# Holographic Composite Higgs

12.7.11



In the context of extra dimensional models, the higgs particle can be embedeed in the degrees of freedom associated to gauge bosons. This mechanism is exploited by Gauge-Higgs-Unification models in $AdS_5$. In these models the higgs particle arises as a Pseudo-Goldstone-Boson from the spontaneous symmetry breaking of the Bulk symmetry, and EWSB is generated radiatively. The same features are also present in 4D composite higgs models, where the global symmetry that gives rise to the higgs particle is broken from strong dynamics. Then it is natural to connect these two approaches using the AdS/CFT duality. This kind of methodology defines the concept of "Holographic composite Higgs".

## Outline

- Big picture summarizing the three different approaches to obtain a 4D phenomenological theory from a 5D start.
- Slice of $AdS_5$ in contrast to the full AdS space
- 5D Gauge-Higgs-Unification (GHU) model
  - A picture of the model
  - The 5D action and boundary conditions
  - Gauge Fixing and the spontaneous arising of the higgs field
- KK reduction of the 5D GHU model
- Holographic approach to the effective 4D action
- GHU and the AdS/CFT duality to composite higgs models
- Conclusions and References

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## From a 5D theory to the phenomenological 4D theory

Duality (if exist)

theory on $AdS_5$ $\longleftarrow - - - - - - - - - \longrightarrow$ CFT in 4D

K-K reduction

4D KK theory SM as the zero modes

4D theory in terms of boundary fields

Dictionary

4D theory in terms of strongly coupled fields plus "gravity" source fields

## Slide of $AdS_5$

In Martin's talk we have seen that there exist a duality between two theories in different dimensions

$$\text{SUGRA on } AdS_5 \times S_5 \xleftrightarrow{\text{Dual}} \mathcal{N}=4 \text{ SYM in 4D}$$

with a useful dictionary which translates computations from one side to each other.

$$e^{S_g[\phi]} = \langle e^{\int \phi_0 \mathcal{O}} \rangle_{CFT}$$

- $\langle \ \rangle_{CFT}$ : average using CFT partition function
- $\mathcal{O}$ : CFT operator
- $\phi_0$ : boundary field, defined as $\phi_0(x) = \phi(x, z_{boundary})$
- $\phi(x,z)$ : 5D gravity field

with this dictionary we can compute CFT correlation functions in terms of functional derivatives in the gravity sector.

$$\frac{\delta [e^{S_g[\phi_0]}]}{\delta\phi_0^1 \delta\phi_0^2 \cdots \delta\phi_0^n} = \langle \mathcal{O}^1 \cdots \mathcal{O}^n \rangle$$

- In the previous case the 5D theory is motivated as a low energy realization of ST. However, is well known that the kind of 5D models that have been useful to reproduce 4D effective theories related to the SM and/or Higgs physics are slightly different to the SUGRA models in 5D.

- One of the big differences corresponds to the AdS background in each case. In the SUGRA case the extra dimension is extended, while in the SM like 5D models this extra dimension is compactified. In the SUGRA case the background geometry arises from N-p branes and in the SM $AdS_5$ we include only two. Then will be useful to clarify some aspects before going further on the develop of the "Holographic composite Higgs" model

$\boxed{\eta_{\mu\nu} = (1,-1,-1,-1)}$

SUGRA $AdS_5$

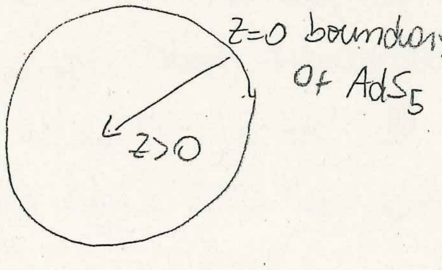
SM $AdS_5$

$$ds^2 = \frac{L^2}{z^2}\{\eta_{\mu\nu}dx^\mu dx^\nu - dz^2\}$$

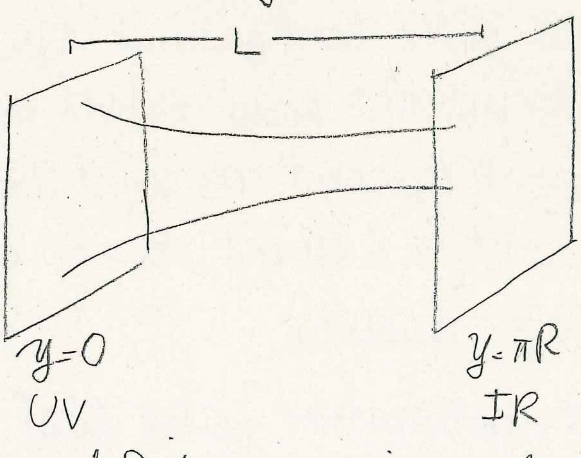
$$ds^2 = e^{-2ky}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2 \} \text{Randall-Sundrum}$$

$0 < z < \infty$

$0 \leq y \leq L$




Euclidean compactification of extended $AdS_5$




usual Picture of Slice of $AdS_5$

- It is convenient to write the SM-$AdS_5$ Metric in terms of "z" variable for this we make the change of variables

identifying $k = \frac{1}{L}$

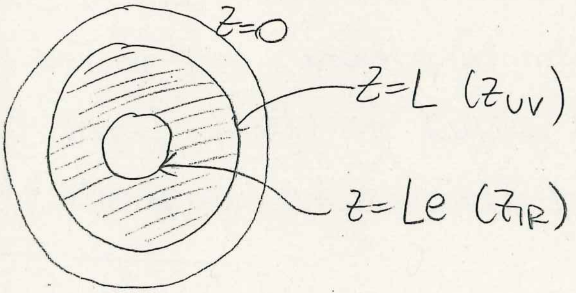
$$\frac{L}{z} = e^{-ky} \quad \Rightarrow ds^2 = e^{-2ky}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2$$

$$\Rightarrow dy = \frac{-dz}{zk} \qquad = \frac{L^2}{z^2}\eta_{\mu\nu}dx^\mu dx^\nu - \frac{dz^2}{z^2k^2} \qquad ds^2 = \frac{L^2}{z^2}\left[\eta_{\mu\nu}dx^\mu dx^\nu - dz^2\right]$$

From the previous identification we can try to localize the branes of the SM-$AdS_5$ in the language of SUGRA-$AdS_5$

$$UV: \; y=0 \;\Rightarrow\; z = Le^{ky} = L$$
$$IR: \; y=L \;\Rightarrow\; z = Le^{kL} -$$
$$z = Le \qquad \boxed{z_{IR} > z_{UV}}$$

Then, a picture of the 5D SM background geometry will be.




Euclidean Compactification of SM-$AdS_5$

From now on we are going to consider the metric with 5D "z" axis and the location of branes at $z=z_{UV}$ and $z=z_{IR}$ correspondingly.

- As we can see in the SM-$AdS_5$ we have not included the $z=0$ boundary. Indeed the $z_{UV}$ brane is the most closer brane to this boundary. This difference in the background AdS space in comparison to SUGRA case, plus the different field content of the 5D model make difficult the search for the correct 4D dual theory of the 5D-SM. This will be, if this exist, a different version of the original CFT side of the AdS/CFT duality.

- The correct identification of the dual CFT theory will be important to finally identify the "Holographic Composite Higgs Model". However we will show that some develop of the theory only considering the 5D SM action, KK reductions, and the Holographic principle can be done without the duality and much of the results will be useful after stablishing the possible correspondence.

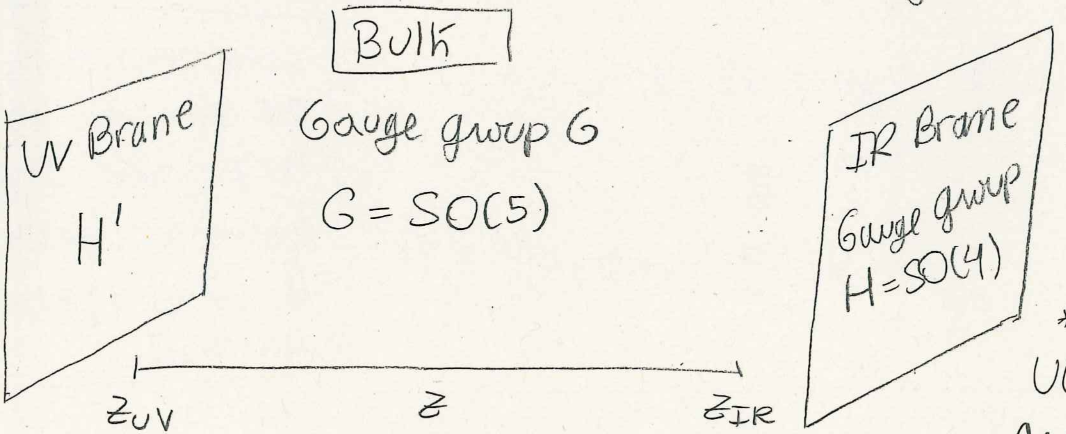
# 5D-Gauge Higgs Unification Model

hep-ph/0610336
hep-th/0703287

- We have stablished the geometrical background of our 5D Theory. Now we have to incorporate the field content of the theory. In particular for this talk we are interesting in some field content which includes the physics of EWSB and/or the Higgs. The model that we are going to consider now, and for the rest of the talk, is 5D-GHU Model, which in principle does not contain neither composite or the Holographic works in the title, but we will see that these concepts will arise naturally in this context.

## Picture of The Model

Fig 1.




$SO(4) \cong SU(2)_L \times SU(2)_R$

$\downarrow$ SM Group

$\downarrow$ Custodial Group

* Also we should add a $U(1)$ factor to each group, but it is not relevant here.

The Bulk symmetry is breaking in the IR brane from boundary conditions of the fields. As the symmetry is "spontaneously" broken the arises of Goldstone bosons is expected.

$\#$ generators $\in SO(5) = 10$, $t^A : A = 1 \ldots 10$

$\#$ generators $\in SO(4) = 6$, $t^a : a = 1, .., 6$

$\#$ generators $\in \frac{SO(5)}{SO(4)} = 4$, $t^{\hat{a}} : \hat{a} = 1, .., 4$

The coset $SO(5)/SO(4)$ is not a group because $[t^{\hat{a}}, t^{\hat{b}}] = f^{\hat{a}\hat{b}c} t^c$

On the other hand $SO(4)$ is by definition a group, thus $[t^a, t^b] = f^{abc} t^c$

- As the symmetry is broken in the direction of the $t^{\hat{a}'}$'s generators we can identify these 4 degrees of freedom with the complex Higgs doublet.

- The group on the UV brane has to be specified, but in principle should not be relevant for our discussion, thus let me take it generically as $H'$

- 5D Action & Boundary conditions

(we will consider only the gauge sector. Fermions are extraordinarily relevant for realistic models)

$$S_{5D} = \frac{1}{2g_5^2}\int d^4x \int_{z_{UV}}^{z_{IR}} dz \sqrt{g}\, g^{MO} g^{NP} \left\{ -\mathrm{Tr}\left\{ F_{MN} F_{OP} \right\} \right\}$$

$$F_{MN} = F^A_{MN} T^A = \partial_M A^A_N - \partial_N A^A_M + i g_5 [A^B_M T^B, A^C_N T^C]$$

$$g_{MN} = \frac{L^2}{z^2}\eta_{MN} \quad \text{with} \quad \eta_{MN} = (1,-1,-1,-1)$$

$$\equiv a^2(z)\eta_{MN} \qquad a(z) = \frac{L}{z}$$

$$g^{MN} = a^{-2}(z)\eta^{MN} \qquad \sqrt{g}\, g^{MO} g^{NP} = a(z)\eta^{MO}\eta^{NP}$$

Sometimes $A_z^{\hat{a}'}$'s are identified as the "possible" Higgs degrees of freedom. We use a different approach

$$\Rightarrow \boxed{S_{5D} = \frac{1}{2g_5^2}\int d^4x \int_{z_{UV}}^{z_{IR}} dz\, a(z) \left\{ -\mathrm{Tr}\left\{ F_{MN} F^{MN} \right\} \right\}} \quad (1)$$

$$S_{5D} = \frac{1}{2g_5^2}\int d^4x \int_{z_{UV}}^{z_{IR}} dz\, a(z) \left\{ -\mathrm{Tr}\left\{ F_{\mu\nu} F^{\mu\nu} \right\} + 2\,\mathrm{Tr}\left\{ F_{\mu z} F^{\mu z} \right\} \right\}$$

$$\delta S = \frac{2}{g_5^2}\int d^4x \int_{z_{UV}}^{z_{IR}} dz\, \mathrm{Tr}\left\{ \delta A^\mu \left( a D^\nu F_{\nu\mu} + D_z (a F_{\mu z}) \right) - \delta A_z \left( a D^\mu F_{\mu z} \right) \right\}$$

$$- \frac{2}{g_5^2}\mathrm{Tr}\left[ a F_{\mu z} \delta A^\mu \right]_{z_{IR}} + \frac{2}{g_5^2}\mathrm{Tr}\left[ a F_{\mu z} \delta A^\mu \right]_{z_{UV}}$$

where $D_M \equiv \partial_M - i[A_M, \cdot]$. To cancel the boundary variations and obtain the non-Abelian Maxwell equations in the Bulk two possibilities are given. At each boundary and for each component $A^A$ of the gauge field either we take Dirichlet ($A_\mu^A = 0$) or Neumann ($F_{\mu z}^A = 0$) boundary conditions

↳ The first choice induces a breaking of the transformations generated by $t^A$, while the symmetry is unbroken in the second case. (This is because the boundary conditions $A_\mu^A = 0$ determine if the zero mode gauge field is present or not)

According to the picture given in Fig 1. The corresponding BC are given by

$$(A)_\mu^{\hat{a}}(x, z_{IR}) = 0 \qquad (F)_{\mu z}^{a}(x, z_{IR}) = 0$$

$(A)_\mu^A(x, z_{UV}) = (B)_\mu^A(x) \to$ generic conditions

$(B)_\mu^A$ will play an important role in the Holographic description

(we are considering the $A_z^A$ components explicitly because in some step we are going to take the gauge $A_z^A = 0$)

It is useful to clarify the meaning of the notation $(A)_M^A$. It means that we are taken the component "A" of the vector $A_\mu \equiv A_\mu^A T^A$. In the practice this can be done using that $\delta^{AB} = 2\,Tr\{T^A, T^B\}$

$$\Rightarrow (A)_M^A = 2\,Tr(A_M t^A) = 2\,Tr(A_M^B t^B t^A)$$

① This will be essential for the next discussion about the gauge fixing

Before passing to the possible approaches to find a 4D equivalent of this 5D theory we are going to reduce the number of free components of the field $A_M^A$ by choosing the gauge $A_z^A = 0$. This will become the next computations more clean. Although the formal gauge fixing is a bit odd.

In order to fix $A_z^g = 0$ we need to use the gauge transformations defined by the Wilson line

$$g(x,z) = \exp\left[-i\int_{z_{UV}}^{z} dz'\, A_z^A(x,z')\, t^A\right]$$

usual gauge transformation of a field in the adjoint rep.

It is direct to see that $A_z^g = g(x,z)\, A_z^A t^A g^\dagger(x,z) + \frac{i}{g_5} g(x,z)\, \partial_z g^\dagger(x,z)$
$= 0$

So, in principle we would apply directly the "g" gauge transformation to the action ①. However we have to take care of the BC first. Given that the IR boundary conditions only consider a subset of the $A^A$ components the previous gauge fixing does not "conserve" the structure of these BC, let say

$$A_\mu^{\hat{a}} = 0 \xrightarrow{\;g\;} (A^g)_\mu^{\hat{a}} \neq 0$$

because in general the transformation $g$ mix all the $A^A$ components

and something similar occur for $(F)_{\mu z}^a = 0$.

To solve this inconvenient we need to introduce the field $\Sigma(x)$ defined as

$$\Sigma(x) = \exp\left[\sigma_{\hat{a}}(x)\, t^{\hat{a}}(x)\right]$$

which is an element of the G group ("The Higgs field")

Thus we define a new set of "rotated" boundary conditions in the IR

$$\left(A^{\Sigma^{-1}}\right)_\mu^{\hat{a}} = 0 \quad \left(F^{\Sigma^{-1}}\right)_{\mu z}^{a} = 0 \quad \Big|_{z=z_{IR}}$$

It can be shown, after some computations using the property ② that these boundary conditions satisfy:

$$\left(A^{\Sigma^{-1}}\right)_\mu^{\hat{a}} = 0 \xrightarrow{\;g\;} \left[\left(A^{\Sigma^{-1}}\right)^g\right]_\mu^{\hat{a}} = 0$$

$$\left(F^{\Sigma^{-1}}\right)_{\mu z}^{a} = 0 \xrightarrow{\;g\;} \left[\left(F^{\Sigma^{-1}}\right)^g\right]_{\mu z}^{a} = 0$$

This means that, even after the $g$ transformation the Gauge group is broken in the same $\hat{a}$ directions.

After this rotation of the boundary conditions and the application of the $g(x,z)$ transformation we get the next system:

$$S^g_{5D} = \frac{1}{2g_5^2}\int d^4x \int_{z_{UV}}^{z_{IR}} dz\, a(z)\left[-\mathrm{Tr}\left[F^g_{\mu\nu}F^{\mu\nu g}\right] + 2\,\mathrm{Tr}\left[\partial_z A^g_\mu \partial_z A^{\mu g}\right]\right]$$

+ Boundary conditions

$$\left[(A^{\Sigma^{-1}})^g\right]^{\hat{a}}_\mu = 0\,, \quad \left[(F^{\Sigma^{-1}})^g\right]^a_{\mu z} = 0 \quad \Big|_{z=z_{IR}}$$

$$(A^g)^A_\mu = B^A_\mu(x)\Big|_{z_{UV}}$$

We see that we can forget the "g" index because all the expansions are written in the same frame. Also we can observe that $S_{5D}$ is invariant under transformations which does not depend on $z$, as for example the one defined by $\Sigma(x)$. Then we can take the transformation

$A \xrightarrow{\Sigma} A^\Sigma$ to permute the rotations from the IR to the UV brane

$$\Rightarrow S_{5D} = \frac{1}{2g_5^2}\int d^4x \int_{z_{UV}}^{z_{IR}} dz\, a(z)\left[-\mathrm{Tr}\left[F_{\mu\nu}F^{\mu\nu}\right] + 2\,\mathrm{Tr}\left[(\partial_z A_\mu)^2\right]\right]$$

+ BC $\quad (A)^{\hat{a}}_\mu = 0\,, \quad (\partial_z A_\mu)^a = 0 \quad \Big|_{z=z_{IR}}$

$$(A)^A_\mu = (B^{\Sigma^{-1}})^A_\mu(x)\Big|_{z_{UV}}$$

This simplified action $S_{5D}[A^A_\mu, A^A_z=0]$ plus the system of IR boundary conditions and the rotated UV ones will be the starting points, or 5D reference theory, for the next 3 chapters

# KK reduction of the 5D GHU model

to start with some explicit calculations using the previous 5D action we are going to take the limit $g_5 \to 0$. The only quadratic terms in the action are relevant

$$S_{5D} \sim \frac{1}{2g_5^2}\int d^4x \int dz\, a(z) \Big\{ -(\partial_\mu A_\nu^A \partial^\mu A^{\nu A} - \partial_\mu A_\nu^A \partial^\nu A^{\mu A}) + (\partial_z A_\mu^A \partial_z A^{\mu A}) \Big\}$$

$$\sim \frac{1}{2g_5^2}\int d^4x \int dz \Big[ A_\nu^A \left[ a(z)(\partial_\rho \partial^\rho \eta_{\mu\nu} - \partial_\mu \partial^\nu)\right] A^{\mu A} - A_\mu^A \partial_z (a(z)\partial_z A^{\mu A})\Big] + \frac{1}{2g_5^2}\int d^4x \left[ a(z) A_\mu^A \partial_z A^{\mu A}\right]\Big|_{UV}$$

Here we are retaining the boundary term in the UV position. This in order to introduce the boundary fields $B_\mu$ and $\Sigma$ in the 4D action. The validity of the Bulk Maxwell equations is then recovered as an equation of motion after validation of $(B^{\Sigma^{-1}})$

Now expanding in KK states we have

$$A_\mu^A(x,z) = \sum_n f_n^A(z) A_\mu^{An}(x)$$

BC: $A_\mu^{\hat{a}}(x, z_{IR}) = 0 \Rightarrow f_n^{\hat{a}}(z_{IR}) = 0$

$\partial_z A_\mu^a(x,z)\big|_{z_{IR}} = 0 \Rightarrow \partial_z f_n^a(z)\big|_{z_{IR}} = 0$

$A_\mu^A(x, z_{UV}) = (B_\mu^{\Sigma^{-1}}(x))^A$

$$S_{5D} \sim \frac{1}{2g_5^2}\int d^4x \int dz \Big[ \sum_n f_n^A(z) A_\nu^{An}(x) \left[ a(z)(\partial_\rho \partial^\rho \eta_{\mu\nu} - \partial_\mu \partial^\nu)\right] \sum_m f_m^A A^{\mu A m} - \sum_n f_n^A(z) A_\mu^{An}(x) \partial_z (a(z)\partial_z \sum_m f_m^A(z) A^{\mu A m}(x))\Big]$$

$$+ \frac{1}{2g_5^2}\int d^4x \left[ (B_\mu^{\Sigma^{-1}}(x))^A \partial_z A^{\mu A}(x,z)\Big|_{z_{UV}}\right] \qquad \boxed{a(z_{UV}) = 1}$$

The equation of motion for $f_n^A(z)$ is obtained from the requirement that $A_\mu^{nA}(x)$ obeys a Proca equation in the absence of interactions

$$\Rightarrow \partial_z\left(a(z)\,\partial_z f_n^A(z)\right) = -a(z)\, m_n^2 f_n^A(z)$$

$$a'\partial_z f_n^A + a\,\partial_z^2 f_n^A + a\, m_n^2 f_n^A = 0$$

$$\left(\partial_z^2 + \frac{a'}{a}\partial_z + m_n^2\right) f_n^A(z) = 0$$

If the right side is chosen as $-\frac{m_n^2}{a(z)} f_n^A(z)$ Then the normalization is $\int f_n f_m = \delta_{nm}$

Assuming that we can solve this equation and using the normalization

$$\int_{z_{IR}}^{z_{UV}} dz\, a(z)\, f_n^A(z)\, f_m^A(z) = \delta_{nm}$$

we get a 4D action

$$S_{4D} \sim \frac{1}{2g_5^2}\int d^4x \left[\sum_n \left(A_\nu^{An}(x)\left[\partial_\rho\partial^\rho \eta_{\mu\nu} - \partial_\mu\partial^\nu\right] A^{\mu An} + m_n^2 A_n^{\mu A} A_\mu^{An}\right)\right]$$

$$+ \frac{1}{2g_5^2}\int d^4x \left[(B_\mu^{\Sigma^{-1}})^A \sum_n \partial_z f^{nA}(z)\Big|_{z_{UV}} A_n^{\mu A}(x)\right]$$

In order to get second order derivatives for the field $\Sigma$ we require

$$\partial_z f_n^A(z)\Big|_{z_{UV}} = f_n^A(z_{UV}) + C_n^A \quad \text{this includes the case } \partial_z f_n^A(z)\Big|_{z_{UV}} = 0$$

$$\Rightarrow S_{4D} \sim \frac{1}{2g_5^2}\int d^4x \left[-\frac{1}{2}\sum_n \left(\partial_\mu A_\nu^{An} - \partial_\nu A_\mu^{An}\right)^2 + m_n^2 A_n^{\mu A} A_\mu^{An}\right]$$

$$+ \frac{1}{2g_5^2}\int d^4x \left[(B_\mu^{\Sigma^{-1}})^A (B_\mu^{\Sigma^{-1}})^A + \sum_n C_n^A (B_\mu^{\Sigma^{-1}})^A A_n^{\mu A}\right]$$

Finally to see an "usual" Higgs sector action we can take a first order expansion of $\Sigma$, given by

$$\Sigma(x) = \exp(\sigma^{\hat a} t^{\hat a}) = 1 + \sigma^{\hat a} t^{\hat a} + \dots \qquad \sigma^{\hat a} \ll 1$$

In this regime of little perturbations $\sigma^{\hat{a}}$ we have that

$$(B_\mu^{\Sigma^{-1}})^A = B_\mu^A + f^{AB\hat{c}} B_\mu^B \hat{\sigma}^c + \frac{\partial_\mu \sigma^A}{g_5} \quad \text{by definition } \sigma^a = 0, \; \sigma^{\hat{a}} \neq 0$$

$$(B_\mu^{\Sigma^{-1}})^A (B_\nu^{\Sigma^{-1}})_A = (B_\mu^A + f^{AB\hat{c}} B_\mu^B \sigma^{\hat{c}} + \partial_\mu \sigma^A) \times (B_\mu^A + f^{AD\hat{e}} B_\mu^D \sigma^{\hat{e}} + \partial_\mu \sigma^A)$$

$$= B_\mu^A B^{\mu A} + \partial_\mu \sigma^{\hat{c}} \partial^\mu \sigma^{\hat{c}} + 2 B_\mu^A \partial^\mu \sigma^A + \ldots$$

$$S_{5D} \sim \frac{1}{g_5^2} \int d^4x \left[ \sum_n -\frac{1}{4} (\partial_\mu A_\nu^{An} - \partial_\nu A_\mu^{An})^2 + \frac{m_n^2}{2} A_\mu^{An} A^{\mu An} \right]$$

$$+ \frac{1}{g_5^2} \int d^4x \left[ \frac{1}{2} \partial_\mu \tilde{\sigma}^{\hat{c}} \partial^\mu \tilde{\sigma}^{\hat{c}} + \frac{1}{2} B_\mu^A B^{\mu A} + B_\mu^{\hat{a}} \frac{\partial^\mu \tilde{\sigma}^{\hat{a}}}{g_5} \right. \qquad \boxed{\tilde{\sigma}^c = \frac{\sigma^c}{g_5}}$$

$$\left. + \sum_n \frac{c_n}{2} \left( B_\mu^A A_n^{A\mu}(x) + \partial_\mu \tilde{\sigma}^{\hat{a}} A_n^{\hat{a}\mu} \right) \right] + \quad \text{Interactions at the order}$$

Notice that $B_\mu^{\hat{a}}$ mix with the rest of gauge bosons through bilinear terms. Also there is not a tree level potential for $\sigma^{\hat{a}}$. However a 1-loop potential can be generated and EWSB is then triggered radiatively. Gauge bosons kinetic mixing with the Goldstones are also usual in this approach.

↳ In fact we have to add fermions to the theory in order to generate EWSB. It seems like only gauge boson contributions are not enough

This is a realization of the Higgs field as a Pseudo-Goldstone-Boson field (PGB), which is also a feature of composite Higgs models. Thus it is natural to focus the study of this theory in the direction of a dual composite model. But first let us introduce the Holographic 4D theory associated to this model

## Holographic Approach

Initially the holographic action for GHU model is defined in analogy with the AdS/CFT recipe, this is

$$Z[B_\mu] \equiv e^{iS_h[B_\mu]} = \int \mathcal{D}A_\mu(x,z)_{\hat{A}_\mu = B_\mu} DA_z(x,z)\, e^{iS[A]}$$

where $\hat{A}_\mu \equiv A(x, z_{UV}) = B_\mu(x)$ and $S[A]$ corresponds to the primitive 5D action of GHU. $S_h$ is denominated "holographic action" because it contains only boundary degrees of freedom. In some sense this means that we can reproduce the 5D theory only in terms of a theory in the boundary.

Following the idea of previous chapter we would like to fix the gauge so that $A_z = 0$, before computing the integration of the bulk degrees of freedom. In the language of path integral this can be done using the Fadeev-Popov procedure. As we have seen before, the fixing of the gauge $A_z = 0$ requires the introduction of the field $\Sigma$, which modifies our initial definition of the holographic action, because this field also has to be integrated. After some few, but deep, steps we arrive to the expression

$$Z^{g.f}[B_\mu] \equiv \int \mathcal{D}\Sigma(x)\, e^{iS_h[B_\mu, \Sigma]}$$

$$= \iint \mathcal{D}\Sigma(x)\, \mathcal{D}A_\mu(x,z)_{\hat{A}_\mu = B_\mu^{\Sigma^{-1}}}\, e^{iS[A_\mu, A_z = 0]}$$

where we see explicitly the condition $\hat{A}_\mu \equiv A_\mu(x, z_{UV}) = B_\mu^{\Sigma^{-1}}(x)$. In this equation we have defined the holographic action $S_h[B_\mu, \Sigma]$. When a dual AdS/CFT interpretation is possible this is the effective action for the goldstone bosons in the presence of sources for the currents. (The last two lines should be better understood after covering the next chapter)

In order to compute the holographic action $S_n[B_\mu, \Sigma]$ we start by solving the linear tree level equations in the bulk with the condition that at the UV boundary; $A(x, z_{UV}) = B^{\Sigma^{-1}}(x)$. As the classic solutions minimize the action we consider that the greatest contribution to

$$S_n[B_\mu, \Sigma] \text{ is just } S[A_{cl}]\Big|_{\hat{A} = B_\mu^{\Sigma^{-1}}}$$

$S[A_{cl}]$: Action evaluated in the classical solutions

taken as starting point the linear action in 5D,

$$S_{5D} \sim \frac{1}{2g_5^2} \int d^4x \int dz\, a(z) \Big[ A_\nu^A \left[ (\partial_\rho \partial^\rho \eta_{\mu\nu} - \partial_\mu \partial^\nu) \right] A^{\mu A} + \left( \partial_z A_\mu^A \partial_z A^{\mu A} \right) \Big]$$

we can write this expression in the fourier space.

$$A_\mu^A(x, z) \propto \int d^4p\, e^{ipx} A_\mu^A(p, z)$$

$$S_{5D} \sim \frac{1}{2g_5^2} \int d^4p \int dz\, a(z) \Big\{ \left[ A_\nu^A \left[ (-1)(p^2 \eta_{\mu\nu} - p_\mu p^\nu) A^{\mu A} \right] (-1) \right] + \left( \partial_z A_\mu^A \partial_z A^{\mu A} \right) \Big\}$$

$$\sim \frac{1}{2g_5^2} \int d^4p \int dz\, a(z) \Big\{ - \left[ A^{\nu A} \left[ p^2 \left( \eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) A^{\mu A} \right] \right] + \left( \partial_z A_\mu^A \partial_z A^{\mu A} \right) \Big\}$$

Making use of the transverse and longitudinal degrees of freedom

$$A_\mu^A = A_{\mu t}^A + A_{\mu L}^A$$

with $A_{\mu t}^A = \left( \eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) A^{\nu A} \equiv P_{\mu\nu T} A^{\nu A}$

$$P_T P_L = 0$$
$$(P_T^2) = P_L$$

$$A_{\mu L}^A = \frac{p_\mu p_\nu}{p^2} A^{\nu A} \equiv P_{\mu\nu L} A^{\nu A}$$

$$S_{5D} \sim \frac{1}{2g_5^2} \int d^4p \int dz\, a(z) \Big\{ -\left[A_t^{\nu A}\, p^2 A_t^{\mu A}\right] + \partial_z A_{\mu t}^{A}\, \partial^z A_t^{\mu A}$$
$$+ \partial_z A_{\mu L}^{A}\, \partial^z A_L^{\mu A} \Big\}$$

$$\delta S_{5D} \sim \frac{1}{2g_5^2} \int d^4p \int dz\, a(z) \Big\{ -\delta A_t^{\mu A}\left[2p^2 A_t^{\mu A}\right] + 2\,\partial_z \delta A_{\mu t}^{A}\, \partial^z A_t^{\mu A}$$
$$+ 2\,\partial_z \delta A_{\mu L}^{A}\, \partial^z A_L^{\mu A} \Big\}$$

Using that $a(z)\partial_z \delta A_{\mu t}^{A}\, \partial^z A_t^{\mu A} = -\delta A_{\mu t}^{A}\, \partial_z\left(a(z)\partial_z A_t^{\mu A}\right) +$
$\partial_z\left(a(z)\, \delta A_{\mu t}^{A}\, \partial_z A_t^{\mu A}\right)$

$$\delta S_{5D} \sim \frac{1}{2g_5^2} \int d^4p \int dz \Big[ -2\delta A_t^{\nu A}\left(a\, p^2 A_t^{\nu A} + \partial_z\left(a(z)\, \partial_z A_t^{\nu A}\right)\right)$$
$$-2\,\delta A_L^{\nu A}\left(\partial_z\left(a(z)\partial_z A_L^{\nu A}\right)\right)\Big] + \text{B.T.}$$

Now we write $A_t^{\nu A}(p,z) = \hat{A}_t^{\nu A}(p)\, f_t^{A}(p,z)$ with $f_t^{A}(z) = 1\Big|_{z_{UV}}$

$A_L^{\nu A}(p,z) = \hat{A}_L^{\nu A}(p)\, f_L^{A}(p,z)$ " $f_L^{A}(z) = 1\Big|_{z_{UV}}$

Thus, the bulk equations of motions that minimize the action are given by

$$p^2 f_t^{A}(z) + \frac{1}{a(z)}\partial_z\left(a(z)\partial_z f_t^{A}(z)\right) = 0 \qquad + \text{BC} \quad f_t^{\hat{a}}(z_{IR}) = 0 = f_L^{\hat{a}}(z_{IR})$$
$$\partial_z f_t^{a}(z_{IR}) = 0$$
$$\frac{1}{a(z)}\partial_z\left(a(z)\partial_z f_L^{A}(z)\right) = 0 \qquad \partial_z f_L^{a}(z_{IR}) = 0$$

Again, we will assume that these equations can be solved. Then we replace back into the actions to get

$$S_h = \frac{-1}{2g_5^2} \int d^4x \sum_A \hat{A}_\mu^{A}\left(\Pi_t^{A} P_t^{\mu\nu} + \Pi_L^{A} P_L^{\mu\nu}\right)\hat{A}_\nu^{A}$$

with $\Pi_{t,L}^{A} = \partial_z f_{t,L}^{A}(p^2, z)\Big|_{z_{UV}}$ where we have to understand $p^2$ as $\partial^2$

the Holographic fields $B_\mu$ and $\Sigma$ only enter the above equation through $\hat{A}$,

$$\hat{A}^A_\mu = (B_\mu^{\Sigma^{-1}})^A = (\Sigma^\dagger B_\mu \Sigma + \Sigma^\dagger \partial_\mu \Sigma)^A$$

$$\Rightarrow S_h[B_\mu, \Sigma] = -\frac{1}{2g_5^2}\int d^4x\, (B_\mu^{\Sigma^{-1}})^A (\Pi_t^A P_t^{\mu\nu} + \Pi_L^A P_L^{\mu\nu})(B_\nu^{\Sigma^{-1}})^A$$

this holographic action only contains as degrees of freedom the boundary field $B_\mu(x)$ and the Goldstone boson $\Sigma$. It can be shown that the induced potential of $\Sigma$ is equivalent to the one that can be computed in the KK approach.

## AdS/CFT duality

In the very beginning of this talk was pointed out the big difference between a SUGRA theory in $AdS_5$ and the GHU in the slide of $AdS_5$. Then was advanced that in the case that a CFT dual theory for GHU exist this has to be very different to the $\mathcal{N}=4$ SYM CFT theory dual to SUGRA. As far as I have understood the approach to find the corresponding CFT dual of GHU starts with the assumption that this exist!!, and then one tries to match the particular features of GHU to dual behaviors on the CFT. Although the complete story is very technical we can point out some natural dualities in the next diagram.

| | dual | |
|---|---|---|
| $\boxed{\text{5D GHU in the slice of } AdS_5}$ | $\longleftrightarrow$ | $\boxed{\text{4D CFT+SM and } \Sigma \text{ composite Higgs}}$ |
| SO(5) Global Symmetry in the Bulk | $\longleftrightarrow$ | SO(5) Global symmetry on the 4D CFT. |
| Spontaneous breaking from BC at the IR | $\longleftrightarrow$ | Spontaneous breaking of SO(5) at low energies from composite dynamics |
| Boundary fields $B_\mu^A$ | $\longleftrightarrow$ | SM source field coupled to CFT operators |
| Goldstone boson $\Sigma$ from BC | $\longleftrightarrow$ | Goldstone boson associated to SSB of SO(5) at the IR. $\Sigma$ is a composite state. |
| Holographic 4D action with well known behavior | $\longleftrightarrow$ | Effective theory after integration of CFT states, expressed in term of form factors |

Now we are going to develop an effective 4D theory with CFT content (always in the gauge sector)

$$\mathcal{L}_{4D} = \underbrace{\mathcal{L}^{SU(N)}_{CFT}}_{\substack{\text{Unknown} \\ \text{Strongly} \\ \text{coupled} \\ \text{theory}}} + \underbrace{\mathcal{L}_{\widetilde{SM}}}_{\substack{\widetilde{SM}\text{ Lagrangean} \\ \text{for the sources} \\ A^A_\mu}} + \underbrace{J^A_\mu}_{\substack{\text{CFT currents} \\ (\mathcal{O}\text{ operators})}} B^{\mu A} \longrightarrow \text{SM sources}$$

$N \to$ Large number of colors

$\widetilde{SM}$: SM theory with $SO(5)$ symmetry.

In order to mimic the break of $SO(5)$ symmetry from IR-BC in the 5D model, we will assume that the global $SO(5)$ symmetry of $\mathcal{L}_{4D}$ is spontaneously broken from the strong dynamics in the CFT sector. to generate the correct q-numbers of the Higgs, this breaking has to occur in the $SO(5)/SO(4)$ directions (index $\hat{a}$), given as a result a set of goldstone bosons defined as $\Sigma(x) = \Sigma_0 \exp(\sigma^{\hat{a}}(x)\, t^{\hat{a}})$. this procedure is analogous to the spontaneous chiral symmetry breaking in strong interactions[1], thus we are going to use this approach to explain schematically the SSB in the CFT sector

- We start with $\mathcal{L}^{SU(N)}_{CFT}$ with $SO(5)$ global symmetry. the corresponding conserved currents and charges are

$$J^A_\mu = \bar{\Psi}\gamma_\mu t^A \Psi \Rightarrow Q^A = \int d^3x\, \bar{\Psi}(x)\gamma_0 t^A \Psi(x)$$

- If we assume that real world is invariant under $SO(5)$, then the vacuum state has to be invariant under the action of the charges

$$Q^A|0\rangle = 0 \Longrightarrow [Q^A, H] = 0 \qquad H: \text{Hamiltonian}$$

- However we know that $SO(5)$ is not a symmetry of the nature, thus we require that the vacuum is not invariant under the complete $SO(5)$ group

$Q^{\hat{a}}|0\rangle \neq 0$ where for convenience we choose the breaking of the symmetry along the $\hat{a}$ directions $\Leftrightarrow \langle 0|\bar{Q}^{\hat{a}} Q^{\hat{a}}|0\rangle = 0$ condensate.

1. Gauge Field Theories (2° edition). Stefan P.

- the spontaneous symmetry breaking of the vacuum generates a set of goldstone bosons associated to each generator that breaks the symmetry. In the case of strongly coupled theories these goldstone bosons are pseudo-scalars, or composite fields, that are parameterized as:

$$\Sigma(x) = \Sigma_0 \exp\{\sigma^{\hat{a}}(x) t^{\hat{a}}\}, \quad \Sigma_0 = (0,0,0,0,1)$$

Once the SO(5) symmetry is spontaneously broken and we integrate out the complete set of CFT bound states, the most general effective Lagrangian for the external fields is in momentum space and at the quadratic level

$$\mathcal{L}_{eff} = \frac{1}{2} P_{\mu\nu t} \left[ \Pi_0(p) \, Tr\left[B^\mu B^\nu\right] + \Pi_1(p) \, \Sigma B^\mu B^\nu \Sigma^+ \right]$$

where the form factors $\Pi(p)$ encode the effect of the strong dynamics and cannot be determined perturbatively in the 4D theory.

This action has to be compared with the holographic action defined in the previous chapter. However this is in principle not direct because the fields $\Sigma$ in each case are written in a different way.

$$\left.\begin{array}{ll} \text{Holographic approach} & \Sigma(x) = \exp\{\sigma^{\hat{a}}(x) t^{\hat{a}}\} \\ \text{Dual approach} & \Sigma(x) = \Sigma_0 \exp\{\sigma^{\hat{a}}(x) t^{\hat{a}}\} \end{array}\right\} \text{Different authors}$$

The match between both theories is done on the SO(4) invariant vacuum, $\Sigma = \Sigma_0$ (or $\sigma^{\hat{a}} = 0$), and considering the gauge $\partial_\mu B^A_\mu = 0$. As a result it is obtained that

$$\Pi_0(p) = \Pi^a_t(p)$$

$$\Pi_1(p) = 2\left[\Pi^{\hat{a}}_t(p) - \Pi^a_t(p)\right]$$

$$\mathcal{L}_{eff} = \frac{1}{2} P_{\mu\nu t} \left[ \Pi^a_t(p) B^{a\mu} B^{a\nu} + \Pi^{\hat{a}}_t(p) A^{\hat{a}\mu} A^{\hat{a}\nu} \right]$$

Computing the two point function $\langle B^\mu B^\nu \rangle$ one can derive the spectra of the composite states using the expansion in poles of $\Pi^{a,\hat{a}}_t(p)$.

We can parameterize the pole expansion as:

$$\Pi_a(p) = p^2 \sum_n \frac{F_{\rho_n}^2}{p^2 + m_{\rho_n}^2}, \qquad \Pi_{\hat{a}}(p) = p^2 \sum_n \frac{F_{a_n}^2}{p^2 + m_{a_n}^2} + \frac{1}{2} f_\pi^2$$

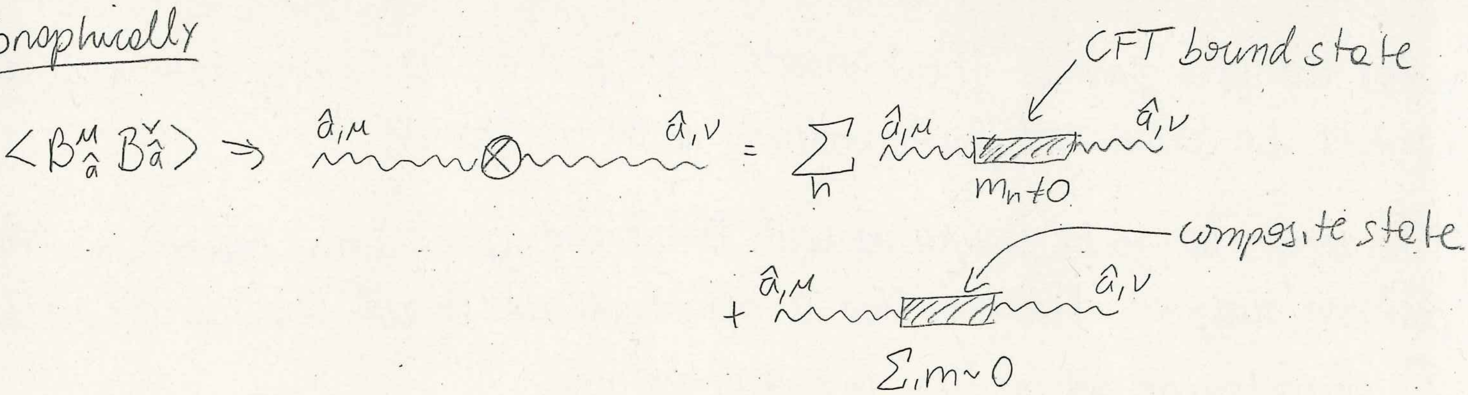
→ contribution of a massless composite state.

and compare with the "exact" results from the 5D theory, obtaining that

$$f_\pi^2 = \frac{4}{g_5^2 k} \frac{1}{L_{IR}^2}, \qquad m_\rho \equiv m_{\rho_1} \simeq \frac{3\pi/4}{\sqrt{1 + 9\pi^2/32 Z_{IR}}} \frac{1}{L_{IR}}, \qquad m_{a_1} \simeq \frac{5\pi}{4} \frac{1}{L_{IR}}$$

with $Z_{IR} = \frac{g_5^2 k}{g_{IR}^2}$

Graphically




Conclusion:

⇒ G.H.U. models generate the Higgs field as a PGB in the effective 4D theory. Since these GHU models use as background geometry a slice of $AdS_5$, a dual description in terms of a strongly coupled CFT Theory is motivated. Once we match both theories we can interpret the PGB Higgs as a composite state. As the duality that we use is Holographic by construction, this kind of approach to obtain a 4D Higgs is denominated "Holographic Composite Higgs" model

References:

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