Universal Extra Dimensions at Colliders

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Elina Fuchs

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1 Universal Extra Dimensions

In the previous talks, we learned about large extra dimensions in the ADD model, the strongly curved extra dimensions in the warped Randall-Sundrum model and the case that only the graviton may travel large distances in the extra dimensions. Now [1] we allow all Standard Model (SM) particles to propagate through the bulk of ($D = 1$ or more) *large, flat* **Universal Extra Dimensions (UED)** which was proposed by Appelquist, Cheng and Dobrescu [2]. We will restrict the phenomenological discussion to the simplest case of $D = 1$.

1.1 Motivation

There exist several compelling motivations for UED:

- proton stability: ADD + RS predict shorter proton lifetimes than observed whereas UED limits higher dimensional operators which lead to proton decay.
- anomaly cancellation: 3 generations of fermions cancel gauge anomaly.
- Dark Matter (DM): Lightest Kaluza-Klein Particle (LKP) is stable, neutral and non-coloured, thus a viable DM candidate.
- collider searches: The severe electroweak bounds on the size of the extra dimensions are relaxed by the introduction of a Kaluza-Klein (KK) parity, resulting in rather light KK states possibly within the range of today's or future colliders.
- effective field theory: strongly coupling, but no-confining at cut-off scale if fermions travel in the UED

1.2 Cut-off

An argument on mass dimensions shows that the theory is not renormalisable for $D > 0$. We denote the non-compact space-time coordinates by $x^\mu,\ \mu = 0, ..., 3$. All D extra dimensions (with coordinates $y^a, a = 1, ..., D$) are flat, accessible by all fields, and compactified at a scale $R^{-1} \equiv M_c$. Capital indices A run from 1 to $3 + D$. The Lagrangian contains a term

$$\mathcal{L}(x^\mu) \supset \int d^D y \frac{1}{4} Tr \left[F_i^{AB}(x^\mu, y^a) F_{iAB}(x^\mu, y^a) \right] \tag{1}$$

Since the action $S = \underbrace{\int d^4 x}_{M^{-4}}\ \underbrace{d^D y \mathcal{L}(x^\mu, y^a)}_{M^4 = M^{-D} M^2 [A]^2}$ is dimensionless, we need $[A] = M^{1+D/2}$. On the other hand, the covariant derivative $D_\mu = \partial_\mu + igA_\mu$ implies $[gA] = 1$. Hence, the gauge coupling has a negative mass dimension $[g] = M^{-D/2}$! We can redefine $A_\mu \to g\tilde{A}_\mu$, so $F_{\mu\nu}F^{\mu\nu} = \frac{1}{g^2}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu}$.
The inverse running coupling with mass dimension

$$\left[\alpha^{-1}\right] \equiv \left[\frac{4\pi}{g^2}\right] = M^{+D} \tag{2}$$

gives rise to a powerlaw in the divergent behaviour in terms of the cut-off parameter Λ:

$$\left[\alpha^{-1}\right]^{(4+D)}_{1-loop} = \left[\alpha^{-1}\right]^{(4+D)}_{tree} + \Lambda^D \tag{3}$$

$$\left[\alpha^{-1}\right]^{(4)}_{1-loop} = \int^{2\pi R} dy^D \left[\alpha^{-1}\right]^{(4+D)}_{1-loop} = (2\pi R)^D \left[\alpha^{-1}\right]^{(4+D)}_{tree} + (2\pi R\Lambda)^D \tag{4}$$

Thus in the case of $D = 1$ there is a linear divergence. Furthermore, the Λ is not well-defined as a linear shift could be absorbed in the coupling. Beyond the cut-off of this effective approach, a UV-completion is needed. Furthermore, Λ limits the highest possible KK mass $m_{n_{max}^{(KK)}} \leq \Lambda$. The product $R\Lambda \equiv \frac{\Lambda}{M_c}$ as the relation of the cut-off and the compactified scale turns out to be a crucial parameter in phenomenological discussions.

1.3 Fields

- The SM fields exist as copies at each KK level, supplemented by the additional components. Also the Clifford algebra of γ-matrices is extended to $\left\{\Gamma^A, \Gamma^B\right\} = 2g^{AB}$. For $D = 1$, this is fulfilled by $\Gamma_\mu = \gamma_\mu$ and $\Gamma_4 = i\gamma_5$.
- Finally, the extra dimensions need to be compactified. In order to obtain *chiral* fermions, an orbifold compactification is required. In general, this works with $\left[(S^1 \times S^1)/Z_2\right]^K$ for an even D or $\left[(S^1 \times S^1)/Z_2\right]^K \times S^1/Z_2$ for D odd (with $K = [D/2]$). Hence, in the simplest case of $D = 1$ it is sufficient to consider S^1/Z_2.
- Because the KK $n = 0$-modes correspond to the SM fields, the dof have to match. Therefore we define which fields are even or odd under the orbifold transformation $y \to -y$. The μ-components of the gauge fields transform evenly, but no additional massless scalar has been observed as a zero-mode, so A_5 is odd. Scalars can be even or odd. In order to allow Yukawa couplings, an even Higgs field H is chosen. So we can decompose the bosonic fields in even and odd parts.

$$(H, A_\mu) = \frac{1}{\sqrt{\pi R}} \left[(H_0, A_{\mu,0})(x_\mu) + \sqrt{2} \sum_{n=1}^{\infty} (H_n, A_{\mu,n})(x_\mu) \cos\left(\frac{ny}{R}\right) \right] \tag{5}$$

$$A_5 = \sqrt{\frac{2}{\pi R}} \sum_{n=1}^{\infty} A_{5,n}(x_\mu) \sin\left(\frac{ny}{R}\right) \tag{6}$$

One Dirac fermion, however, requires two 5-dimensional fermionic fields, $\psi_{L,n}$ and $\psi_{R,n}$, out of which the 4 μ-components are projected to build $\psi^{SM} = P_L \psi_{L,0} + P_R \psi_{R,0}$, and in general,

$$\psi(x^\mu, y) = \frac{1}{\sqrt{\pi R}} \left[\psi^{SM}(x^\mu) + \sqrt{2} \sum_{n=1}^{\infty} P_L \psi_{L,n}(x^\mu) \cos\left(\frac{ny}{R}\right) + P_R \psi_{R,n}(x^\mu) \sin\left(\frac{ny}{R}\right) \right] \tag{7}$$

1.4 Kaluza-Klein Parity

- Similar to the R-parity in SUSY, the KK-parity specifies even and odd KK modes. Its group is the remaining symmetry after orbifolding, which destroys the invariance under translations along the extra dimensions, and thus extra-dimensional momentum conservation. As a result, the KK-number n is not conserved, but the KK-parity (e.g. a Z_2 symmetry for one compactified extra dimension on S^1/Z_2) - at least if we assume a corresponding UV completion. KK-number violating interactions compete with the KK-number conserving ones, but the former enter through loops and exist only on the boundaries, resulting in volume-suppressed couplings.
- In the following, we focus on $D = 1$ and a conserved KK-parity $\boxed{P = (-1)^n}$ so that only the odd modes are charged under the Z_2. This implies that the LKP is stable and, analogously to the LSP, an interesting DM candidate. As another phenomenological consequence, $n =$ *odd* KK-mode can only be pair produced whereas even ones also alone.

2 Phenomenological Consequences

2.1 Spectrum

Due to the quantised momenta in the extra dimension, as we saw previously, the mass of particle X in the nth KK-mode is at tree-level given by

$$m^2_{X^{(n)}} = \underbrace{m^2_{X^{(0)}}}_{m^2_{X_{SM}}} + \frac{n^2}{R^2} \tag{8}$$

- For a large compactification scale M_C, i.e. a small radius R, the SM mass $m^2_{X^{(0)}}$ is negligible compared to the KK excitation, $\frac{n}{R}$. This leads to highly degenerate mass spectra at each KK-level n since the KK contribution is universal.
- However, if loop corrections [3] are taken into account, the degeneracy is alleviated because the contributions distinguish between the particle species. Furthermore, while at tree level all momentum conserving decays happen exactly at threshold, radiative corrections have a crucial impact on the decay kinematics by enlarging the phase space or closing some channels completely.

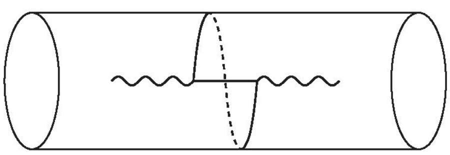

Figure 1: Lorentz-violating loop wrapped around the circle of the extra dimension such that it cannot be contracted to a point [3].

One subtlety is now the calculation of the loops although the theory is not renormalisable (see section 1.2).

1. Loops in the compactified dimension are different from those on the boundaries because the Lorentz symmetry is broken upon compactification. An internal loop around the circle of the compactified dimension (as in figure 1) cannot be shrunk to a point. This non-local effect leads to a well-defined, n-independent and UV-finite contribution (because Lorentz invariance is restored at small distances $\ll R$). Fermions do not receive such corrections to the mass [3].
2. On the other hand, for all fermions and bosons logarithmic divergences [4] arise from loops on the boundaries: $\propto \ln \frac{\Lambda}{\mu} = 2 \ln \Lambda R$ for the (arbitrary) choice of the renormalisation scale $\mu = R^{-1}$.

The exact results can be found in [3], which are summarised in the following structure:

$$\delta m^2_{V_i^{(n)}} = \frac{g_i^2}{16\pi R^2}\left(v_{,i1}\zeta(3) + v_{i,2} n^2 \ln \Lambda R\right) \tag{9}$$

$$\delta m_{F_j^{(n)}} = \frac{n}{16\pi R}\left(f_{j,1} g_3^2 + f_{j,2} g^2 + f_{j,3} g'^2\right) \ln \Lambda R \tag{10}$$

$$\delta m^2_{H^{(n)}} = \frac{n^2}{16\pi^2 R^2}\left(h_1 g_2^2 + h_2 g'^2 - 2\lambda_H\right) \ln \Lambda R + \bar{m}_H^2 \tag{11}$$

Here, $V_i = B, W, g$ denote the gauge bosons and ${}_jF = Q, u, d, L, e$ the fermions as $SU(2)$ doublets or singlets. g_i are the respective gauge couplings and v_i, f_i are numerical coefficients.

- It is remarkable that $v_{i,1} < 0 \forall i$. For $B^{(n)}$, also $v_2 < 0$ so that the overall correction is negative and it stays the lightest particles.
- Among the gauge bosons, the gluon mass receives the largest correction.
- The corrections to the KK quark masses are larger than those to the KK leptons.
- Each KK-level comprises $H^{\pm(n)}, H^{0(n)}, A^{0(n)}$. The fifth component of the gauge fields and these Higgs KK modes are eaten by $W^{(n)}, Z^{(n)}$ to become massive.
- KK gravitons receive only negligible radiative corrections.

However, the KK photons and Z's are a mixture of $B^{(n)}$, $W_3^{(n)}$, also the fermions mix. The tree level and the loop-corrected spectrum at the KK-level $n = 1$ are shown for one choice of Λ and R in figure 2.

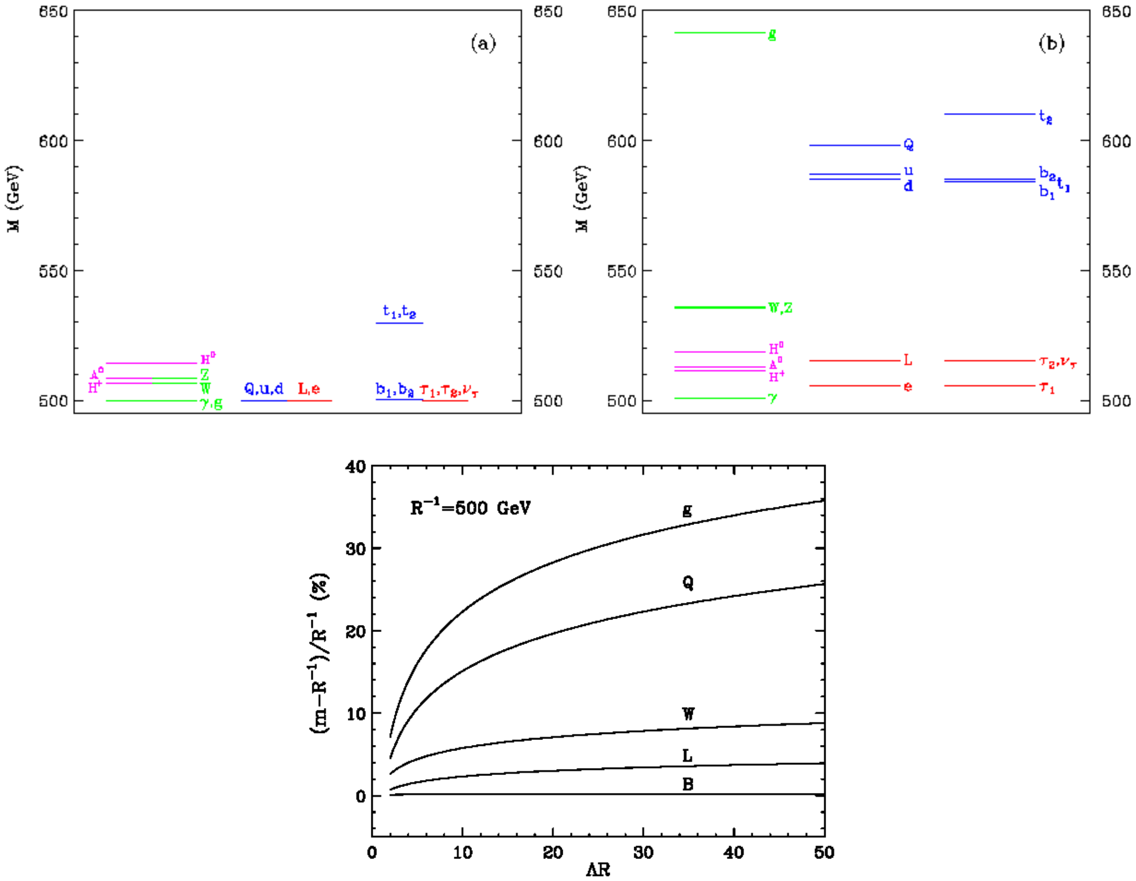

Figure 2: Mass spectrum of the $n = 1$ KK modes for $R = (500\,\mathrm{GeV})^{-1}$, $\Lambda = 1\,\mathrm{TeV}$, *up left:* at tree level, *up right:* including bulk and boundary 1-loop corrections (figures from [3]). *down:* Relative radiative corrections for $n = 1$ KK states, depending on ΛR [5].

2.2 Electroweak Precision Observables

- Although KK states can only be directly produced if $\sqrt{s} \geq \frac{2}{R}$, which corresponds either to the pair production of two $n = 1$ states or a single $n = 2$ state, KK modes also enter SM observables via loops.
- A set of electroweak precision observables (EWPO) has been calculated and measured to an excellent agreement within the SM.

- If only gauge bosons are allowed to travel in the bulk, their masses are restricted to be at least around several TeV. In UED, however, these constraints from LEP measurements at the Z-pole are relaxed.
- The strongest constraints come from the following two parameters, shown for the more sensitve $\hat{T}$ in figure 3.

$$\begin{aligned}\hat{T} &\equiv \frac{1}{m_W^2}\left(\Pi_{W_3W_3}(0) - \Pi_{W^\pm W^\pm}(0)\right) &&= \frac{g^2 m_t^4}{96 m_W^2 m_{KK}^2} - \frac{5g^2 \sin^2\theta_W m_h^2}{1152\cos^2\theta_W m_{KK}^2} \\ \hat{S} &= \frac{g}{g'}\Pi_{W_3B}(0) &&= \frac{g^2 m_t^2}{576 m_W^2 m_{KK}^2} - \frac{g^2 m_h^2}{2304 m_{KK}^2}\end{aligned} \quad (12)$$

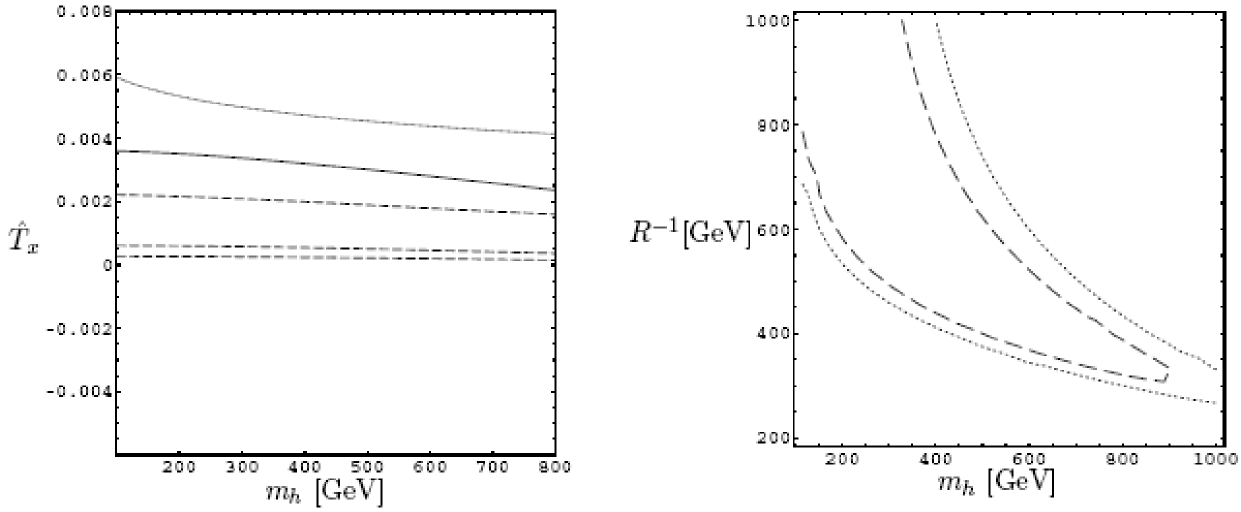

Figure 3: *left:* Contributions to $\hat{T}$ for $m_{KK} = 400\,\mathrm{GeV}$ from $n = 1, 2, 3$ (dashed), sum of $n = 1, ..., 10$ (solid) and only Higgs term (dotted). *right:* Exclusion regions in the $m_h - R^{-1}$ plane at 95% (dashed) and 99% (dotted) confidence level, for heavy a Higgs of $m_h \gtrsim 300\,\mathrm{GeV}$ the allowed range of R^{-1} is bounded from below and above. (from [6])

3 UED Searches at Colliders

KK parity conservation implies that odd KK states can only be pair produced. If two $n = 1$ states are within the range of the collider energy, also one $n = 2$ state can be produced.

3.1 Hadron Colliders

Figure 4 shows the discovery reach of the Tevatron and LHC. The production cross sections of coloured particles are enhanced. So if $\sqrt{s}$ is sufficient to produce pairs, then the LHC will first be sensitive to strongly interacting $N = 1$ KK states. Typical decay signatures are

- jets + $\not{E}_T$ which suffers from a large SM background.
- multi-leptons+jj+$\not{E}_T$ which results from decay chains as in figure 5 and was in the focus of the Tevatron KK searches.

The Tevatron Run IIB can exclude the range $R^{-1} < 540\,\mathrm{GeV}$. With $10\,\mathrm{fb}^{-1}$ the LHC will be able to probe the compactification scale up to $1.5\,\mathrm{TeV}$.

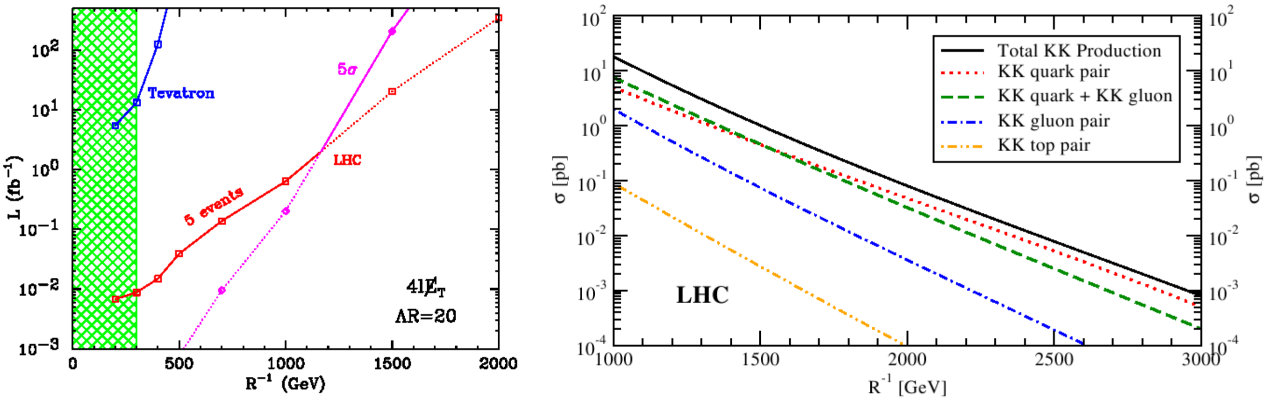

Figure 4: *left:* Discovery potential for UED depending on the compactification scale R^{-1} and the integrated luminosity per experiment (combined searches not considered here) in the $4l + \not{E}_T$ channel. 5 observed events or a 5σ excess might be a faster discovery criterion at the LHC, according to R^{-1} [5]. *right:* Pair production cross sections of coloured $n = 1$ KK modes at the LHC [1].

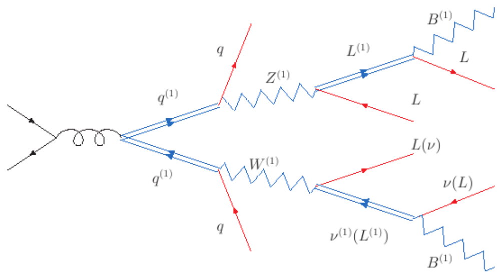

Figure 5: Quark-antiquark annihilation for KK $q^{(1)}q^{(1)}$ production both of which decay into a SM quark, 2 leptons and a LKP, leading to a $jjl^+l^+l^-l^- + \not{E}_T$ signature [1].

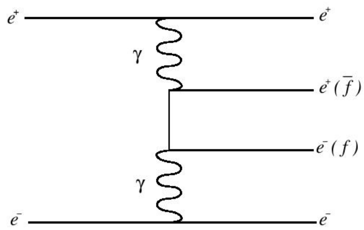

Figure 6: Background at e^+e^- colliders in the dilepton + missing E_T channel if the initial e^+ and e^- escape undetected down the beam pipe after soft initial state photon radiation. This would be avoided at an e^-e^- machine [7].

3.2 Lepton Colliders

- Despite the lower centre-of-mass energy, accurate measurements of mass, couplings and spin of KK states require a high-energy ILC with clean signals in the multilepton+$\not{E}_T$ channel, especially with the purpose to distinguish UED from other BSM scenarios as in section 4.
- A serious background to $l^+l^- + \not{E}_T$ searches at an e^+e^- are diphoton events from very soft initial state radiation (ISR) as shown in figure 6. While the 2 ISR-photons go to oppositely charged leptons, the initial electron and positron escape detection by vanishing in the beam pipe.
- This problem (especially for small mass splittings compared to R^{-1}) can be bypassed by a like-sign dilepton $+ \not{E}_T$ search at an e^-e^- collider [7] which provides the possibility to study
  1. KK electrons
  2. possibly lepton flavour violating interactions in UED.
- Handles to suppress **SM backgrounds** are e.g.
  1. an acoplanarity[1] cut as the $\gamma^*\gamma^* \to e^+e^-$ are mostly soft and coplanar with the beam axis
  2. the forward-backward asymmetry $A_{FB} := (\sigma_F - \sigma_B)/(\sigma_F + \sigma_B)$ which is 0 for the SM and depends on R^{-1} in UED while polarisation and ΛR have a minor impact on A_{FB} [8].

[1] Acoplanarity is defined as $\phi_1 - \phi_2 - \pi$ with the azimuthal angles ϕ_i of the final state particles.

$n = 2$ searches: A promising chance to distinguish UED from competing BSM scenarios would be the detection of a nearly (up to different radiative corrections, see eq. 11) repeated spectrum shifted to a higher scale, which would allude to the $n = 2$ level. The resolution of the narrow, close resonances of $\gamma^{(2)}$ and $Z^{(2)}$ requires the clean ILC. Decay modes of an $n = 2$ KK state are summarised in table 1. As long as no KK-parity violating interactions are introduced explicitely, the decay $(2) \to (1)(0)$ is forbidden. By contrast KK-number violating decays are possible, but suppressed by the boundary-to-bulk ratio of the couplings because the tree-level couplings are forbidden by discrete 5-momentum conservation. The exact radiative mass corrections decide whether a specific KK-number conserving decay channel is open.

Decay	P_{KK}	n_{KK}	phase space
$(2) \to (0)(0)$	✓	× (suppressed by boundary-to-bulk ratio)	✓
$(2) \to (1)(0)$	×	×	✓
$(2) \to (1)(1)$	✓	✓	× (suppressed or forbidden)
$(2) \to (2)(0)$	✓	✓	× (suppressed or forbidden)

Table 1: Decay channels of a $n = 2$ KK-mode. Criteria are the KK-parity P_{KK}, KK-number n_{KK} and the available phase space.

4 UED or SUSY?

Assuming the LHC finds evidence for physics beyond the SM, the crucial question will be where this new signal originates from. In fact, SUSY and UED decay chains often yield the same observed final state - just that the $\not{E}_T$ comes from a LSP ($\tilde{\chi}_0^4$) a LKP $\left(\gamma^{(1)}\right)$, as shown in the left part of figure 7.

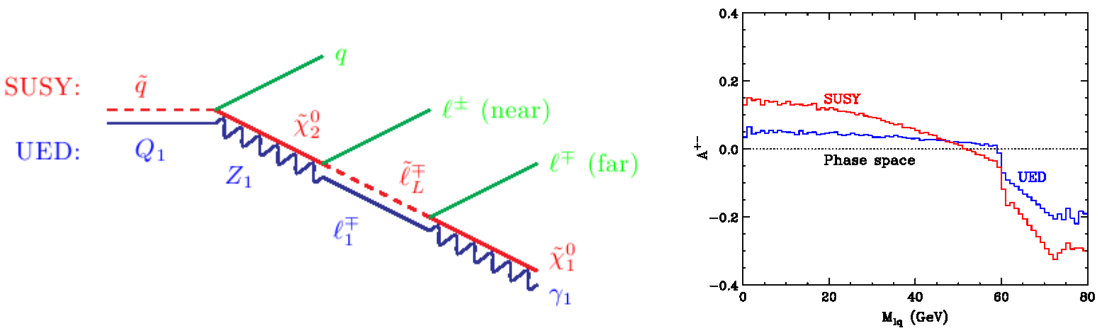

Figure 7: *left:* Cascade decay resulting in the final state $ql^+l^-\not{E}_T$ where $\not{E}_T$ belongs to a $\gamma^{(1)}$ (UED $Q^{(1)}$ decay) or $\tilde{\chi}_1^0$ (SUSY $\tilde{q}$ decay). *right:* The charge asymmetry $A^{\pm}$ is smaller in UED with $R^{-1} = 500\,\text{GeV}$ than in SUSY with a parameter choice such that the spectra correspond approximately [9].

Luckily, there are 3 distinguishing features between these two models:

1. **Spin:** KK-copies of the SM fields carry the same spin - unlike the superpartners. Hence, angular distributions of final state leptons can work as discriminating observables.
2. **Higgs sector:** The minimal UED scenario does not comprise an analogue of the heavy MSSM Higgs sector which do not carry R-parity. So the $n = 1$ UED states $A^{(1)}, H^{(1)}, H^{\pm(1)}$ rather resemble the MSSM higgsinos.
3. **Mass spectrum:** While some SUSY particles are close in mass (depending on the parameters), the KK spectrum is nearly completely mass degenerate - up to important radiative corrections. Furthermore, repeated spectra at larger masses would indicate higher KK-modes.

- In order to unravel the spin of the intermediate particle in figure 7 (left), one could analyse the invariant mass distribution of the near lepton. But since the detector does not see a difference between the near and the far lepton, invariant dilepton masses are calculated instead. Finally, the remaining charge asymmetry

$A^{\pm}$ (due to different $q/\bar{q}$ distributions at the LHC as a pp collider) helps distinguish between UED and SUSY (figure 7, right) with

$$A^{\pm} := \left(\frac{dN(ql^{+})}{dM_{ql}} - \frac{dN(ql^{-})}{dM_{ql}}\right) / \left(\frac{dN(ql^{+})}{dM_{ql}} + \frac{dN(ql^{-})}{dM_{ql}}\right) \tag{13}$$

- At a multi-TeV $e^{+}e^{-}$ collider, the differential cross section of $e^{+}e^{-} \to \mu^{+}\mu^{-}$ for the μ-scattering angle θ_{μ} looks different:

$$\left(\frac{d\sigma}{d\cos\theta_{\mu}}\right)_{UED} \propto 1+\cos^{2}\theta_{\mu} \qquad \left(\frac{d\sigma}{d\cos\theta_{\mu}}\right)_{SUSY} \propto 1-\cos^{2}\theta_{\mu} \tag{14}$$

While the MSSM contains 105 free parameters, the Minimal UED (MUED) confronts us only with 3: R^{-1}, Λ, m_h.

5 Conclusion

5.1 Summary

In Universal Extra Dimensions, all SM fields can propagate in the bulk. This feature relaxes the constraints on the compactification scale $M_c \equiv R^{-1}$ from electroweak precision measurements. Neglecting SM masses compared to R^{-1}, the spectrum is exactly degenerate for each n whereas the seizable radiative corrections are specific for the particles. While loops around the extra dimensions are finite, the boundary contributions diverge logarithmically in ΛR (with the cut-off parameter Λ).
Searches at colliders look for signatures with large missing transverse energy from the LKP which have to be distinguished from SUSY and SM backgrounds. With 10 fb^{-1} the LHC will be able to probe R^{-1} up to 1.5 TeV, but precise measurements will require the ILC.

5.2 Outlook

So far there exists only few literature about UED in more than one extra dimension. One complication in $4+D$ dimensions is the KK-level as a vector $\vec{n} = (n_1, ..., n_D)$ which leads to more combinatorics in the decay channels of higher KK-modes. The KK-parity reads then $P = (-1)^{\sum_{j=1}^{D} n_j}$. Some consequences of $D=2$ on dark matter are mentioned in [1] where also KK DM in $D=1$ is discussed in detail.

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