Orbifold GUTs

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References

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   hep-ph/0711.1582v1
   - Intro to Orbifold GUTs
4. M. Quirós – New Ideas in Symmetry Breaking
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   - SUSY and gauge symmetry breaking by orbifolding
5. Arthur Hebecker, John March-Russell (1)
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   - Concise overview of different aspects of field theory on orbifolds
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1. Introduction

- Notation in this section follows Ross.

1.1. Motivation

- SM is a remarkably successful gauge theory
  - gauge group: $G_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$
  - 3 families of matter with following content:
    $M: (3,2,\frac{1}{3}) + (\bar{3},1,4/3) + (\bar{3},1,-\frac{2}{3}) + (1,2,-1) + (1,1,-2)$
  - Vector bosons
    $V: (8,1,0) + (1,3,0) + (0,0,0)$
  - Higgs doublet
    $H: (1,2,1)$
- Questions:
  - Why is G_{SM} semi-simple and not simple?
    (Group G semi-simple if its only normal subgroups are the trivial group e and G itself. Normal subgroup N of G: $\forall g \in G$ and $n \in N$ $gng^{-1} \in N$. Product of simple groups is a semi-simple group)
  - Why is the charge quantized, or why are the hypercharges rational multiples of one another?
    $\Rightarrow$ all el. charges are rat. multiples of one another.
- Possible Answer: (Georgi, Glashow '74)
  - Underlying gauge symmetry of nature is a simple group, broken at some high energy scale to G_{SM}
  - Hypercharge is embedded in a non-abelian group factor
- Additional support:
  - In SUSY gauge couplings unify at the scale
    $M_{GUT} \simeq 2 \cdot 10^{16}$ GeV

1.2 SU(5) – The prototype GUT

- G_{SM} : 4 diagonal generators $(T_3^c, T_8^c, T_3^L, Y) \Rightarrow$ rank 4
- Each $G \supset G_{SM}$ should have at least rank 4
- SU(N) groups have rank $N-1$.
  SU(5) has rank 4 and is the unique rank 4 candidate for grand unification, since other possible groups $[SU(2)]^4$, $[O(5)]^2$, ... fail some criteria.

- SU(5) : defined by the adjoint representation: 24 5x5 complex unitary matrices with determinant equal to 1.
  Generators : $L^1, \dots, L^{24}$
- Convenient basis: $L^{1,\dots,8} = \begin{pmatrix} \lambda^a & 0 \\ 0 & 0_{2\times2} \end{pmatrix}$, λ^a: Gell-Mann Zweig matrices

$$L^{9,10} = \begin{pmatrix} 0_{3\times3} & 0 \\ 0 & \sigma_{1,2} \end{pmatrix}, \quad \sigma_i : \text{Pauli matrices}$$

<u>gauge bosons</u>: $V_\mu = V_\mu{}^a L^a$, $V^{1,\dots,8}$: gluons

$\frac{1}{\sqrt{2}}(V_\mu^9 \pm V_\mu^{10})$ are $W^\pm$ bosons

Additionally 2 further diagonal gen. of SU(5) chosen proportional to the third component of weak-isospin and hypercharge

$$L^{11} = \begin{pmatrix} 0 & & & & \\ & 0 & & & \\ & & 0 & & \\ & & & 1 & \\ & & & & -1 \end{pmatrix}, \quad L^{12} = \frac{1}{\sqrt{15}} \begin{pmatrix} -2 & & & & \\ & -2 & & & \\ & & -2 & & \\ & & & 3 & \\ & & & & 3 \end{pmatrix}$$

V_μ^{11}, V_μ^{12} are W^3 and B respectively

- 12 generators $L^{13}, \dots, L^{24}$ produce new gauge bosons <u>coupling quarks to leptons</u>

SU(5) : Matter

- 5 of SU(5) transforms as $(3,1)+(1,2)$ under $SU(3)\times SU(2)$
  - $(3,1)$ could be d_R^i or u_R^i ; $(1,2)$ are $\binom{\nu}{e}_L$
  - Therefore $\psi_5 = \begin{pmatrix} q^1 \\ q^2 \\ q^3 \\ e^c \\ -\nu^c \end{pmatrix}_R$; since photon is also a gauge boson of SU(5), charge generator Q must be identified with one of the traceless generators of SU(5)

- Q applied to ψ_5 and traceless condition requires:
  $3Q_q + Q_{e^c} + Q_{\nu^c} = 0$

$3Q_q$ since all 3 quarks in one SU(3) triplet and charge commutes with SU(3)

$\Rightarrow Q_q = -\frac{1}{3}e$, $\psi_5 = \begin{pmatrix} d^1 \\ d^2 \\ d^3 \\ e^c \\ -\nu^c \end{pmatrix}_R$; $Q = \mathrm{diag}(-\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, 1, 0)$
$\stackrel{!}{=} \frac{1}{2}(L^{11} + \sqrt{\frac{5}{3}} L^{12})$

$\Rightarrow$
- charge is quantized as expected
- quarks must carry third integer charges because they come in 3 colours (prediction of SU(5))

- Rest of the matter : Take <u>products of two 5</u>.

$5\times5 = 10 + 15$, 10 antisymm. product

$\chi^{ij} = \frac{1}{\sqrt{2}}(a^i d^j - d^j a^i)$ $i,j = 1,\dots,5$

- $i,j = 1,2,3$ a^i and a^j transform each as $(3,1)$
  $3\times3 = 6 + \bar{3}$
  $\Rightarrow$ <u>χ^{ij}, $i,j = 1,2,3$</u> transform as $(\bar{3},1)$: <u>u_L^c quarks</u>

- $i = 1,2,3$ $j = 4(5)$: colour triplet with 3rd. component of weak isospin $\frac{1}{2}$ $(-\frac{1}{2})$
  $(3,1)\times(1,2) = (3,2)$
  $\Rightarrow$ <u>$\chi^{i4} = (u^i)_L$, $\chi^{i5} = (d^i)_L$</u>

- χ^{45} is singlet under $SU(3)\times SU(2)$: e^+_L

- $$\chi = \begin{pmatrix} 0 & u^c_3 & -u^c_2 & -u^1 & -d^1 \\ -u^c_3 & 0 & u^c_1 & -u^2 & -d^2 \\ -u^c_2 & -u^c_1 & 0 & -u^3 & -d^3 \\ u^1 & u^2 & u^3 & 0 & -e^+ \\ d^1 & d^2 & d^3 & e^+ & 0 \end{pmatrix}_L = (3,2)+(\bar{3},1)+(1,1)$$

- The combination $(\bar{5}+10)$ is anomaly free

- <u>$SU(5)$: Higgs</u>

$$H = \begin{pmatrix} h^1 \\ h^2 \\ h^3 \\ h^+ \\ -h^0 \end{pmatrix} \quad \text{in } 5 = (3,1)+(1,2)$$

- Assuming: $V(H) = -\frac{1}{2}v^2|H|^2 + \frac{1}{4}\lambda(|H|^2)^2$, $v^2, \lambda > 0$

$\langle (H)^5 \rangle = \langle -h^0 \rangle = v_0 \quad \curvearrowright$ EWSB

BUT : h^i triplet acquires the same mass as SM Higgs!
They have baryon number violating couplings to quarks and leptons and will mediate proton decay far too fast if they are so light.

- <u>Doublet-triplet splitting problem 1</u> (can be solved)

- Additional gauge bosons of $SU(5)$ mediate proton decay by effective dim 6 operators.

- <u>Supersymmetric $SU(5)$</u>

- Renormalizable superpotential

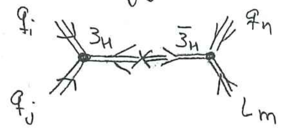
$$W_{GUT} \supset -(y_d)_I\, \psi^{Ii}\, \chi_{ij}\, H_d^j + \mu H_u H_d$$
$$-(y_u)_{IJ}\, \varepsilon^{ijklm}\, \chi^I_{ij}\, \chi^J_{kl}\, H_u$$

$\Rightarrow W_{GUT} \supset Y_d H_d^T D^c U^c + Y_d H_d^T L Q + Y_u H_u^T U^c E^c$
$+ Y_u H_u^T Q Q + M_H H_u^T H_d^T$

$\Rightarrow$ Integrate out heavy triplet Higgs

<u>In SUSY case $\Rightarrow$ Effective dim 5 operators</u> $\frac{LQQQ}{M_H}$, $\frac{E^c UUD}{M_H}$

'scalar - fermion - scalar - fermion' interactions via Higgsino exchange mediating <u>proton decay</u>



- Doublet - triplet splitting problem II

$\Rightarrow$ - Challenge: "Why does matter appear in complete GUT representations while the Higgs multiplets are incomplete (split)?"

$\Rightarrow$ Most appealing solution to this problem is obtained in higher dimensional extensions of the SM.

$\rightarrow$ <u>Orbifold GUTs</u>

2. Extra dimensions and symmetry breaking

2.1. Compactification

- $D = 4 + d$ dim. theory with action

$$S_D = \int d^D z \, \mathcal{L}_D [\phi(z)]$$

$z^M = (x^\mu, y^m)$, $(\mu = 0,1,2,3;\ m = 1, \dots, d)$

- Assume theory is defined on the space $\mathbb{R}^4 \times \mathcal{C}$, where $\mathcal{C}$ is a smooth compact space.
- Theory has a gauge symmetry with gauge group G.
- Compactification to effective 4d theory

$$\mathcal{L}_4 = \int d^d y \, \mathcal{L}_D [\phi(x^\mu, y^m)]$$

If the energy of the interactions on the scale on which we work
$E \ll l_c^{-1}$; l_c : typical size of $\mathcal{C}$
heavy fields can be integrated out
⇒ Effective 4d theory of massless fields with higher-dim. operators.

* It is in general possible to write

$$\mathcal{C} = M/K$$

M: covering space (non-compact manifold) ; K: discrete group acting freely on M
$K: M \to M;\ y \mapsto K[y]$

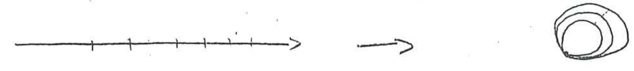
K acts freely: only operator corresponding to identity leaves M invariant. $y' = K[y] = y$

$\mathcal{C}$ is constructed by identification of points $y \sim K[y]$

- Example: real line $\mathbb{R}$; identify $x \sim x'$, $x' = x + 2\pi n R$, $n \in \mathbb{Z}$

⇒ $\mathbb{R}/\mathbb{Z} = S^1$: circle with radius R and lengths $2\pi R$

- fundamental domain: $(-\pi R, \pi R]$ no double counting.



* What should fields on M obey?

- Physics should depend only on $\mathcal{C}$ (orbits of K) and not on points in M.

$$\Rightarrow \mathcal{L}_D [\phi(x,y)] = \mathcal{L}_D [\phi(x, K[y])]$$

- sufficient condition $\phi(x, K[y]) = \phi(x,y)$ "untwisted fields"
- necessary and sufficient $\Phi(x, K[y]) = T_g \phi(x,y)$ "twisted fields"

T_g belongs to a global symmetry group of the theory!
T_g are representations of K acting on field-space.

⇒
- Allowed field configurations on M are restricted.
- Group K should be symmetry of M and of the theory.
- Fields are functions on the covering space M.
- In untwisted case fields are also functions on $\mathcal{C}$.
- In twisted case fields are not single-valued functions on $\mathcal{C}$.

2.2 The meaning of "modding out"

- QFT with gauge group G defined on $\mathbb{R}^4 \times M$
- Suppose M and QFT possess a symmetry under the discrete group K
- Action of K:
  1) Geometrical action on M
     $K: (x,y) \mapsto (x, K[y])$
  2) Action in field space
     $K: \Phi_i \to (T_g)_{ij} \Phi_j \quad , g \in K$
     Φ_j: vector of all fields $\qquad$ T_g: matrix rep. of K.
- "Modding out": only field configurations invariant under combined action of 1) & 2) are physical.
  (Not a construction of equivalent classes)

A) $K: (x,y) \to (x, K[y])$ $\quad$ acts freely in space
$K: \Phi_i \to \Phi_i$ $\quad$ trivial action in field space

$\Rightarrow$ $C = M/K$ is smooth

$\Rightarrow$ Gauge symmetry G of the theory the same, theory defined simple on smaller physical space C.
"untwisted case" in previous section

B) $K: (x,y) \to (x,y)$
$K: \Phi_i \to (T_g)_{ij} \Phi_j$

$\Rightarrow$ Field configurations should be invariant under this transformations in every point in space

$\Rightarrow$ Same physical space M

$\Rightarrow$ Reduced gauge symmetry ! $\quad$ Only centralizer $N_K(G)$ in G unbroken.
later in more detail. $\quad$ $N_K(G) = \{ g \in G \, ; gk = kg \}$

$\Rightarrow$. Theory on M with smaller gauge group $\mathcal{H} = N_K(G)$.

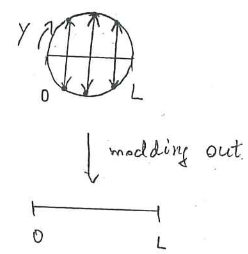
- A+B) "Twisted case" of section 2.1.
  $\Rightarrow$ QFT with full gauge group G on C.
  $\Rightarrow$ However possible conf. of fields on M are restricted.

3. Orbifolds

- UP TO NOW K ACTED FREELY BUT WHAT IF
  $K[y] = y$ for K being not the trivial action?
  $\Rightarrow$ $\mathcal{C} = M/K$ is not a manifold but orbifold
  "orbit-manifold"
  $\Rightarrow$ Fixed points y_i, $K[y_i] = y_i$ are singular points of the orbifold.

Example: $M = S^1$ and $K = \mathbb{Z}_2$

$K: (y) \to (-y)$



"Mod out" in space $\quad$ $y \sim y' ; y' = -y$

- $\left.\begin{array}{l} y = 0 \; ; \; K[0] = 0 \\ y = L \; ; \; K[L] = L \end{array}\right\}$ fixed points.

- Field space

$\Phi_i(x,y) = (T_g)_{ij} \Phi_j (x, K[y]) = (T_g)_{ij} \Phi_j (x, -y) \quad y \neq 0, L$

$\Phi_i(x,0) = (T_g)_{ij} \Phi_j (x,0)$

$\Phi_i(x,L) = (T_g)_{ij} \Phi_j (x,L)$

Φ is restricted on lower half-circle by its values on the upper half circle.

If. for example for some i,j $(T_g)_{ij} = -1 \cdot \delta_{ij}$
Field Φ_i has to vanish on the boundaries.

3.1. Example 1

Orbifold $S^1/\mathbb{Z}_2$ with a theory with gauge group $SU(3)$, here non-SUSY

- $K = \mathbb{Z}_2$ as a subgroup of $SU(3)$; $(T_g)_{ij}^2 = \mathbb{1}$ $\Rightarrow$ eigenvalues are ± 1.
- Choose $(T_g) = P = \begin{pmatrix} -1 & & \\ & -1 & \\ & & 1 \end{pmatrix}$ in the fundamental representation
- Scalar bulk fields transforming as 3 of $SU(3)$

$$\phi(x,-y) = P\phi(x,y)$$

$\Rightarrow$ boundary condition

- $\phi(x,0) = P\phi(x,0)$ on the interval
- $\phi(x,L) = P\phi(x,L)$

- Work with eigenstates of P. Field ϕ decomposes into two pieces, eigenstates of P with eigenvalues $+1$ and -1, denoted $\phi_\pm$ respectively.

$$\Rightarrow \quad \underline{\phi_\pm(x,-y) = P\phi_\pm(x,y) = \pm\phi(x,y)}$$

- 5 dimension is compact: Expand eigenstates in terms of Fourier modes
  - $\phi_+ = \sum_{n=0}^{\infty} \frac{1}{\sqrt{2^{\delta_{n0}}\pi R}} \phi_+^{(n)}(x) \cos\left(\frac{ny}{R}\right)$
  - $\phi_- = \sum_{n=1}^{\infty} \frac{1}{\sqrt{\pi R}} \phi_-^{(n)}(x) \sin\left(\frac{ny}{R}\right)$

  R: radius S^1, $L = R\pi$.

- In effective 4d theory: Fourier modes are massive states with masses $\frac{n}{R}$.
- Only ϕ_+ possesses a zero-mode. $\Rightarrow$ At energies below $1/R$ EFT with only ϕ_+ zero-modes as dynamical degrees of freedom.

- Gauge fields: K acts via adjoint action on the generators of G: $(K \subset G)$ $T^a \to K T^a K^{-1}$.

  P: K in fundamental reps.

$$\underline{A^a_\mu(x,-y)T^a = A^a_\mu(x,y) P T_a P^{-1}}$$

- This condition ensures the invariance of the action.

  $S_D[A_M] = S_D[A'_M]$, $A'_\mu = A_\mu(x, K[y]) P T P^{-1}$

  as can be seen from the covariant derivative.

  $D_\mu = \partial_\mu - ig \underline{A^a_M T^a}$; (A^a_5 gets an additional − sign)

- Only the gauge bosons of an $SU(2) \times U(1)$ subgroup of $SU(3)$ have zero-modes in 4D. (only matrices λ_i with $\underline{i = 1,2,3,8}$ $\underline{\text{commute with } P}$)
- $SU(2) \times U(1)$ is centralizer $N_{\mathbb{Z}_2}(SU(3))$ in $SU(3)$.

$\Rightarrow$ Aspects of orbifold compactifications

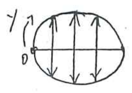
- Gauge symmetry gets reduced (in 4d EFT)
- Bulk fields furnishing representations under the (larger) bulk gauge symmetry survive the projection conditions only partially.

SAME MECHANISM THAT BREAKS GAUGE SYMMETRY LEADS TO APPEARANCE OF SPLIT MULTIPLETS

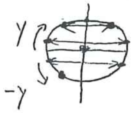
3.2. Example 2

$S^1/_{\mathbb{Z}_2 \times \mathbb{Z}_2'}$ – Orbifold

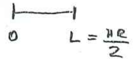
• P: $y \sim y'$, $y' = -y$



• P': $y \sim y'$, $y' = -y - \frac{\pi R}{2}$



After bothidentifications:



• Second equivalence introduces no new fixed points

- All bulk fields Φ have to obey

$$P: \phi(x,y) = P\phi(x,-y)$$
$$P': \phi(x,y) = P\phi(x,-y-\tfrac{\pi R}{2})$$

• $y=0$ is fixed point under P

$y \sim L = \frac{\pi R}{2}$ is fixedpoint under P'

⇒ Fields have to fulfill different boundary conditions at both fixed points.

• At $y=0$ for bulk fields:

$$A^a_\mu(x,0)T_a = A^a_\mu(x,0)PT_aP^{-1}$$
$$\phi(x,0) = P\phi(x,0)$$

$y = L$

$$A^a_\mu(x,L)T_a = A^a_\mu(x,L)P'T_aP'^{-1}$$
$$\phi(x,L) = P'\phi(x,L)$$

• Denote eigenstates of P and P' by $\phi_{\pm\pm}$, expand these in terms of Fourier eigenstates.

$$\phi_{++}(x,y) = \sum_{n=0}^{\infty} \frac{1}{\sqrt{2^{\delta_{n,0}}\pi R}} \phi_{++}^{(2n)}(x)\cos\left(\frac{2ny}{R}\right)$$

$$\phi_{+-}(x,y) = \sum_{n=0}^{\infty} \frac{1}{\sqrt{\pi R}} \phi_{+-}^{(2n+1)}(x)\cos\left(\frac{(2n+1)y}{R}\right)$$

$$\phi_{-+}(x,y) = \sum_{n=0}^{\infty} \frac{1}{\sqrt{\pi R}} \phi_{-+}^{(2n+1)}(x)\sin\left(\frac{(2n+1)y}{R}\right)$$

$$\phi_{--}(x,y) = \sum_{n=0}^{\infty} \frac{1}{\sqrt{\pi R}} \phi_{--}^{(2n+2)}(x)\sin\left(\frac{(2n+2)y}{R}\right)$$

• The only state leading to a 4D zero-mode is ϕ_{++}.

• <u>4D ZERO-MODES ARE MODES THAT SURVIVE ALL PROJECTION CONDITIONS SIMULTANEOUSLY!</u>

3.2.1 SUSY $SU(5)$ on $S^1/_{\mathbb{Z}_2 \times \mathbb{Z}_2'}$

: Minimal SUSY in 5D: <u>8 real supercharges</u>

- This follows from the dimension of spinor repr.

$n = 2^{(d-1)/2}$ if d odd

⇒ $n = 4$ for 5 dimensions. The representation is not reducible
⇒ 4 complex = 8 real degrees of freedom.

• $N=2$ SUSY in 4d has also 8 supercharges.

• $N=1$ SUSY in 5d appears as $N=2$ SUSY in 4d.

• Consider a bulk hypermultiplet in 5 of $SU(5)$.

→ Hypermultiplet of $N=2$ SUSY consists of two chiral multiplets of $N=1$ SUSY in conjugated representations of the gauge group.

• $\mathcal{H} = (\overset{5}{\Sigma_i^1}, \overset{\bar{5}}{\Sigma_i^2})$

• $5_{hyper} \to [(3,1) \oplus (\bar{3},1) \oplus (1,2)_1 \oplus (1,2)_{-1}]_{chiral}$

- Choose $P = \mathrm{diag}(1,1,1,1,1)$, $P' = \mathrm{diag}(-1,-1,-1,1,1)$
- P' breaks $SU(5)$ to $SU(3) \times SU(2) \times U(1)$ at $y = L$,
- P' is allowed since $-P'$ acts the same way on the adjoint representation
- Transformations of Σ^1 and Σ^2 differ by an overall sign

$$\Sigma'^1(x,0) = P\Sigma^1(x,0)$$
$$\Sigma^1(x,L) = P'\Sigma^1(x,L)$$
$$\Sigma^2(x,0) = -P\Sigma^2(x,0)$$
$$\Sigma_1'^2(x,L) = -P\Sigma^2(x,L)$$

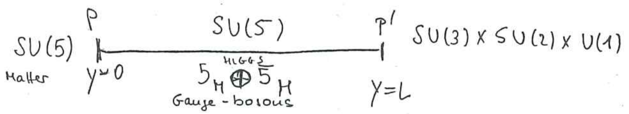
$\Rightarrow$ Only the doublet $(1,2)_1$ has a zero-mode

(P,P')	4d superfield	4d mass
$+\,+$	$\Sigma'^1_{1\,\mathrm{DOUBLET}}$	$\frac{2n}{R}$
$+\,-$	$\Sigma'^1_{1\,\mathrm{TRIPLETT}}$	$\frac{2n+1}{R}$
$-\,+$	$\Sigma'^2_{1\,\mathrm{TRIPLET}}$	$\frac{2n+1}{R}$
$-\,-$	$\Sigma'^2_{1\,\mathrm{DOUBLET}}$	$\frac{2n+2}{R}$

$\Rightarrow$ • The same mechanism that is used for gauge symmetry breaking leads to doublet-triplett splitting

- 4d $N=2$ SUSY is broken to 4d $N=1$ SUSY on both branes fixedpoints.

$\Rightarrow$ Higher-dimensional models offer an intuitive explanation of the presence of complete and split multiplets at the same time



Since SM matter can be confined to live at $y=0$ in the extra dimension and it appears in complete representation of local $SU(5)$.

- SM matter can be confined to live only at $y=0$ in the extra dimension; it lives on a brane and fields confined to $y=0$ do not care for what's going on at $y=L$.

- In this example mass partners of the triplets do not couple to SM matter, and therefore the triplet exchange diagram does not exist.

- It is possible to confine Higgs to G_{SM} brane at $y=L$. Here it is not necessary to introduce the triplet at all and therefore there is no triplet exchange.

$\Rightarrow$ However SM matter cannot live on $SU(5)$ brane, since then it would not be able to interact with electro-weak Higgs.

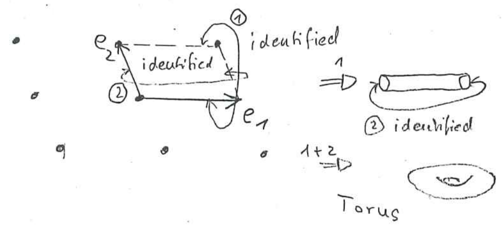
$\Rightarrow$ This ruins the intuitive explanation of SM matter in terms of complete $SU(5)$ representations.

- Summary
  - Fields in the bulk : 4d zero-modes should survive all projection conditions simultaneously
  - Fields at branes : Appear in complete representations of the local group.

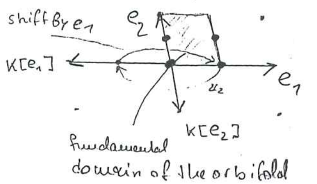
3.3. Higher-dimensional orbifol[illegible]

- Important example $\mathbb{T}^2/\mathbb{Z}_2$

- Torus $\mathbb{T}^2$ defined by 2 linear**y** independent lattice vectors e_1 and e_2 spanning the fundamental domain
- Points differing by lattice vectors are identified.



- The $\mathbb{Z}_2$ acts as reflection about an arbitrary lattice node.
- Certain points are mapped under the orbifold action onto themselves (up to lattice translations); these are orbifold fixed points.
- Identify points in the fundamental domain of the torus related by $\mathbb{Z}_2$ $\Rightarrow$ fundamental domain of the orbifold.



fix points shown as bold dots

K: action of $\mathbb{Z}_2$

- Fold the fundamental domain along the line connecting the upper two fixed points and glue adjacent edges together
$\Rightarrow$ "Pillow" or "Ravioli" object smooth everywhere except for the corners.

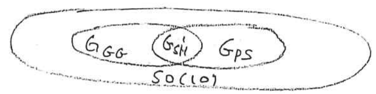
bold dots are orbifold fix-points

3.3.1. A[illegible]ka – Buchmüller – Covi model

- SO(10) in the bulk
- Orbifold parities chosen such that SO(10) is broken to important subgroups, whose phenomenology has been studied in great detail:
  - Pati-Salam $G_{PS} = SU(4) \times SU(2)_L \times SU(2)_R$
  - Georgi-Glashow $G_{GG} = SU(5) \times U(1)$
  - Flipped SU(5) $G_{Fl} = SU(5) \times U(1)$

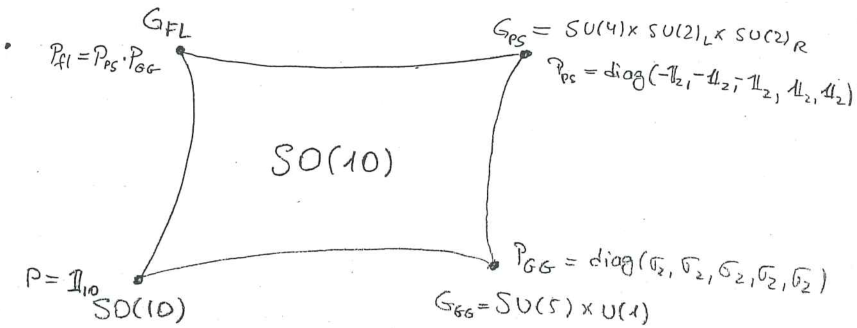
at three of the four fixed points while the gauge embedding is trivial at the fourth fixed point.

- Remarkable (group-theoretic) fact:
  Intersection of G_{GG} and G_{PS} in SO(10) yields the standard model G_{SM} up to U(1).



$G_{SM}' = SU(3) \times SU(2) \times U(1)^2$

-



- One generation of matter at each of G_{PS}, G_{Fl}, G_{GG} fixed points.
- Those localized fields have to furnish complete reps. of local gauge groups.
- Higgs from the bulk 10-plets – <u>only electroweak doublet survives.</u>

- Results:
  - 'non-trivial gauge group topography'
    bulk group gets broken to different 'local groups' at different fixed points
  - Low energy gauge group = Intersection of local groups in the bulk gauge group
  - Localized matter comes in complete reps. of the local gauge groups
  - Bulk fields get split, i.e. partially projected out.

- Still more questions:
  - Why SO(10), i.e why matter generations fit into 16-plets of SO(10)?
  - What is the rule behind the field content?
  - What is the theory behind the field couplings?
  - Here EFT → need UV completion
  - What about gravity?

⇝ <u>String derived orbifolds</u> ...

<u>Final remarks</u>

- <u>Gauge breaking by orbifolding</u>

If the discrete group K is to be a symmetry of the gauge action, then in general it acts as a linear transformation on the Lie algebra:
$A^a(x,y)T^a \to A^a(x,K[y])M^{ab}T^b$ that preserves the structure constants:

$$M^{ad}M^{be}f^{def} = f^{abc}M^{cf} \quad \text{(from invariance of the action)}$$

⇒ Action of K is an automorphism of the Lie-algebra.

- There are 2 classes of automorphisms: inner, outer
  - Inner: can always be written as $T^a \to gT^ag^{-1}$ (group conj.)
    - They were considered up to now. <u>Rank preserving</u>
  - Outer: cannot be written this way.
    <u>Do not preserve the rank</u>

⇒ In both cases allowed breaking patterns are constrained. (Quiros, Hebecker, Russel 1)

- There is a relation between orbifold breaking and Wilson-line breaking of gauge symmetries. (Hebecker Russell 1)