonic, which is a severe problem.

Ways out of that

- consider additional bulk fields B that couple to the visible sector (sleptons) as well as the hidden sector and induce additional mass contributions.
- consider additional Higgs fields with large yukawa couplings. On first sight they make the β function for the gauge couplings worse, but the tachyonic behavior of the sleptons might be overcome by the behavior of β_y.
- introduce SUSY violating thresholds as discussed above
- introduce an additional gauge symmetry for sleptons and Higgses that is asymptotically free (along the lines of Technicolor), in that case the leptons and sleptons are composite fields.

The μ-term problem

In Anomaly mediated setups one cannot simply have a μ term in the superpotential, since such a term is accompanied with the Weyl compensator

$$\mathcal{L}_\mu = \int d^2\theta \mu \phi H_u H_d \ , \tag{51}$$

whose F-term would induce a too large $B\mu \sim \mu F_\phi$ term.

Potential solutions:

- Consider rather the NMSSM.
- Inducing the μ term via a D-term. Consider a chiral superfield X that is invariant under a shift symmetry with a coupling to the Higgs sector

$$\mathcal{L} = \int d^4\theta \frac{\overline{\phi}}{\phi} \frac{1}{M} \overline{X} H_u H_d + h.c. \tag{52}$$

that will induce a a μ term

$$\mathcal{L}_\mu = \int d^2\theta \frac{\overline{F_X}}{M} \phi H_u H_d \tag{53}$$

without inducing a $B\mu$ term.