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Moduli stabilization

Abstract

Metric moduli generically arise in theories with extra dimensions. If not stabilized by some mechanism, these massless scalar fields can cause serious phenomenological problems. In this talk, we discuss two mechanisms of moduli stabilisation, namely stabilization via flux potentials and gaugino condensation.

Contents

1 Motivation and Outline **2**
1.1 What are moduli and why do we want to stabilize them? 2
1.2 The idea 3

2 Setting **4**
2.1 Type IIB supergravity in $d = 10$ 4
2.2 Calabi-Yau compactification 6

3 Moduli Stabilization via Fluxes **7**
3.1 A no-go-theorem and how to circumvent it 7
3.2 Scalar potential 10
3.3 Consequences for complex structure moduli 12

4 Stabilization of Volume Moduli **13**
4.1 D-Branes 14
4.2 Stabilization via gaugino condensate 15

5 Summary **16**

A p-forms **17**

1 Motivation and Outline

1.1 What are moduli and why do we want to stabilize them?

In previous talks, we have encountered massless scalar fields, so called modulus fields, which frequently arise in theories with extra dimensions. The vevs of these fields, called moduli, are thus not constrained to any value by the scalar potential. A simple example is the scalar component of the 5D metric g_{55}, which sets the sets the scale of the extra dimensions models with one extra dimension compactified on a circle. In more complicated models fluctuations of the metric give rise to further massless scalars (hence referred to as metric or geometric moduli).

In the context of Calabi-Yau compactification, non-trivial fluctuations of the metric which preserve the Calabi-Yau condition $R_{ij} = 0$ lead to two types of metric moduli (s. Christoph's talk) : (1,1)-forms which leave the Kähler structure of the metric invariant (volume/ Kähler moduli) and (2,1)-forms which modify the Kähler structure (complex structure moduli):

$$\delta g = \underbrace{\delta g_{i\bar{j}}}_{\to h^{1,1}\text{ Kähler moduli}} dz^i d\bar{z}^{\bar{j}} + \underbrace{\delta g_{ij}}_{\to h^{2,1}\text{ complex structure moduli}} dz^i dz^j + h.c. \tag{1}$$

If not stabilized by some mechanism, these moduli can cause serious problems:

- Typically the predictions of the theory crucially depend on the vevs of the moduli. However, since these have no potential, their vev can be chosen arbitrary and hence the theory looses predictivity. In other words, the moduli can be understood as further continuous free parameters of the theory. In the context of type IIB string theory, this implies literally hundreds of free parameters instead of one (string scale g_s).

- Moreover, these parameters can be time-dependent, which is in conflict with observations.

- Massless scalars can mediate long range forces (typically roughly the strength of gravity). This would lead to deviations from Newton's law, which have not been observed.

- One might argue, that scalar fields typically obtain large loop corrections to the bare mass (as happens, e.g. to the Higgs in the SM), and hence the moduli are stabilized automatically. However, in supersymmetric theories, the mass scale is set by the SUSY breaking scale and hence only relatively small masses can be expected to be produced. In the early universe, light scalar fields can obtain vevs of $\mathcal{O}(M_P)$. Their subsequent oscillations and decays pose serious cosmological problems (overclosure of the universe and entropy production which endangers successful BBN and baryogenesis). This goes under the name of Polonyi problem.

- Note that there is a crucial difference between these massless scalars and the familiar massless Goldstone bosons. The origin of the Goldstone mode in symmetry breaking implies that the physics in any vacuum connected by the Goldstone mode is the same, since all these vacua are related by the symmetry. Moduli, however, can arise without a symmetry, and hence in general physics will depend on their values. Thus one finds a family of physically distinct vacua (the moduli space), which are connected by varying massless fields.

- plus: stabilizing the moduli can also help with a completely different question. How can parameters such as SM masses arise from a fundamental theory with no free parameters? The mechanism of moduli stabilization discussed here will yield a "string landscape", hence helping to attack this question and maybe even the cosmological constant problem.

The goal of this talk is to show how the metric moduli (in type IIB supergravity with Calabi-Yau compactification) can be stabilized.

1.2 The idea

The metric moduli are directly related to the non-trivial topological objects of the Calabi-Yau manifold. E.g. if you imagine the Calabi-Yau manifold as a Swiss cheese, then the the complex structure moduli describe the effect of deforming the holes in the cheese, whereas the volume moduli simply describe a rescaling.

On the other hand, topologically non-trivial objects can lead to non-vanishing fluxes[1] when integrating a field strength over a closed surface which encloses the topological defect, as happens, e.g., for magnetic monopols:

$$\text{Flux} = \int_\Sigma \text{field strength}\,, \qquad \Sigma = \text{topologically non-trivial cycle}^2\,. \tag{2}$$

These non-vanishing fluxes add a non-vanishing energy contribution to the Lagrangian. This energy will in general depend on the complex structure moduli (since they determine the properties of the cycles over which the integration is performed), and hence we can hope to create non-flat contribution for the effective scalar potential $\mathcal{V}$ of these moduli:

$$\mathcal{V} \ni \int_{\mathcal{M}^6} \sqrt{-g}\ (\text{field strength})^2\,. \tag{3}$$

Later on we will see, that evaluating this term indeed yields expressions proportional to the flux over the topologically non-trivial cycles of the manifold, thus confirming the chain of thought explained above.

[1]In order to preserve Lorentz-invariance, we will focus on fluxes in the extra dimensions.

[2]More precisely, the homology group H(M) should be non-trivial and Σ a non-trivial element of the homology.

However, the technical problem of computing this quantity is difficult at the best, since there is no closed form expression known for any Ricci-flat metric on a campact Calabi-Yau manifold. At many points, we will therefore use qualitative arguments inferred from holomorphy and dualities of the setting, instead of explicit computation.

Nevertheless, we can argue that we would expect energy scales such as the Planck mass, the string tension or the inverse compactification radius to enter into the additional contributions to the energy density, and hence the resulting masses could well be very heavy and thus the moduli indeed stabilized. Section 3 is dedicated to demonstrating this mechanism of moduli stabilization via generalized fluxes in some detail.

From Eq. (2), we can infer that a successful realization of the idea described above requires the dimensions of the modulus related to the topologically non-trivial cycle Σ and and field strength to 'match'. Hence in order to stabilize the complex structure moduli of type IIB supergravity with Calabi-Yau compactification described by (1,2)-forms, we need 3-form field strengths. As we have seen in Felix's talk on supergravity, we can find such objects in type IIB $d = 10$ supergravity. However, in order to stabilize the volume moduli described by (1,1)-forms by this mechanism, we require a 2-form field strength, which cannot be found in the type IIB supergravity spectrum. In order to stabilize all metric moduli, we thus need to introduce a further (non-perturbative) mechanism. This is the topic of Section 4.

2 Setting

→ Christoph's and Felix's talks

2.1 Type IIB supergravity in $d = 10$

- Top-down approach: Type IIB string theory is a chiral N=2 supersymmetric superstring theory in 10 dimensions. The mass spectrum of the one-particle states is 'tower-like', with quasi-degenerate levels separated by M^2_{string}. The lowest level (= low energy limit) can be described by chiral N=2 $d = 10$ Supergravity.
- Bottom-up approach: Searching for a 'maximal supergravity' yields a unique $d = 11$ supergravity theory, and from that different types of $d = 10$ supergravity. Here we will focus on type IIB supergravity in $d = 10$.
- The bosonic field content is given by the symmetric tensor g^{MN} (10d metric, graviton), the 2-form field $B_{(2)}$ (antisymmetric tensor), the dilaton ϕ and the 0-, 2-, and 4-form fields $C_{(0)}$, $C_{(2)}$ and $C_{(4)}$, also referred to as potentials. From these, the field strengths can be calculated as

$$F_{(2p+1)} = dC_{(2p)} \qquad H_{(3)} = dB_{(2)} \tag{4}$$

Note that while we obtain two 3-form field strengths $F_{(3)}$ and $H_{(3)}$, we do not obtain any 2-form field strengths. This indicates that the fluxes in this setting will help stabilizing the complex structure moduli, but not the volume moduli.

- Furthermore, there are non-perturbative objects, so called D(p)-branes (Dirichlet branes) and O(p)-branes (Orientifold branes). In particular the former will be of interest for this discussion. They are p+1 dimensional objects, with $p \rightarrow 2p-1$ in type IIB supergravity. In the context of string theory, they can be interpreted as the boundaries of open strings, in the context of SUGRA, they can be understood as solitonic objects. They source gauge fields in 10d and carry the tension T_p and charge μ_p.
- A powerful analogy: electromagnetism (see Tab. 1). Comparing the p-form language of type IIB 10d supergravity to Maxwell's electromagnetism yields a far-reaching analogy and helps to gain an intuitive understanding of this abstract language. In Tab. 1, the first column describes electromagnetism in the 'conventional' language, the second describes electromagnetism in the p-form language (see Sec. A) and the third shows the generalization to 10d supergravity.

	Electromagnetism		10d SUGRA type IIB
(vector) potential	A^μ	$A_{(1)}$	$C_{(2p)}$
field strength	$F^{\mu\nu} = \partial^{[\nu} A^{\mu]}$	$F_{(2)} = dA_{(1)}$	$F_{(2p+1)} = dC_{(2p)}$
action (pure gauge)	$\int d^4x F^{\mu\nu} F_{\mu\nu}$	$\int F_{(2)} \wedge *F_{(2)} \sim \int \sqrt{-g}\, F_{(2)}^2$	$\int \sqrt{-g}\, F_{(2p+1)}^2$
sources of gauge field	e^-	D0-brane	D(2p-1)-brane
action (source - gauge field)	$q \int d\xi^\nu A_\mu(\xi)$	$q \int_{\mathcal{C}} A_{(1)}$	$\mu_p \int_{\mathcal{W}} C_{(2p)}$
flux	$\int dS^{\mu\nu} F_{\mu\nu}$	$\int F_{(2)}$	$\int F_{(2p+1)}$

Table 1: Analogy between 10 dimensional Supergravity and Maxwell's electromagnetism. $\mathcal{C}$ denotes the worldline of a one-dimensional charged particle parametrized by ξ, $\mathcal{W}$ the 2p dimensional worldvolume of a D(2p-1)-brane, and $dS^{\mu\nu}$ parametrizes the surface through which the flux is measured.

Note that in electromagnetism, integrating over a closed surface yields a vanishing flux - unless we encounter topologically nontrivial objects inside that surface, i.e. magnetic monopoles. This also holds for the generalized situation in 10d SUGRA. Luckily, when compactifying the 10 dimensional theory on a Calabi-Yau manifold (see below), we obtain such topologically non-trivial objects. Hence we can hope to be able to introduce non-vanishing fluxes, as required in Section 1.2.

- 10d SUGRA effective action (bosonic part):

$$S_{IIB} = \frac{1}{2\,\kappa_{10}^2}\int dx^{10}\sqrt{-g_s}\left\{e^{-2\phi}\left[R_s + 4(\nabla\phi)^2\right] - \frac{1}{2}F_{(1)}^2 - \frac{1}{12}G_{(3)}\cdot\bar{G}_{(3)} - \frac{1}{4\cdot 5!}\tilde{F}_{(5)}^2\right\}$$
$$+\frac{1}{8\,i\,\kappa_{10}^2}\int e^{\phi}C_{(4)}\wedge G_{(3)}\wedge\bar{G}_{(3)} + S_{\text{loc}}\,. \tag{5}$$

Here $C_{(2p)}$ denote the 2p-form potentials of type IIB string theory and $F_{(2p+1)} = dC_{(2p)}$ the respective field strengths, with $F^2 = (-g)^{-1/2}F\wedge *F$. The compact notation of eq. (5) is a result of the following definitions:

$$\tau \equiv C_{(0)} + ie^{-\phi}\,, \tag{6}$$
$$G_{(3)} \equiv F_{(3)} - \tau H_{(3)}\,, \tag{7}$$
$$\tilde{F}_{(5)} \equiv F_{(5)} - \frac{1}{2}C_{(2)}\wedge H_{(3)} + \frac{1}{2}B_{(2)}\wedge F_{(3)}\,, \tag{8}$$

with $B_{(2)}$ the antisymmetric 2-tensor of type IIB supergravity and τ referred to as axio-dilaton. κ_{10}, g_s and R_s are the string coupling constant, the string metric and the Ricci scalar, respectively. In Eq. (5), the first term describes the action of Einstein gravity, the dilaton and the field strengths of type IIB $d = 10$ supergravity (s. Tab. 1), whereas the second term is a metric-independent Cern-Simons-term. Finally, S_{loc} denotes the action of localized objects, such as D-branes. For example, the action of a p-brane wrapped on a (p-3) cycle Σ of the manifold $\mathcal{M}_6$ is given by

$$S_{\text{loc}} = -\int_{R^4\times\Sigma} d^{p+1}\xi\,T_p\sqrt{-g} + \mu_p\int_{R^4\times\Sigma} C_{(p+1)}\,, \tag{9}$$

where T_p and μ_p denote the brane tension and charge, respectively. The first term describes the coupling of the D-brane to gravity, the second term is a metric-independent Cern-Simons-term and describes the coupling of the worldvolume of the brane to the background potential (s. Tab. 1).

2.2 Calabi-Yau compactification

In order to recover a realistic scenario in our 3+1 dimensions, we need to compactify six dimensions of the 10 dimensional theory discussed above. The 10 dimensional space is described as

$$R^{1,3}\times\mathcal{M}^6\,, \tag{10}$$

with $R^{1,3}$ describing the four dimensional Minkowski space and $\mathcal{M}^6$ describing the compact, six dimensional, compact manifold unobservable at very low energies. Here, we choose $\mathcal{M}^6$ to be a Calabi-Yau manifold CY^3 with three complex dimensions (s. Christoph's talk). Among other properties, this implies

- CY^3 is a complex manifold, i.e. we have two sets of coordinates, associated with d^3z and $d^3\bar{z}$ and we can define holomorphic functions.

- On a general CY 3-fold, there are following harmonic forms:

$$\Omega\,[(3,0)-\mathrm{form}]\,,\ \chi_a\,[(2,1)-\mathrm{form}]\,,\ \bar{\chi}_{\bar{a}}\,[(1,2)-\mathrm{form}]\,,\ \bar{\Omega}\,[(0,3)-\mathrm{form}]\,, \qquad (11)$$

  with $a = 1,..,h^{2,1}$, $\bar{a} = 1,..,h^{1,2}$. χ_a is imaginary self dual (ISD): $*_6\chi_a = i\chi_a$, whereas Ω is anti imaginary self dual (AISD): $*_6\Omega = -i\Omega$.

- CY^3 is Kähler, i.e. its metric can be expressed as derivatives of a Kähler potential.

- CY^3 is not unique but comes in families, parametrized by the metric moduli.

Moduli in CY - compactifications:

Metric moduli:
The Calabi-Yau metric is not unique, but comes in families parametrized by vacuum expectation values of metric fluctuations which preserve the CY condition. In the effective 4d theory, these metric fluctuations appear as massless scalar fields, i.e. modulus fields. In the setting discussed here, this leads to two types of metric moduli: (1,1)-forms which leave the Kähler structure of the metric invariant (volume/ Kähler moduli) and (2,1)-forms which modify the Kähler structure (complex structure moduli):[3], see Eq. (1). The number of volume (complex structure) moduli is given by the Hodge number $h^{1,1}$ ($h^{2,1}$), which the denotes the dimension of the respective co-homology group and is typically $\mathcal{O}(100)$.

Further moduli:
Further moduli, not originating from fluctuations of the metric, include the dilaton ϕ (parametrizing the string coupling constant) and $\int C_{(0)}$, $\int C_{(2)}$ and $\int B_{(2)}$, sometimes referred to as axions. In general, further mechanisms (beyond the scope of this talk) are required to stabilize these moduli.

3 Moduli Stabilization via Fluxes

3.1 A no-go-theorem and how to circumvent it

Before turning on fluxes to stabilize moduli, we need to check if the theory provides any constraints for this case. As it turns out, the Einstein equations indeed impose severe constraints on the existence of fluxes in compact manifolds. However, allowing for localized objects with negative tension, so called orientifold branes, these can be evaded.

A no-go theorem for fluxes and warped solutions on compact manifolds

[3] related to the co-homology groups $H^{1,1}(\mathcal{M})$ and $H^{2,1}(\mathcal{M})$.

Our starting point on the way to examine the effect of fluxes on the Calabi-Yau moduli is the effective low-energy type IIB SUGRA action Eq. (5). Deriving the resulting Einstein equations, we obtain serious constraints for the existence of non-trivial warped solutions and fluxes on compact manifolds. Since the calculation is quite tedious, I here merely roughly outline the actual calculation before presenting the result. More elaborate explanations can be found e.g. in [1].

- Reformulate eq. (5) in the Einstein frame ($\rightarrow$ Felix's talk)
- Looking for solutions with 4d Poincaré symmetry, choose the metric ansatz

$$ds_{10}^2 = \underbrace{e^{2A(y)}\eta_{\mu\nu}}_{g_{\mu\nu}} dx^\mu dx^\nu + e^{-2A(y)}\tilde{g}_{m,n} dy^m dy^n \,, \tag{12}$$

  with μ, $\nu = 0..3$ and m, $n = 4...9$, i.e. x the coordinates of our 4 dimensional space and y the coordinates of the compactified 6 dimensional manifold. $A(y)$ is called warp factor.
- Express $\tilde{F}_{(5)}$ as $\tilde{F}_{(5)} = (1+*)\,[d\alpha \wedge dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3]$ with $\alpha = \alpha(y)$.
- Calculate 10d trace reversed Einstein equations $R_{MN} = \kappa_{10}^2(T_{MN} - \frac{1}{8}g_{MN}T)$ (Christoffel connection $\rightarrow R_{MN}$, $T_{MN} \sim (-g)^{-1/2}\delta S/\delta g_{MN}$)

This finally yields the equation

$$\tilde{\nabla}^2 e^{4A} = e^{2A}\frac{G_{(3)} \cdot \bar{G}_{(3)}}{12 Im\tau} + e^{-6A}(\partial_m e^{4A}\partial^m e^{4A} + \partial_m\alpha\partial^m\alpha) + \frac{1}{2}\kappa_{10}^2 e^{2A}(T_m^m - T_\mu^\mu)^{\text{loc}} \,, \tag{13}$$

where $\tilde{\nabla}$ denotes the use of $\tilde{g}_{mn}$. These equations imply stringent constraints on the existence of fluxes and non-trivial warp factors on compact manifolds. Note that integrating the left side of eq. (13) over the compact six dimensional manifolds yields zero, whereas the first two terms on the RHS are positive definite. Hence in the absence of localized sources (third term), the first two terms must vanish independently, i.e. $G_{(3)} = 0$ and e^A =constant. Hence in particular, their can be no non-vanishing fluxes due to 3-form field strengths, which is however just the type of fluxes required to stabilize the (2,1)-form complex structure moduli, as discussed in Section 1.2.

A second constraint can be derived from combining this result with the Bianchi identity for $\tilde{F}^5$. This restricts the allowed type of localized objects and in particular yields the condition

$$*_6 G_{(3)} = i G_{(3)} \,, \tag{14}$$

i.e. requires $G_{(3)}$ to be imaginary self-dual (ISD).

...and a way around it

In order to rescue the idea discussed in Section 1.2, our only hope in evading the no-go-theorem above lies with the third term in Eq. (13), which describes the effect of localized sources. String theory automatically supplies us with localized sources, so called p-branes[4]. These are non-perturbative p+1 dimensional objects. The action of a p-brane wrapped on a (p-3) cycle Σ of the manifold $\mathcal{M}_6$ is given by Eq. 9. From this, we can calculate the stress-energy tensor

$$T^{\rm loc}_{MN} = -\frac{2}{\sqrt{-g}}\frac{\delta S_{\rm loc}}{\delta g^{MN}}\,. \tag{15}$$

Exploiting the formal relation $\frac{\delta\phi(y)}{\delta\phi(x)} = \delta(x-y)$ for functional derivatives and $\delta g = -g\, g_{MN}\,\delta g^{MN}$, we obtain

$$\begin{aligned}\frac{\delta S_{\rm loc}}{\delta g^{MN}} &= \int_{R^4\times\Sigma} d^{p+1}\xi\,\frac{1}{2}\,T_p\,(-g)^{-1/2}\frac{\delta g(\xi)}{\delta g^{MN}(x,y)}\\ &= \frac{1}{2}\int_{R^4\times\Sigma} d^{p+1}\xi\,T_p\,\sqrt{-g}\;\underbrace{g_{MN}\,\frac{\delta g^{MN}(\xi)}{\delta g^{MN}(x,y)}}_{\delta^d(\xi-x,y)\Pi_{MN}}\end{aligned}$$

The projector Π_{MN} arises due difference between the external metric and the induced metric on the manifold (similar to the Jacobian determinant). Hence for $M,N=\mu,\nu$, the projector is trivial $\Pi_{\mu\nu}=g_{\mu\nu}$ since on R^4 the induced metric is just the external metric. However, on the compact manifold $\mathcal{M}_6$, i.e. for $M,N=m,n$, the induced metric depends on the specification of the manifold. Finally the integration of the d-dimensional delta-distribution over $R^4\times\Sigma$ yields a $d-[4+(p-3)] = d-p+1$ dimensional delta distribution in the y-coordinates transverse to Σ, hence describing the position of the cycle ($\delta(\Sigma)$). With this, we obtain

$$T^{\rm loc}_{\mu\nu} = -T_p g^{\mu\nu}\delta(\Sigma)\,,\qquad T^{\rm loc}_{mn} = -T_p\Pi^{\Sigma}_{mn}\delta(\Sigma)\,, \tag{16}$$

and hence

$$\begin{aligned}T^{\mu}_{\mu} = g^{\mu\nu}T_{\nu\mu} &= -\underbrace{4}_{dimR}T_p\delta(\Sigma)\\ T^{m}_{m} = g^{mn}T_{nm} &= -T_p\underbrace{(p-3)}_{dim\Sigma}\delta(\Sigma)\end{aligned} \tag{17}$$

Regarding the third term on the RHS of Eq. (13), this implies

$$(T^m_m - T^\mu_\mu)^{\rm loc} = (7-p)T_p\delta(\Sigma) \tag{18}$$

From this we see, that for $p<7$ we can circumvent the no-go theorem from Section 3.1 if we have localized objects with negative tension in the theory[5]. String theory has such

[4]Exploiting the Bianchi identity for $\tilde{F}_5$, one obtains that type IIB supergravity in $d=10$ allows Dirichlet branes D3, D7 and orientifold branes O3.

[5]$p<7$ is not really a constraint, due to the equivalence $C_{(p)}\sim *C_{(d-p)}$.

objects, e.g. O3-branes (3+1 dimensional orientifold branes). Here, we will not be concerned about the details of these objects, but merely state, that if the theory contains such objects, we can turn on non-trivial 3-form fluxes without violating the constraints derived from the Einstein equations.

3.2 Scalar potential

Having learned that we can indeed turn on fluxes, we now proceed by examining the scalar potential for the moduli fields generated by such fluxes. Our strategy will be to calculate a treelevel approximation to the 4d effective potential via dimensional reduction of $d = 10$ supergravity. In general, we will have to add possible quantum corrections and nonperturbative contribution to this expression. We will come back to this point in Section 4.

Rewriting eq. (5) in the Einstein frame yields ($\rightarrow$ Felix's talk)

$$\begin{aligned} S_{IIB} = \frac{1}{2\kappa_{10}^2} \int d^{10}x\sqrt{-g} \left\{ R - \frac{\partial_M \tau \, \partial^M \tau}{2(Im\tau)^2} - \frac{G_{(3)} \cdot \bar{G}_{(3)}}{12 Im\tau} - \frac{\tilde{F}_{(5)}^2}{4 \cdot 5!} \right\} \\ + \underbrace{\frac{1}{8i\kappa_{10}^2} \int \frac{1}{Im\tau} C_{(4)} \wedge G_{(3)} \wedge \bar{G}_{(3)}}_{\text{contains } G_{(3)} \text{ but no metric fluctuations}} + S_{\text{loc}} \end{aligned} \tag{19}$$

Following the idea introduced in Section 1.2, we are interested in the flux potential of the symbolic form $\int \sqrt{-g}$ (field strength)^2. S_{IIB} contains two such terms, one for $G_{(3)}$ and one for $F_{(5)}$. Since CY moduli are expressed by (1,1) and (2,1) forms, we will focus on the $G_{(3)}$-term in the following, since the dimension of the non-trivial cycle related to the modulus and the flux threading this cycle must match. Hence the only term which can potentially yield a scalar potential for the metric fluctuations generated by fluxes, is

$$\begin{aligned} S_G &= -\frac{1}{2\,\kappa_{10}^2} \int d^{10}x\,\sqrt{-g}\,\frac{G_{(3)} \cdot \bar{G}_{(3)}}{12\,Im\tau} \\ \rightarrow \mathcal{L}^{4d} &= -\frac{1}{24\,\kappa_{10}^2} \int_{\mathcal{M}} d^6y \sqrt{-\tilde{g}}\,\frac{G_{(3)} \cdot \bar{G}_{(3)}}{Im\tau} \\ &= -\underbrace{\frac{1}{12\,\kappa_{10}^2\,Im\tau} \int_{\mathcal{M}} d^6y \sqrt{-\tilde{g}}\, G_{(3)}^{+} \cdot \bar{G}_{(3)}^{+}}_{\mathcal{V}} - \underbrace{\frac{i}{4\,\kappa_{10}^2\,Im\tau} \int_{\mathcal{M}} G_{(3)} \wedge \bar{G}_{(3)}}_{\text{contains } G_{(3)} \text{ but no metric fluctuations}} \quad , \end{aligned} \tag{20}$$

with $G_{(3)} = G_{(3)}^{+} + G_{(3)}^{-}$, where $G_{(3)}^{+}$ ($G_{(3)}^{-}$) are AISD (ISD) components of $G_{(3)}$: $G_{(3)}^{\pm} = (G_{(3)} \pm i *_6 G_{(3)})/2$. Here we are working in the approximation of weak warping, $A(y) = const.$. Note that Lorentz invariance in 4d requires $G = G(y)$. The contribution to the scalar potential we are interested in is hence given by

$$\begin{aligned}\mathcal{V} &= -(12\,\kappa_{10}^2\,Im\tau)^{-1}\int_{\mathcal{M}} d^6y\sqrt{-\tilde{g}}\,G^+_{mnp}\bar{G}^{+\,mnp}\\ &= -(2\,\kappa_{10}^2\,Im\tau)^{-1}\int_{\mathcal{M}} G^+_{(3)}\wedge\underbrace{*_6\bar{G}^+_{(3)}}_{-i\bar{G}^+_{(3)}}\end{aligned}\tag{21}$$

where we used the definition of the Hodge dual in six dimensions for the last equality. Expanding[6] the AISD 3-form $G^+_{(3)}$ spanning three real dimensions in the complex CY manifold, i.e. in terms of the harmonic AISD forms $\Omega\,[(3,0)-\text{form}]$ and $\bar{\chi}_{\bar{a}}\,[(1,2)-\text{form}]$ with $\bar{a}=1,2,..h^{1,2}$,

$$G^+_{(3)} = \alpha\Omega + \bar{\beta}^{\bar{a}}\bar{\chi}_{\bar{a}}\,,\tag{22}$$

we can determine the coefficients α and β as follows. Keeping in mind that it is possible to express p-forms with *anticommuting* differentials, see Eq. (46), we observe that the integration over the compact six dimensional manifold $\mathcal{M}$ over a wedge product of two p-forms is only non-vanishing if the resulting product of differentials contains exactly three dz s and three $d\bar{z}$ s. Hence evaluating $\int_{\mathcal{M}} G^+_{(3)}\wedge\bar{\Omega}$ and $\int_{\mathcal{M}} G^+_{(3)}\wedge\chi_b$ yields

$$\begin{aligned}\alpha &= \frac{\int_{\mathcal{M}} G^+_{(3)}\wedge\bar{\Omega}}{\int_{\mathcal{M}}\Omega\wedge\Omega}\,,\\ \bar{\beta}^{\bar{a}} &= \frac{\int_{\mathcal{M}} G^+_{(3)}\wedge\chi_b}{\int_{\mathcal{M}}\bar{\chi}_{\bar{a}}\wedge\chi_b} = K^{\bar{a}b}\frac{\int_{\mathcal{M}} G^+_{(3)}\wedge\chi_b}{\int_{\mathcal{M}}\Omega\wedge\bar{\Omega}}\,,\qquad K^{\bar{a}b} = \frac{\int_{\mathcal{M}}\Omega\wedge\bar{\Omega}}{\int_{\mathcal{M}}\bar{\chi}_{\bar{a}}\wedge\chi_b}\,.\end{aligned}\tag{23}$$

With this, inserting Eq. (22) into Eq. (21), thereby taking into account the anticommuting relations for the differentials, yields

$$\mathcal{V} = (2\kappa_{10}^2 Im\tau)^{-1}\frac{\int_{\mathcal{M}} G^+_{(3)}\wedge\bar{\Omega}\int_{\mathcal{M}}\bar{G}^+_{(3)}\wedge\Omega + \mathcal{G}^{a\bar{b}}\int_{\mathcal{M}} G^+_{(3)}\wedge\chi_a\int_{\mathcal{M}}\bar{G}^+_{(3)}\wedge\bar{\chi}_{\bar{b}}}{\int_{\mathcal{M}}\Omega\wedge\bar{\Omega}}\,.\tag{24}$$

Note that since $\int_{\mathcal{M}} G^-_{(3)}\wedge\bar{\Omega}$ and $\int_{\mathcal{M}} G^-_{(3)}\wedge\chi_a$ vanish, we can replace $G^+_{(3)}\to G_{(3)}$ in Eq. (24). How does this expression depend on the moduli fields? Terms of the type $\int_{\mathcal{M}}\bar{G}_{(3)}\wedge\Omega$ can be rewritten exploiting the decomposition in terms of the dual cycles A^a and B^a (canonical homology baisis for $H_3(\mathcal{M},\mathbb{Z})$):

$$\int_{\mathcal{M}}\bar{G}_{(3)}\wedge\Omega = \sum_a\underbrace{\int_{B^a} G_{(3)}}_{\text{flux } N_a}\underbrace{\int_{A^a}\Omega}_{z^a} + \sum_a\underbrace{\int_{A^a} G_{(3)}}_{\text{flux } M_a}\underbrace{\int_{B^a}\Omega}_{\Pi^a(z^b)}\tag{25}$$

The integral of the holomorphic 3-form Ω over the topologically non trivial cycles A^a, $z^a=\int_{A^a}\Omega$, can be identified as the projective coordinates of the complex structure moduli

[6] $G^-_{(3)}$ then in turn can be expressed in terms of the (0,3) and the (2,1) ISD forms ($\bar{\Omega}$, χ_a).

on the CY threefold [4]. Proving this relation is beyond the scope of this talk, however from a qualitative point of view we can observe that $\int_{A^a}\Omega$ projects the the structure of the topologically non-trivial cycle onto a scalar, which should therefore be related to scalar the modulus describing the cycle A^a. In other words, the complex structure of $\mathcal{M}$ is entirely determined by the z^a. The integral over the dual cycles B^a can be interpreted as the "dual" of the complex structure moduli, so called periods. The integral of the 3-from fieldstrength $G_{(3)}$ over the topologically non-trivial cycles is just the flux threaded through this cycles. Hence we have confirmed our earlier argument, that non-vanishing fluxes over topologically non-trivial objects generates a potential for the complex structure moduli. Thus Eq. (24) yields a scalar potential for the modulus fields, as required. In order to see, that this actually stabilizes the moduli (and is not a flat or run-away potential), we could try and express Eq. (24) in terms of the complex structure moduli using Eq. (25). In practice, this is not quite as easy as it sounds since the $\Pi^a(z^b)$ are in general complicated functions of the z^b, and therefore we will take a shortcut via the introduction of the superpotential and Kähler potential.

3.3 Consequences for complex structure moduli

In supergravity, the scalar potential is given by

$$V = \frac{1}{2\kappa_{10}^2} e^K (K^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} - 3|W|^2) \tag{26}$$

with W the superpotential, K the Kähler potential, $K^{i\bar{j}}$ the inverse Kähler metric and $D_i W \equiv \partial_i W + W \partial_i K$. From this, we can check that the effective 4d theory

$$\begin{aligned} W &= \int_{\mathcal{M}} G_{(3)} \wedge \Omega\,, & (27)\\ K &= -3\ln(T+\bar{T}) - \ln(-i(\tau-\bar{\tau})) - \ln(-i\int_{\mathcal{M}} \Omega\wedge\bar{\Omega})\,, & (28) \end{aligned}$$

with T denoting a volume modulus field, yield the scalar potential given by Eq. (24), e.g.:

$$\begin{aligned} D_T W &= 0 + W\frac{-3}{T+\bar{T}} = \frac{-3}{T+\bar{T}}\int_{\mathcal{M}} G_3\wedge\Omega\,, & (29)\\ D_a W &= \int_{\mathcal{M}} G_3\wedge(k_a\Omega+\chi_a) - \frac{\int_{\mathcal{M}} G_3\wedge\Omega}{\int_{\mathcal{M}}\Omega\wedge\bar{\Omega}}\left(k_a\int_{\mathcal{M}}\Omega\wedge\bar{\Omega} + \underbrace{\int_{\mathcal{M}}\chi_a\wedge\bar{\Omega}}_{=0}\right) & \\ &= \int_{\mathcal{M}} G_3\wedge\chi_a\,, & (30)\\ D_\tau W &= \frac{1}{\bar{\tau}-\tau}\int_{\mathcal{M}}\bar{G}_3\wedge\Omega\,, & (31)\\ K^{T\bar{T}} &= (\partial_{\bar{T}}\partial_T K)^{-1} = \frac{1}{3}(T+\bar{T})^2\,, \qquad \text{etc.} & (32) \end{aligned}$$

where we used $\frac{\partial\Omega}{\partial z^a} = k_a\Omega + \chi_a$ [4].

Hence we have found a 4d effective supergravity theory for the original 10d setting. Let's have a closer look at this theory and its implications.
The way T enters in K implies $K^{T\bar{T}}D_T W D_{\bar{T}}\bar{W} = -3|W|^2$, and hence the resulting treelevel scalar potential is independent of T (so called no-scale potential) and positive semi definite:

$$V \sim K^{\tau\bar{\tau}}D_\tau W D_{\bar{\tau}}\bar{W} + K^{a\bar{a}}D_a W D_{\bar{a}}\bar{W}\,, \tag{33}$$

with a running over the complex structure moduli. Hence vacua minima of the potential are obtained for [7]

$$D_i W = 0 \quad \text{for } i = \tau, a\,. \tag{34}$$

This is system of $h^{2,1}+1$ equations for $h^{2,1}+1$ variables for each choice of integral flux, with $h^{2,1}$ denoting the number of complex structure moduli. Hence for sufficient generic fluxes, we will find isolated points satisfying Eq. (34) which (due to the positive definiteness of the potential) correspond to minima of the potential with positive masses for all directions. Thus generic fluxes will fix all of the complex structure moduli as well as the axio-dilaton τ.

4 Stabilization of Volume Moduli

In the last section, we demonstrated how the complex structure moduli arising in the Calabi-Yau compactification of type IIB supergravity can be stabilized by an effective treelevel potential generated by fluxes. Note however, that no effective potential for the volume moduli was generated at the same time (no-scale form of the potential!). This can be understood as follows: The volume (Kähler) moduli are (1,1)-forms, hence following the arguments above we would require a 2-form flux, i.e. a 1-form potential in order to generate an appropriate potential. However, type IIB supergravity in $d = 10$ only yields potentials 2p-form potentials with 2p even, and hence this technique of stabilization can not work.[8]
Hence we must consider a different mechanism to stabilize the volume moduli in this setting. As mentioned above, we need to take quantum corrections of the Kähler potential (the superpotential is protected by a non-renormalization theorem) and non-perturbative contributions into account when calculating the full effective scalar potential. The main ingredient we will focus on here will be non-perturbative D-branes. As we will see, a stack of these can generate a SYM gauge group whose coupling constant is related to the volume modulus of the cycle the stack of branes is wrapped around[9]. Considering a pure gauge

[7] Inserting Eqs. (31) and (30), Eq. (34) implies that at the minimum $G_{(3)}$ is a (2,1) form, i.e. ISD. This is consistent with Eq. (14), derived in the 10d formalism using the Bianchi identity. This is a further confirmation of our "guess" of W and K for the effective 4d theory.

[8] Note that in type IIA theories, p-form potentials with odd p arise and hence volume moduli can be stabilized by flux potentials.

[9] In type IIA theories, the gauge coupling is related to the complex structure moduli.

theory on the brane leads to gaugino condensation and thereby to an effective potential for the volume moduli.

4.1 D-Branes

Until the mid 1990s, it was widely believed that compactifications of type II string theories could never yield any SM - like phenomenology at low energies. This changed significantly after the discovery of the so called Dirichlet (D)-branes (Polchinsky, 1995). D-branes provide a new origin for non-abelian gauge symmetries, can localize chiral matter representations at their intersection locus and can preserve some but not all of the supersymmetry of the respective type II compactification, hence allowing $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$ supersymmetry for the low energy effective theory.

T-Duality:
An open string theory compactified on a small cycle is T-dual to a compactification on large cycle with open string endpoints restricted to lie on hyperplanes. The latter are called D-branes. Higher dimensional Dp-branes are obtained by dualizing further (compact) dimensions. The resulting Dp-branes fill all four Minkowski dimensions, the remaining $p-3$ spatial dimensions are 'wrapped' around a cycle of the complex manifold. Viable type IIB compactifications include D7-branes.

Mass Spectrum:
In the context of one dualized dimension, consider the massless spectrum of open strings, which arises by considering strings with zero winding number and with both endpoints confined to the same D-brane. If all D-branes are located at distinct locations (in the remaining coordinates), we obtain

- d-1 states with tangent polarization $\rightarrow$ gauge field living on hyperplane.
- 1 state with perpendicular polarization with respect to the hyperplane $\rightarrow$ collective coordinate for the shape of the hyperplane.

Hence we find that the D-brane has become a dynamical object of the theory (described by the latter dof) with a U(1) gauge symmetry living on the brane. If N branes coincide, i.e. have the same location, the resulting massless spectrum of open strings consists of

- N^2 massless vectors (tangent polarization) $\rightarrow$ non-abelian $U(N) + SU(N) \times U(1)$ gauge symmetry on the coinciding branes.
- N^2 massless scalars (perpendicular polarization) $\rightarrow$ after gauge transformation: N collective coordinates describing the fluctuations of the N D-branes.

The same qualitative behavior is found for higher dimensional Dp-branes.

Phenomenology

From the discussion above, we see that non-Abelian gauge groups can be realized on stacks of branes. The gauge coupling is given by

$$g_{YM}^2 = \frac{g_s\, l_s^{p-3}}{Vol(\Sigma)}\,, \tag{35}$$

where g_s denotes the string coupling, l_s the string length and $Vol(\Sigma)$ is the volume of the cycle wrapped by the brane under consideration. Generally, different gauge groups arise from stacks of branes wrapping different cycles with different volumes, so the couplings have no reason to be equal. The branes responsible for the SM dofs can be localized to a small subregion of the compact manifold, allowing its energy scales to be influenced by warping.

Analogy to Electromagnetism:
A Dp-brane can be understood as the analogue of an electron in electromagnetism. The electron, a point-like particles, sources a 1-form potential (A) and its action is given by coupling its one-dimensional wordline to this 1-form potential. The Dp-brane sources a p+1-form potential $C_{(p+1)}$ and its action is given by coupling its p+1 dimensional world-sheet to this p+1-form potential.

4.2 Stabilization via gaugino condensate

The the non-abelian $U(N)$ gauge symmetry (with N large) on the coinciding branes shares many features of the non-abelian $SU(3)$ gauge symmetry of QCD.

- Due to RGE, the coupling constant increases for decreasing energy. Hence there is a cutoff scale Λ, below which the perturbation theory is no longer valid but the physics is governed by non-perturbative effects.
- In analogy to quark confinement below the cutoff scale Λ_{QCD} in QCD, this leads to gaugino condensation: $\langle\lambda\lambda\rangle \sim \Lambda^3$.
- The physics below the scale Λ can be described by an effective field theory. In QCD, the "elementary" particles of this theory are e.g. protons and pions and we can write down effective interactions and an effective potentials for effective dofs. In the $U(N)$ SYM theory on the brane, we can also formulate such an effective theory, and in particular an effective scalar potential and superpotential.

The action describing the SU(N) gauge theory on the brane is given by

$$\int d^2\theta \frac{1}{g^2} W^\alpha W_\alpha\,, \tag{36}$$

with g the SU(N) coupling constant in the high energy limit. In type IIB supergravity, this coupling constant is promoted to a dynamical (modulus) field T (see Eq. (35))

$$g^2 \sim Vol(\Sigma)^{-1} \sim T^{-1}\,. \tag{37}$$

We can thus rewrite the action as

$$\sim \int d^2\theta\, T\, W^\alpha W_\alpha \supset F_T \lambda\lambda\,, \tag{38}$$

exploiting that the lowest order (θ^0) contribution of $W^\alpha W_\alpha = \lambda\lambda$ projects out the F-term of the modulus T. Analogous to quark confinement in QCD, the strong coupling regime of this theory will induce gaugino condensation

$$\lambda\lambda \to \langle\lambda\lambda\rangle \sim \Lambda^3\,, \tag{39}$$

where Λ is the scale where the coupling becomes strong, given by the running of the gauge coupling

$$g^2(\mu) = \frac{1}{\frac{1}{g_{IR}^2} + \frac{b}{8\pi^2}\ln\left(\frac{M_{IR}}{\mu}\right)} \qquad \to \qquad \Lambda = M_{IR}\exp\left(-\frac{8\pi^2}{g_{IR}^2\cdot b}\right) \tag{40}$$

with b determined by the structure of the gauge group and g_{IR} the gauge coupling of the low energy effective theory, governed by the modulus T. Plugging this into Eq. (38), we obtain a contribution to the action of the form

$$\sim F_T \exp(-\frac{2\pi}{N}T). \tag{41}$$

In the 4d effective theory this can be expressed as

$$W(T) = W_0 + Ae^{-\frac{2\pi}{N}T}\,, \tag{42}$$

where W_0 is the perturbative treelevel superpotential and N explicitly shows the dependence on the size of the D7-stack. Large N allow for to fix T at large values, hence justifying the supergravity assumption of large volumes.

The effective contribution to the scalar potential obtained from Eq. (42) yields a positive squared mass contribution for the Kähler modulus T (quadratic term in $W \to$ mass term). This demonstrates the basic idea of how non-perturbative effects can stabilize the Volume moduli.[10] It can be extended to the case of more than one volume modulus.

So finally, including fluxes and non-perturbative effects, we have demonstrated a loose proof-of-principle argument, that it is possible to stabilize all metric moduli.

5 Summary

In theories with extra dimensions, fluctuations of metric generically give rise to massless scalars in the effective 4d theory. If not stabilized by some mechanism, these so-called moduli fields can cause serious phenomenological problems.

[10]The effective scalar potential derived from Eq. (42) yields $V^{min} < 0$, i.e. and AdS vacuum. This can be uplifted into a dS vacuum by introducing D3-branes.

In type IIB supergravity in 10 dimensions with Calabi-Yau compactification, the complex moduli can be stabilized using flux potentials. These fluxes, obtained by integrating a 3-form field strength over topologically non-trivial objects parametrized by the complex structur moduli, generate an effective scalar potential for the moduli. Starting form the IIB sugra action in $d = 10$, we derived an expression for the 4d scalar potential, which can be mimicked by an effective 4d supergravity theory characterized by a superpotential and a Kähler potential. Examining the latter theory, we found that for sufficiently generic fluxes, it allows for a stabilization of all complex structure moduli.
Considering non-perturbative effects, in particular gaugino condensation on D7-branes, can stabilize the volume moduli. Large stacks of D-branes wrapped around topologically non-trival cycles source gauge groups with the coupling constant related to the volume of the respective cycle and hence to a volume modulus. Gaugino condensation on the brane yields a contribution to the low energy effective theory, which stabilizes the volume moduli.

A p-forms

- p-form $A_{(p)}$: completely antisymmetric p-index tensor $A_{\mu_1,\ldots\mu_p}$ with indices omitted.
- wedge product ($\sim$ diadic product, tensor product):

$$(A_{(p)} \wedge B_{(q)})_{\mu_1,..\mu_{p+q}} = \frac{(p+q)!}{p!q!} A_{[\mu_1..\mu_p} B_{\mu_{p+1}..\mu_{p+q}]} \,, \tag{43}$$

  with [..] denoting antisymmetrisation.
- exterior derivative takes p-form to p+1 form:

$$(dA_{(p)})_{\mu_1..\mu_{p+1}} = (p+1)\partial_{[\mu_1} A_{\mu_2..\mu_{p+1}]} \,. \tag{44}$$

- Integral: $\int A_{(d)} \equiv \int d^d x A_{0,..,d-1}$.
- Stokes theorem: $\int_{\mathcal{M}} dA_{(p-1)} = \int_{\partial\mathcal{M}} A_{(p-1)}$.
- Hodge star:

$$*A_{\mu_1..\mu_{d-p}} = \frac{\sqrt{-g}}{p!} \epsilon^{\nu_1..\nu_p}_{\mu_1..\mu_{d-p}} A_{\nu_1..\nu_p} \,, \tag{45}$$

  i.e. the hodge star converts a p-form into its Hodge dual, a d-p-form. In the complex Calabi-Yau three-fold, the situation is slightly more complicated. Here the hodge star converts a (q,p)-form into a (3-p, 3-q) form and the 6d epsilon tensor is replaced by two 3d epsilon tensors.
- Representation with anticommuting differentials dx^μ

$$A_p = \frac{1}{p!} A_{\mu_1..\mu_p} dx^{\mu_1}..dx^{\mu_p} \,. \tag{46}$$

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