Experimental Tests of the Gravity Sector

DESY Werkstattseminar "Physics of Extra Dimensions", Summer Term 2011

Kai Schmitz

April 26, 2011

Abstract

Theories with extra spatial dimensions unavoidably lead to new physics in the gravitational sector that may be probed in tabletop laboratory experiments, in the sky as well as with particle accelerators. Positive measurements would possibly provide insights into the higher-dimensional nature of spacetime and the quantum theory of gravity.

Contents

1 Extra Dimensions

Phenomenological consequences of extra spatial dimensions:

- Success of the standard model (SM) of particle physics up to the electroweak scale, corresponding to distances of order $(100\,\mathrm{GeV})^{-1} \sim 10^{-3}\,\mathrm{fm}$, implies that SM fields cannot propagate large distances in the extra dimensions. They are either confined to a 3-brane, a $(3+1)$-dimensional subspace of spacetime, or some higher-dimensional subspace whose additional dimensions are compactified on a length scale $\ell \lesssim (1\,\mathrm{TeV})^{-1}$.

- By contrast, gravity necessarily propagates in all dimensions as it is the dynamics of spacetime itself. Depending on the specifics of the higher-dimensional theory this might lead to clear signatures of new gravitational physics in a variety of different physical environments and experiments.

- Topics covered in this talk: deviations from Newton's inverse square law that are searched for in tabletop laboratory experiments, bounds on the thermal production and late-time decay of gravitons from cosmology, cooling and heating of astrophysical objects and finally graviton emission and exchange as well as the production of microscopic black holes at high-energy colliders.

1.1 Large compact extra dimensions

Last week's seminar talk introduced the concepts of large compact as well as (finite and infinite) warped extra dimensions. We will focus on the former scenario as it has richer phenomenological implications at low energies and can, hence, be constrained on the basis of more physical phenomena.

- Simplest framework to study extra dimensions proposed by Arkani-Hamed, Dimopoulos and Dvali (ADD) in Refs. [1, 2, 3].
- Idea: Solve the hierarchy problem by unifying gravity and the gauge forces at the electroweak scale which is assumed to be the only fundamental scale in nature. Gravity appears weak because it can also propagate in the bulk of a higher-dimensional spacetime with n large compact extra dimensions. The dilution of the graviton wavefunction on the SM 3-brane is controlled by the volume of the extra dimensions $V_{(n)}$.
- Consider Einstein-Hilbert action on a $(4+n)$-dimensional factorizable spacetime $\left(M_4 \times Y_n, g_{(4+n)}\right)$ where Y_n is an n-dimensional compact space. Restricting all fields to be constant in the extra dimensions (setting all moduli to their vacuum expectation values in the language of string theory), we can compactify to 4 dimensions,

$$\begin{aligned} S_{(4+n)} \supset & -\frac{1}{2}\int d^{4+n}x\, \hat{M}_{(4+n)}^{2+n}\sqrt{g_{(4+n)}}R_{(4+n)} \\ \to & -\frac{1}{2}\int d^4x\, V_{(n)}\hat{M}_{(4+n)}^{2+n}\sqrt{g_{(4)}}R_{(4)} \subset S_{(4)}\,. \end{aligned} \tag{1}$$

  We identify the 4-dimensional reduced Planck mass as $\hat{M}_{(4)}^2 = V_{(n)}\hat{M}_{(4+n)}^{2+n}$. The hierarchy problem is nullified when the higher-dimensional reduced Planck mass is close to the electroweak scale, $\hat{M}_{(4+n)} \sim 1\,\mathrm{TeV}$, and $V_{(n)}$ is correspondingly large such that $\hat{M}_{(4)} \simeq 2.4 \times 10^{18}\,\mathrm{GeV}$.

- Upon compactification the ordinary massless graviton $G^{(0)}$ becomes supplemented by a tower of Kaluza-Klein (KK) modes $G^{(j)}$. Each graviton by itself represents a canonically normalized perturbation of the 4-dimensional metric around flat Minkowski space. Suppressing Lorentz indices, we may write

$$g_{(4)} = \eta_{(4)} + \frac{1}{\hat{M}_{(4)}}\sum_{j=0}^{\infty} G^{(j)}\,. \tag{2}$$

  where j runs collectively over all n sets of KK modes. The Lagrangian accounting for the coupling to SM matter reads

$$\mathscr{L}_{(4)} \supset -\left[\frac{1}{\hat{M}_{(4)}}G_{\mu\nu}^{(0)} + \frac{1}{\hat{M}_{(4)}}\sum_{j=1}^{\infty} G_{\mu\nu}^{(j)}\right]T^{\mu\nu}\,. \tag{3}$$

  with $T^{\mu\nu}$ being the SM energy-momentum tensor in 4 dimensions. Notice that the KK gravitons couple with the same strength as the massless zero-mode.

- Compactify on n circles with a common radius $r_{(n)}$ such that $V_{(n)} = L_{(n)}^n = \left(2\pi r_{(n)}\right)^n$. The momenta in the extra dimensions $q_i = \left(q_4, q_5, ..., q_{(3+n)}\right)$ then have to be integer multiples of $r_{(n)}$.

$$q_i = \frac{2\pi}{\lambda_i}\,, \quad j_i\lambda_i = 2\pi r_{(n)} \quad \Rightarrow \quad q_i = \frac{j_i}{r_{(n)}}\,. \tag{4}$$

  which results in KK gravitons $G^{(j)}$ with masses $m_j^2 = \sum_i j_i^2/r_{(n)}$ and uniform mass splitting $\Delta m = 1/r_{(n)}$.

- The characteristic length scale of the torus compactification is $r_{(n)}$ rather than $L_{(n)}$. Therefore, the actual mass scale that we should require to be of order TeV is $M_{(4+n)}^{2+n} = (2\pi)^n \hat{M}_{(4+n)}^{2+n}$ (formally, we also define $M_{(4)}^2 = \hat{M}_{(4)}^2$).

$$M_{(4)}^2 = r_{(n)}^n M_{(4+n)}^{2+n} \quad \Rightarrow \quad r_{(n)} = 2 \times 10^{31/n-16}\,\mathrm{mm} \left(\frac{1\,\mathrm{TeV}}{M_{(4+n)}}\right)^{1+2/n} . \tag{5}$$

For $M_{(4+n)} = 1\,\mathrm{TeV}$, one finds $r_{(1)} = 2 \times 10^9\,\mathrm{km}$, $r_{(2)} = 600\,\mu\mathrm{m}$, $r_{(3)} = 4\,\mathrm{nm}$ and so on. The case $n = 1$ is, thus, excluded as it implies deviations from Newtonian gravity over solar system distances. $r_{(2)}$ is just in the range of distances that can be probed in laboratory experiments. But: Is the hierarchy problem not simply replaced by the question why (some of) the extra dimensions are compactified on length scales much larger than the electroweak scale? Fortunately, string theory may provide a plausible answer to that question in the context of moduli stabilization (in a couple of weeks we will have a talk exclusively devoted to that topic).

- There are at least two ways how the bounds on $r_{(n)}$ in Eq. (5) can be alleviated:
  - Asymmetric compactification with, for instance, n very large dimensions of radius $r_{(n)} \sim \left(10^{-3}\,\mathrm{eV}\right)^{-1}$ and N medium sized dimensions of radius $R_{(N)} \sim (1\,\mathrm{TeV})^{-1}$ [4]. For $n = 2$ and $N = 4$ the bound on $M_{(6)}$ increases quadratically when $M_{(10)}$ is taken to larger values,

$$M_{(4)}^2 = M_{(6)}^4 r_{(2)}^2 = M_{(10)}^8 r_{(2)}^2 R_{(4)}^4 \quad \Rightarrow \quad M_{(6)} \sim \left(\frac{M_{(10)}}{1\,\mathrm{TeV}}\right)^2 \mathrm{TeV}. \tag{6}$$

    Note that in such a setup even a scenario with one large extra dimension becomes viable again ($n = 1$, $N = 5$, $M_{(10)} = 100\,\mathrm{TeV}$).
  - In string theory rather the string scale m_s than the higher-dimensional Planck mass $M_{(4+n)}$ represents the fundamental mass scale. In Ref. [2] ADD consider a T-dualized version of type I string theory with n large compact dimensions. In the low-energy effective field theory in 4 dimensions they identify

$$M_{(4+n)} = \left(\frac{2}{\alpha_g^2}\right)^{1/(2+n)} m_s\,, \quad M_{(4)} = \left(\frac{2}{\alpha_g^2}\frac{V_{(6)}}{(2\pi/m_s)^6}\right)^{1/2} m_s\,, \quad \alpha_g = \frac{\lambda}{4}\,. \tag{7}$$

    Here, λ is the string coupling and α_g the gauge coupling at the string scale. In order to solve the hierarchy problem we need $\lambda \sim 1$ and very large $V_{(6)}$ in units of m_s^{-6}. For $\alpha_g \lesssim 1$ the higher-dimensional Planck mass exceeds the string scale and values $M_{(4+n)} \gtrsim 1\,\mathrm{TeV}$ become feasible.

1.2 Warped extra dimensions

Randall and Sundrum also attempted to solve the hierarchy problem by introducing new spatial dimensions [5, 6]. Their scenario does, however, not imply new gravitational physics below the TeV scale which is why we shall devote less attention to it in this talk.

- Idea: The most general solution to Einstein's equations in 5 dimensions that respects 4-dimensional Poincaré invariance admits the possibility of having warp factors in the metric (the ADD scenario then represents the special case without warping). The hierarchy problem is solved by observing that physical mass parameters on the SM 3-brane arise from parameters of the higher-dimensional theory that are exponentially redshifted due to warping. Gravity is weak because the massless graviton mode $G^{(0)}$ is localized in the extra dimension. The dilution of its wavefunction on the SM 3-brane is controlled by the curvature scale of the extra dimension k.

- Setup: Consider one extra spatial dimension compactified on the orbifold S^1/Z_2 with two 3-branes with non-vanishing tension embedded at $\phi = 0$ (Planck brane) and $\phi = \pi$ (SM TeV brane). The 5-dimensional metric reads

$$ds^2 = e^{2kr_c\phi}\eta_{\mu\nu}dx^\mu dx^\nu + r_c^2 d\phi^2\,, \tag{8}$$

which corresponds to a slice of 5-dimensional anti-de Sitter space. r_c is the compactification radius of the extra dimension. In string theory it corresponds to the vacuum expectation value of some modulus field R, sometimes referred to as the radion. The electroweak scale $v_{\text{EW}} \sim 100\,\text{GeV}$ is related to the corresponding higher-dimensional parameter $v_0 \sim M_{(4)}$ through the warp factor, $v_{\text{EW}} = e^{-kr_c\pi}v_0$. The enormous hierarchy between the electroweak and the Planck scale then boils down to a rather mild hierarchy between k and r_c^{-1}. Choosing $kr_c \simeq 12$ successfully generates TeV mass parameters from Planck scale inputs. The radion then typically acquires a mass $m_R \sim 100\,\text{GeV}$.

- It turns out that the spectrum of KK gravitons can be described as the set of eigenfunctions of an analogous non-relativistic problem in quantum mechanics (QM); recall the volcano potential from last week's talk. Interestingly, the KK masses m_j are unevenly spaced,

$$m_j = cx_jM_\star\,, \quad c = \frac{k}{M_{(4)}} \sim 10^{-2} \div 10^0\,, \quad J_1\left(x_j\right) = 0\,, \quad M_\star = e^{-kr_c\pi}M_{(4)}\,. \tag{9}$$

  where J_1 is the Bessel function of the first kind of order 1 and x_j denotes its j^{th} root. Given that $kr_c \simeq 12$, we find that the KK gravitons have masses and mass splittings of order TeV. This is why warped extra dimensions can practically only be probed at colliders. The modification of Newtonian gravity at laboratory distances is negligible, astrophysics and cosmology do not constrain the model as long as only processes at temperatures $T \lesssim 1\,\text{TeV}$ are pertained.

- The ADD scenario is characterized by a large multiplicity of KK graviton states whose coupling to matter is suppressed by the Planck scale. Now we deal with significantly less KK gravitons that couple with TeV-scale suppressed strength to SM matter.

$$\mathscr{L}_{(4)} \supset -\left[\frac{1}{M_{(4)}}G_{\mu\nu}^{(0)} + \frac{1}{M_\star}\sum_{j=1}^{\infty}G_{\mu\nu}^{(j)}\right]T^{\mu\nu}\,. \tag{10}$$

  At the Large Hadron Collider (LHC) KK gravitons may, thus, be produced in Drell-Yan processes such as $q\bar{q} \to G^{(j)} \to \ell^+\ell^-$ and $gg \to G^{(j)} \to \ell^+\ell^-$. Each KK mode would show up as an isolated resonance in the corresponding differential cross section allowing for a one-by-one identification of the excited graviton states. The spin-2 nature of the resonances could be confirmed from analyzing angular distributions in the event sample. If, due to the large involved masses, only $G^{(1)}$ were seen at LHC, such an analysis would be crucial to support the interpretation of the data in the context of warped extra dimensions.

- Just as in the ADD case, the Randall-Sundrum model also promises signatures of quantum gravity at the TeV scale. The fundamental string scale m_s^0 in the higher-dimensional theory translates into a redshifted scale $m_s \sim 1\,\text{TeV}$ in 4 dimensions such that string excitations are expected to be appear at the LHC.

2 Laboratory

In models with large extra dimensions the exchange of light KK gravitons between massive objects modifies Newton's law of universal gravitation at small distances. In the static limit this interaction can be accounted for by adding a Yukawa potential to the ordinary Newtonian potential,

$$V(r) = V_N(r) + V_Y(r) = V_N(r)\left(1 + \alpha e^{-r/\lambda}\right)\,, \quad V_N(r) = -G_{(4)}\frac{m_1m_2}{r}\,, \tag{11}$$

where α parametrizes the relative strength, $\alpha = G_Y/G_{(4)}$, and λ determines the range of the Yukawa force. Notice that, in fact, any interaction mediated by a boson ϕ with mass $m_\phi = 1/\lambda$ can be parametrized in the non-relativistic limit in this way. Such additional contributions to V_N are searched for in laboratory experiments. As long as none are found, these measurements allow to constrain the properties of extra dimensions (and theories with new light bosons).

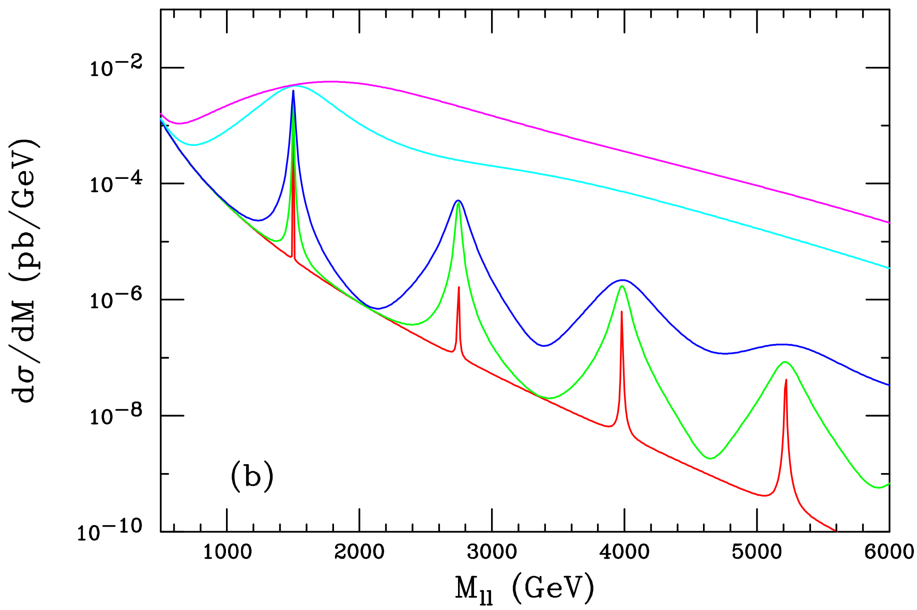

Figure 1: Signature of warped extra dimensions at the LHC: Differential cross section for the Drell-Yan production of a 1500 GeV KK graviton and its subsequent tower states as function of the dilepton invariant mass $M_{\ell\ell}$. From top to bottom, the curves are for $c = k/M_{(4)} = 1$, 0.5, 0.1, 0.05, and 0.01, respectively. Figure taken from Ref. [7].

2.1 Yukawa potential

Before we turn to the actual experiments that have been performed in the recent past, let us review how the exchange of a scalar particle ϕ between two fermions actually induces a Yukawa potential [8]. For higher-spin bosons the calculation is similar. The resulting potentials only differ in their overall sign: Spin-0 and spin-2 bosons mediate attractive, spin-1 bosons repulsive forces.

- Consider the scattering of two distinguishable fermions f_1 and f_2 via the exchange of a scalar ϕ in the t-channel,

$$f_1\left(\vec{p},r\right) f_2\left(\vec{q},s\right) \rightarrow \phi(\vec{k}) \rightarrow f_1\left(\vec{p}^{\,\prime},r'\right) f_2\left(\vec{q}^{\,\prime},s'\right) \,, \quad \vec{k} = \vec{p} - \vec{p}^{\,\prime} \,. \tag{12}$$

  In the non-relativistic limit, the tree-level amplitude $i\mathcal{M}$ reads

$$i\mathcal{M} = \frac{ig^2}{\vec{k}^2 + m_\phi^2} \,, \tag{13}$$

  where g denotes the Yukawa coupling constant.

- This amplitude can be compared to the Born approximation for the scattering amplitude in non-relativistic QM perturbation theory. For scattering an incident particle off a fixed target we have

$$\langle \vec{p}^{\,\prime} |\, iT \,| \vec{p} \rangle = (2\pi)\, \delta\left(E_p - E_{p'}\right) \left(-i\tilde{V}_Y(|\vec{k}|)\right) \quad \Rightarrow \quad \tilde{V}_Y(|\vec{k}|) = \frac{-g^2}{\vec{k}^2 + m_\phi^2} \,. \tag{14}$$

  With its Fourier transform $\tilde{V}_Y$ at hand, we can compute the Yukawa potential V_Y,

$$V(r) = -\frac{g^2}{4\pi} \frac{e^{-m_\phi r}}{r} \,, \tag{15}$$

  which is of the form of V_Y in Eq. (11).

- The range of the Yukawa interaction is set by the Compton wavelength $\lambda = 1/m_\phi$ of the scalar particle ϕ. In the 1930's Yukawa used a potential of this type to describe the binding force between nucleons through the exchange of pions. From the approximate range of the force (~ 1 fm) he could predict the mass of the pion (~ 200 MeV).

- There are a couple of hypothetical scalars that could induce a similar Yukawa potential as light KK gravitons. In general, any light boson of mass $m_\phi \sim 1\,\text{meV}$ leads to deviations from Newtonian gravity at separations $\lambda \sim 100\,\mu\text{m}$. Coherent interactions of ϕ with SM matter may arise from various dimension-5 operators. The scalar couplings to quarks and electrons presumably break chiral symmetry and are, hence, suppressed. We thus expect the coupling to the gluon field strength to be the dominant one.

$$\mathscr{L} \supset \frac{\phi}{f_\phi} G^a_{\mu\nu} G_a^{\mu\nu} \tag{16}$$

with f_ϕ being some characteristic mass scale.The operator in Eq. (16) results in a Yukawa potential with $\lambda = 1/m_\phi$ and $\alpha \simeq M_N^2/\big(4\pi f_\phi^2\big)$, where M_N is the mass of a nucleon. Candidate particles for such an interaction are: String moduli, the dilaton, the radion, scalars from hidden gauge sectors and so on. The axion is special in the sense that it is a pseudoscalar particle and, hence, does not mediate a force between unpolarized bodies. It is by non-perturbative instanton effects that the axion receives a scalar coupling to matter which, however, ends up being quite suppressed.

- In the ADD scenario the gravitational potential between two masses m_1 and m_2 reads

$$V(r) = G_{(4)} \frac{m_1 m_2}{r} \sum_j e^{-m_j r}\,, \quad m_j^2 = \sum_i j_i^2 / r_{(n)}\,. \tag{17}$$

In passing we mention that in the limit $r \ll r_{(n)}$ this potential turns into what is expected from Gauss's law in $4+n$ dimensions,

$$r \ll r_{(n)}\ :\quad V(r) \to G_{(4+n)} \frac{m_1 m_2}{r^{1+n}}\,, \quad G^{-1}_{(4+n)} = 2 S_{(3+n)} M^{2+n}_{(4+n)} / \left(2\pi\right)^n\,, \tag{18}$$

where $S_{(3+n)}$ is the surface area of the unit sphere in $3+n$ spatial dimensions (the relation between $G_{(4+n)}$ and $M_{(4+n)}$ follows from calculating the gravitational force law directly from the $(4+n)$-dimensional action). The largest contributions to the potential in Eq. (17) come from the massless graviton and the lowest-lying KK mode,

$$V(r) = V_N \left(1 + \frac{8n}{3} e^{-r/r_{(n)}} + \ldots\right)\,. \tag{19}$$

which corresponds to a Yukawa correction with $\alpha = 8n/3$ and $\lambda = r_{(n)}$. A factor $2n$ contributing to α stems from the multiplicity of the KK graviton with mass $1/r_{(n)}$. The remaining $4/3$ are due to the different couplings of the various graviton polarization states to matter in the non-relativistic limit.

- In the case of two large extra dimensions we expect a Yukawa interaction with $\alpha = 16/3$ and $\lambda \sim 100\,\mu\text{m} \div 1\,\text{mm}$. This is exactly the parameter range current laboratory experiments are sensitive to.

2.2 Torsion oscillators

Torsion oscillator experiments allow to measure the gravitational attraction between two macroscopic test bodies down to separations $r \sim \text{few} \times 10\,\mu\text{m}$ [9, 10]. The constraints on the Yukawa parameters α and λ that follow from such measurements are summarized in Fig.3.

- General principle: A rotating / vibrating attractor mass exerts a periodically varying differential torque on a torsion oscillator. This causes an angular deflection of the torsion oscillator which can be monitored by various techniques. Varying the separation between attractor and oscillator, one can then deduce the distance dependence of the gravitational force acting on the oscillator and look for non-Newtonian contributions. Likewise, such experimental setups can also be used to search for axion-like particles, test the equivalence principle or look for violations of Lorentz symmetry.

- Eöt-Wash group at University of Washington (Adelberger, Hoyle, Kapner et al. [11]):

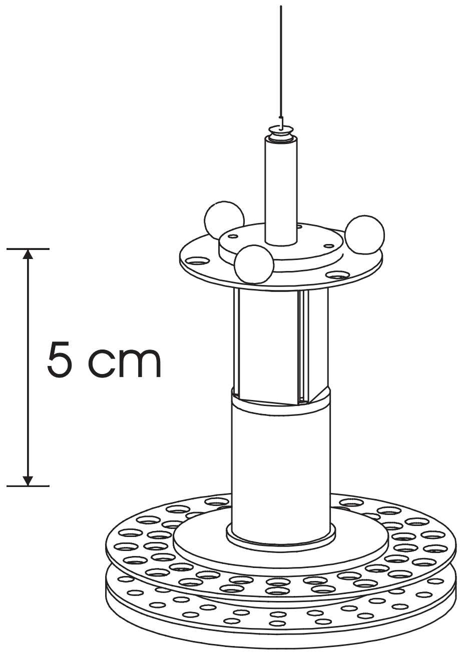

Figure 2: Scale drawing of the detector and attractor used in the experiment of the Eöt-Wash group [11]. The electrical shield of the detector is not shown.

- Similar to the experiment performed by Hungarian physicist Eötvös around 1900 who developed a torsion pendulum with which he tested the equivalence between the inertial and the gravitational mass claimed by Einstein (null test: the crucial observation was that his torsion balance did not rotate.)
- Experimental setup (shown in Fig. 2): Torsion pendulum (detector) suspended above a rotating attractor. The test masses are 42 holes bored into the detector disk such that it exhibits a 21-fold rotational symmetry. The attractor consists of two disks with 42 (upper disk) and 21 (lower disk) holes, respectively. The lower disk is supposed to cancel the Newtonian torque resulting from the upper disk (exact cancellation only occurs for a specific separation, the experiment is thus a partial-null test).
- Gravitational interaction between missing masses leads to oscillatory torques with frequencies that are integer multiples of 21ω where ω is the rotational frequency of the attractor (low-frequency torsion oscillator experiment, $\omega/(2\pi) \sim 1\,\mathrm{mHz}$). These torques twist the suspension fiber of the pendulum which can be monitored by a laser beam reflected from a mirror mounted on the oscillator. By construction, the high harmonics of the detector signal (42ω, 63ω,...) lie above the resonance frequency ω_0 of the pendulum. In this way, one optimizes the signal-to-noise ratio (the main noise source being of thermal origin).
- The Eöt-Wash group puts the most restrictive bound on the size of the extra dimensions in the ADD scenario with $n = 2$: For $\alpha = 16/3$, they find $\lambda = r_{(2)} \lesssim 44\,\mu\mathrm{m}$ which corresponds to $M_{(6)} \gtrsim 3.2\,\mathrm{TeV}$.

- University of Colorado (Long et al.):
  - Vertical oscillations of an attractor with the shape of a diving board drive a compound high-frequency torsion oscillator ($\omega/(2\pi) \sim 1\,\mathrm{kHz}$) consisting of two rectangles that counter-rotate about their torsional axis.
  - Due to its planar geometry (the diving board is parallel to the rectangles of the detector) this experiments constitutes a null test: The Newtonian force between two parallel, infinite planes is independent of their separation and so, in the absence of additional Yukawa forces, no variation in the detector response should be seen when scanning over different distances between attractor and oscillator.

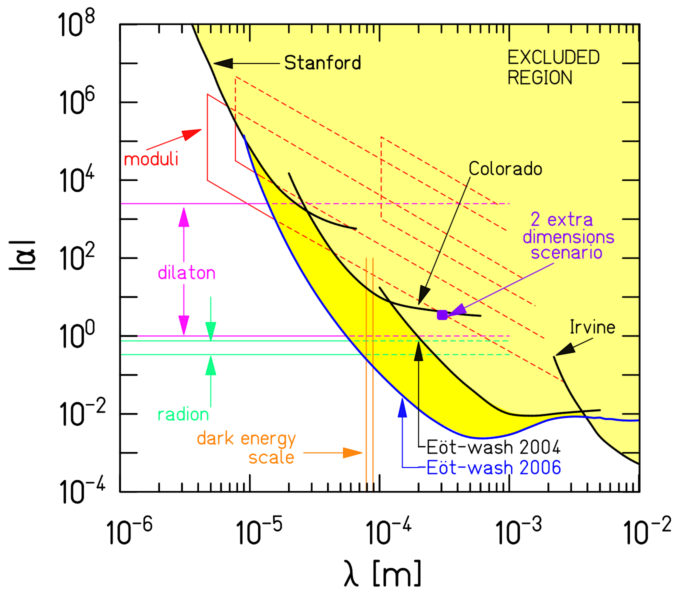

Figure 3: Constraints on Yukawa violations of the gravitational inverse square law derived from dedicated experiments. The shaded region is excluded at the 95 % confidence level. Lighter lines show various theoretical expectations. Figure taken from Ref. [11].

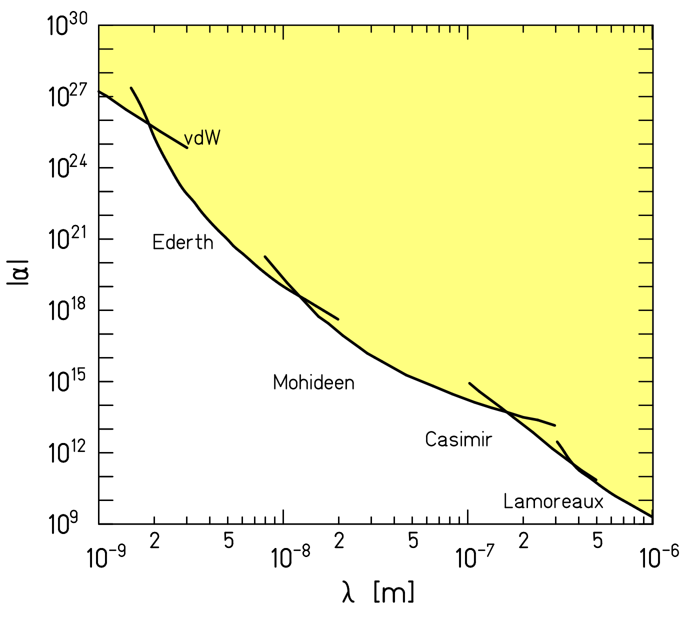

Figure 4: Constraints on Yukawa violations of the gravitational inverse square law derived from measurements of the Casimir force. Figure taken from Ref. [9].

- Stanford University (Chiaverini et al. [12]): Microcantilever (resonant frequency $\omega_0 \sim 300\,\mathrm{Hz}$) with gold test mass mounted on its free end driven by a horizontally oscillating silicon-gold attractor. The separation of the cantilever and the attractor can be reduced down to $25\,\mu\mathrm{m}$.

2.3 Casimir force

The Casimir force is a fundamental background to tests of Newton's inverse square law. Usually it needs to be reduced by means of electrostatic shields and other techniques so that one can focus on probing Yukawa forces only. On the other hand, measurements of the Casimir force itself also allow to constrain the strength and the range of additional interactions. Such analyses are, however, subject to limitations: Corrections to the Casimir force for finite temperature, finite conductivity and surface roughness still need to be understood in greater detail.

- A Casimir force emerges when the zero-point modes of the electromagnetic field are subject to certain boundary conditions. For instance, consider two grounded, perfectly conducting, smooth, infinite, parallel plates at zero temperature. Between the plates the mode spectrum (k^{in}) then becomes discrete as also the quantum field has to satisfy the classical boundary conditions. Outside the plates the spectrum (k^{out}) remains continuous,

$$k^{\mathrm{in}} = m\frac{2\pi}{d}\,, \quad m = 1, 2, ... \quad k^{\mathrm{out}} \in \mathbb{R}\,, \tag{20}$$

  where d is the distance between the two plates. This difference in the mode spectrum leads to a net pressure of virtual photons from the outside such that the two plates are pushed together,

$$\frac{F_C}{A} = \frac{\pi^2 \hbar c}{240 d^4}\,. \tag{21}$$

  For two plates of thickness 1 mm and density $\rho = 10\,\mathrm{g/cm^3}$ the Casimir forces exceeds the Newtonian one for separations smaller than $d \simeq 13\,\mu\mathrm{m}$.

- In the recent past the Casimir forces has been measured by a couple of experiments, the first of which was performed by Lamoreaux at University of Washington [13]. As a by-product these experiments also obtained bounds on the Yukawa parameters α and λ, cf. Fig. 4. Most of these constraints were, however, obtained by assuming that

$$|F_Y| \lesssim |F_N - F_C|\,. \tag{22}$$

  Actually, the data should be fit to a total force $F = F_N + F_C + F_Y$ which would lead to less stringent bounds on α.

3 Cosmology

Cosmological considerations constrain the size of the extra dimensions in the ADD scenario even more than laboratory experiments [3, 14]. In order to not spoil the success of the theory of primordial nucleosynthesis (BBN), require that at temperature $T_{\mathrm{BBN}} \simeq 1\,\mathrm{MeV}$ the extra dimensions are frozen and empty of energy density. Otherwise the expansion rate during BBN would receive inadmissible additional contributions. Let us denote the highest temperature at which the presence of the extra dimensions is negligible in the above sense and at which all SM fields are in thermal equilibrium by $T_\star$. In certain scenarios $T_\star$ may correspond to the reheating temperature T_{RH} after a stage of inflation of the SM 3-brane and subsequent inflaton decay into SM fields. BBN now provides a lower bound on $T_\star$

$$T_\star \gtrsim T_{\mathrm{BBN}} \simeq 1\,\mathrm{MeV}\,. \tag{23}$$

In the following we shall show how this bound can be translated into bounds on $M_{(4+n)}$.

3.1 Graviton overclosure

- During the hot thermal phase of the early universe the graviton and its KK excitations are produced in pair annihilation processes of all relativistic species present in the thermal bath,

$$\ell^+\ell^-, \nu_\ell\bar{\nu}_\ell, \gamma\gamma \to G^{(j)} \,. \tag{24}$$

- Each graviton mode only couples with gravitational strength to SM matter. But due to the enormous multiplicity of KK states a large number of gravitons can actually be produced. Let us be a bit more general for a moment and consider any high-energy process at an energy scale E. The number of KK gravitons that can participate in such a process is then given as $(E/\Delta p)^n = \left(E r_{(n)}^n\right)^n$. The production rate of KK gravitons is proportional to this multiplicity factor,

$$\Gamma_G^{\mathrm{prod}} \propto \frac{1}{M_{(4)}^2}\left(E r_{(n)}^n\right)^n = \frac{E^n}{M_{(4+n)}^{2+n}} = \frac{1}{M_{(4+n)}^2}\left(\frac{E}{M_{(4+n)}}\right)^n \,, \tag{25}$$

where we have used the relation between $M_{(4)}$ and $M_{(4+n)}$ in Eq. (5). So, after all, the interactions of the KK gravitons are only suppressed by $M_{(4+n)} \sim 1\,\mathrm{TeV}$. Notice that the production rate in Eq. (25) can also be motivated in the higher-dimensional theory which only features one massless $(4+n)$-dimensional graviton that couples with $M_{(4+n)}$-suppressed strength to matter: In this case, Γ_G^{prod} directly follows from dimensional analysis. In the early universe the largest accessible energies are of order the temperature, $E \sim T$,

$$\Gamma_G^{\mathrm{prod}}(T) \propto \frac{1}{M_{(4+n)}^2}\left(\frac{T}{M_{(4+n)}}\right)^n \,. \tag{26}$$

- Setting $T_\star = T_{\mathrm{BBN}}$ and requiring that gravitons be not produced too abundantly yields lower bounds on $M_{(4+n)}$. To start with, gravitons must not overclose the universe. That is, Ω_G^0, the present-day graviton energy density in units of today's critical energy density, must be smaller than unity,

$$\Omega_G^0 \leq 1 \,. \tag{27}$$

Neglecting graviton decay, the authors of Ref. [14] translate this requirement into

$$n = 2 \;: \quad M_{(6)} > 6.5/\sqrt{h}\,\mathrm{TeV}\,, \quad r_{(2)} < 15\,h\,\mathrm{\mu m}\,. \tag{28}$$

3.2 Cosmic diffuse gamma ray background

- MeV KK gravitons, that are produced before BBN, decay on timescales much longer than the age of the universe. Dimensional analysis tells us

$$\Gamma_G^{\mathrm{decay}} \sim \frac{m^3}{M_{(4)}^2} \quad \Rightarrow \quad \tau_G(E) \sim 10^{10}\,\mathrm{yr}\left(\frac{100\,\mathrm{MeV}}{m}\right)^3 \,. \tag{29}$$

This long lifetime reflects the low probability of the $(4+n)$-dimensional graviton to return to the SM 3-brane or, equivalently, the weak $M_{(4)}$-suppressed coupling of each individual KK mode. Recall that the stronger coupling of the KK gravitons in the context of graviton production arose as an collective effect due to the large multiplicity of KK states.

- The late-time decay of KK gravitons into photons may lead to distortions in the MeV spectrum of the cosmic diffuse gamma ray background. So far, no bumps have been observed. Based on data from the COMPTEL instrument on board of NASA's Compton gamma-ray observatory (predecessor of the Fermi satellite in the 1990's), the authors of Ref. [14] are able to derive the following bounds:

$$n = 2 \;: \qquad M_{(6)} > 110\,\mathrm{TeV}\,, \qquad r_{(2)} < \; 0.05\,\mathrm{\mu m}\,. \tag{30}$$
$$n = 3 \;: \qquad M_{(7)} > 5\,\mathrm{TeV}\,, \qquad r_{(3)} < \; 0.3\,\mathrm{nm}\,. \tag{31}$$

Here, the most stringent constraints are obtained for photon energies $E_\gamma \simeq 4\,\mathrm{MeV}$.

4 Astrophysics

Graviton production is subject to the same constraints as the production of other light particles. Just as the axion or neutrinos, gravitons may be copiously produced in an astrophysical object, carry away energy from it and escape into the extra dimensions. In this way gravitons may accelerate the cooling dynamics of stellar objects. Likewise, the decay of a cloud of KK gravitons will heat up all objects within the cloud resulting in an observable excess heat. We are, hence, able to put bounds on the properties of extra dimensions by considering the thermodynamics of objects like the sun, supernovae or neutron stars.

4.1 Cooling of the sun and supernovae

- In the sun gravitons are produced by three different processes: photon pair annihilation, gravi-bremsstrahlung and the gravi-Primakoff effect.

$$\gamma + \gamma \to G\,, \quad \gamma + e \to e + G\,, \quad \gamma + \text{EM field from nucleus}\, Z \to G \tag{32}$$

  Here, pair annihilation (which is also most important in the early universe) contributes the most to graviton production. During the core collapse of a supernova the dominant process is nucleon-nucleon bremsstrahlung,

$$N + N \to N + N + G\,. \tag{33}$$

  This process requires a high temperature $T \lesssim m_\pi$ in order for the strong interactions to become active and thus does not play a role in less energetic environments. The next-to-most important process in supernovae is the gravi-Primakoff effect.

- The coupling of the KK gravitons to matter is TeV-scale suppressed, $M_{(4+n)} \sim 1\,\text{TeV}$. When comparing the production of gravitons to that of axions we notice that it is absolutely impossible to accommodate such a scale in an axion model. An axion decay constant $f_a \sim 1\,\text{TeV}$ corresponds to an axion mass of $m_a \sim 10\,\text{keV}$ which is ruled out as it is incompatible with neutrino data from SN 1987A and the lifetime of globular cluster stars. In the case of gravitons a TeV suppression of the coupling might, however, be admissible at first sight. An axion production rate proportional to $1/f_a^2$ namely translates into a graviton production rate as in Eq. (26) such that the actual relation between the scales of axion and graviton coupling should read [3]

$$f_a \leftrightarrow \left(\frac{M_{(4+n)}}{T}\right)^{n/2} M_{(4+n)}\,. \tag{34}$$

  This rule of thumb can help us get an intuition of how relevant graviton production can become in certain astrophysical situations.

- The temperature in the sun is $T \sim 1\,\text{keV}$. For $M_{(4+n)} \sim 1\,\text{TeV}$ and $n = 2$ this results in an effective scale $f_a \sim 10^{12}\,\text{GeV}$ which is completely safe. Axion helioscopes such as CAST at CERN probe axion decay constants up to $f_a \sim 10^7\,\text{GeV}$. For larger values of f_a the energy loss due to axion cooling is irrelevant. Graviton production therefore also does not affect the cooling dynamics of the sun.

- During its collaps the core of SN 1987A reached a temperature of $T \sim 30\,\text{MeV}$. Hence, the equivalent of the graviton coupling strength for $M_{(4+n)} \sim 1\,\text{TeV}$ and $n = 2$ is a axion decay constant of $f_a \sim 3 \times 10^7\,\text{GeV}$. This value is ruled out by the neutrino data from SN 1987A: For such large f_a cooling is too efficient resulting in a shortened duration of the neutrino burst. In order for the ADD scenario to not be in conflict with SN 1987A a Planck mass $M_{(4+n)}$ larger than a TeV is required. We obtain a lower bound on $M_{(4+n)}$ by demanding that the graviton luminosity $\mathcal{L}_G$ (energy loss due to gravitons per unit time) be smaller than the total observed luminosity $\mathcal{L}_{\text{SN}}$,

$$\mathcal{L}_G \lesssim \mathcal{L}_{\text{SN}} \sim 10^{56}\,\text{GeV}\,\text{s}^{-1}\,. \tag{35}$$

The most stringent bound comes from contributions to $\mathcal{L}_G$ from nucleon-nucleon bremsstrahlung. In Ref. [3] ADD obtain

$$M_{(4+n)} > 10^{\frac{15-4.5n}{n+2}}\,\mathrm{TeV}\,, \tag{36}$$

or more explicitly,

$$n=2\ : \qquad M_{(6)} > 30\,\mathrm{TeV}\,, \qquad r_{(2)} < \ 0.7\,\mu\mathrm{m}\,. \tag{37}$$

$$n=3\ : \qquad M_{(7)} > 2\,\mathrm{TeV}\,, \qquad r_{(3)} < \ 1\,\mathrm{nm}\,. \tag{38}$$

- The production of gravitons in supernovae may also leave its imprint in the cosmic diffuse gamma-ray background. In Ref. [15] Hannestad and Raffelt calculate the expected contribution to the MeV gamma-ray spectrum from the two-photon decay of all KK gravitons that were emitted by supernovae throughout the history of the universe. In doing so, they assume that typically a fraction of 0.5 to 1 % of the total energy of a supernova is converted into gravitons. From the non-observation of MeV bumps in the photon spectrum (as measured by the EGRET telescope on board NASA's Compton satellite) they derive the following bounds:

$$n=2\ : \qquad M_{(6)} > 84\,\mathrm{TeV}\,, \qquad r_{(2)} < \ 0.09\,\mu\mathrm{m}\,. \tag{39}$$

$$n=3\ : \qquad M_{(7)} > 7\,\mathrm{TeV}\,, \qquad r_{(3)} < \ 0.2\,\mathrm{nm}\,. \tag{40}$$

4.2 Heating of neutron stars

- In a core collapse supernova most KK gravitons are produced close to the kinematic threshold. That is, for a core temperature of $T \sim 30\,\mathrm{MeV}$ the typical KK graviton mass is of order 100 MeV. Most gravitons leave the supernova core with rather non-relativistic velocities ($v \simeq 0.5\,c$) such that a large fraction of them ends up being gravitationally retained in a cloud around the neutron star remnant. Trapped in this cloud the KK gravitons subsequently decay into pairs of neutrinos, electrons and photons on a time scale comparable to the age of the universe. Because of that, one expects that neutron stars should shine brightly in 100 MeV gamma-rays. According to the data taken by EGRET this is not the case. In Ref. [16] Hannestad and Raffelt deduce the following bounds for the ADD scenario:

$$n=2\ : \qquad M_{(6)} > 500\,\mathrm{TeV}\,, \qquad r_{(2)} < \ 3\,\mathrm{nm}\,. \tag{41}$$

$$n=3\ : \qquad M_{(7)} > 30\,\mathrm{TeV}\,, \qquad r_{(3)} < \ 0.01\,\mathrm{nm}\,. \tag{42}$$

- The KK graviton decay also causes an excess heating of neutron stars. Such an effect should have been seen by the Hubble Space Telescope which is able to observe the thermal emission from the surface of neutron stars. The lack of such an excess heat leads to the currently most stringent bounds on the ADD scenario [16],

$$n=2\ : \qquad M_{(6)} > 1700\,\mathrm{TeV}\,, \qquad r_{(2)} < \ 0.2\,\mathrm{nm}\,. \tag{43}$$

$$n=3\ : \qquad M_{(7)} > 60\,\mathrm{TeV}\,, \qquad r_{(3)} < \ 5\,\mathrm{pm}\,. \tag{44}$$

  In view of these results one has to admit that two or three large compact dimensions are most likely ruled out as possible explanations to the hierarchy problem. Analyses similar to the ones discussed in this section, but based on data from the Fermi Large Array Telescope are expected to update the bounds on the ADD scenario in the near future. Perhaps they will finally close the case.

5 Colliders

At TeV colliders the coupling of the KK gravitons to SM fields becomes unsuppressed opening up the possibility of directly probing the new gravitational physics. The main processes relevant to collider searches are graviton emission and graviton exchange. On top of that, if the fundamental Planck scale really is of order TeV, quantum and strong gravitational effects such as string excitations or the generation of microscopic black holes may be observable in high-energy particle collisions.

5.1 Graviton emission and exchange

- The inclusive cross section for graviton production at colliders scales like

$$\sigma_G(E) \propto \frac{E^n}{M_{(4+n)}^{2+n}} \times \text{multiplicity}(n) \tag{45}$$

where the model-dependent multiplicity factor accounts for all new degrees of freedom in the gravity sector (gravitons, graviphotons, ...). From Eq. (45) it is clear that graviton emission may be seen at colliders if the energy scale E in the scattering process is of order $M_{(4+n)} \sim 1\,\text{TeV}$. The emission of a graviton into the bulk of the higher dimensions would manifest itself in signatures such as

$$\gamma + E_T^{\text{miss}}\,, \quad \text{jet} + E_T^{\text{miss}}\,, \quad \text{initial / final state gravi-bremsstrahlung} \tag{46}$$

- If $M_{(4+n)}$ is the scale of quantum gravity / string theory, stringy effects should also be seen in TeV colliders. One would then expect Regge excitations of all SM particles to show up as narrow resonances and sharp masses. SM particles would, however, not escape into the extra dimensions as long as they are considered to correspond to open strings attached to a 3-brane. These open strings could ultimately wind around the extra dimensions. But the masses of these winding modes would be given as integer multiples of $r_{(n)} m_s^2$ (with m_s being the string scale) making them inaccessible at TeV colliders.

- Both experiments at TeVatron, CDF and D0, have performed searches for signals of graviton emission. At the parton level the relevant processes are

$$q\bar{q} \to qG\,, \quad qg \to qG\,, \quad gg \to gG\,, \quad q\bar{q} \to \gamma G\,, \tag{47}$$

the first three of which resulting in a jet plus missing transverse energy and the last resulting in a photon and missing transverse energy. The main background to these processes are

$$\text{jet} + (Z \to \nu\nu)\,, \quad \text{jet} + (W \to \ell\nu, \ell\,\text{lost})\,, \quad \text{jet} + (W \to \ell\nu, \ell \to \gamma\,\text{misidentified}) \tag{48}$$

CDF and D0 looked for signatures of graviton emission after collecting the following amounts of data (integrated luminosity):

$$\gamma + E_T^{\text{miss}}\,:\, 2\,\text{fb}^{-1}\,(\text{CDF})\,, 1.05\,\text{fb}^{-1}\,(\text{D0})\,, \quad \text{jet} + E_T^{\text{miss}}\,:\, 1.1\,\text{fb}^{-1}\,(\text{CDF})\,. \tag{49}$$

As the data agrees well with the expectation from the standard model CDF and D0 are able to constrain the higher-dimensional Planck mass in the ADD scenario,

$$n = 2\,: \qquad M_{(6)} > 921\,\text{GeV}\,(\text{D0})\,, \qquad M_{(6)} > 1400\,\text{GeV}\,(\text{CDF})\,. \tag{50}$$

$$n = 3\,: \qquad M_{(7)} > 877\,\text{GeV}\,(\text{D0})\,, \qquad M_{(7)} > 1150\,\text{GeV}\,(\text{CDF})\,. \tag{51}$$

- D0 also looked for signatures of virtual KK graviton exchange in the di-electromagnetic (di-EM) channel,

$$pp \to G \to e^+e^-, \gamma\gamma\,. \tag{52}$$

Here, the background processes are SM multijet events and events with a photon and a jet that are misidentified as di-EM events. With $1.05\,\text{fb}^{-1}$ of data the D0 collaboration is able to derive the following bounds:

$$n = 2\,: \quad M_{(6)} > 2.09\,\text{TeV}\,. \tag{53}$$

$$n = 3\,: \quad M_{(7)} > 1.94\,\text{TeV}\,. \tag{54}$$

- In a first study with LHC data the authors of Ref. [18] compared the full graviton-exchange amplitude for one to six extra dimensions to dijet events recorded by ATLAS,

$$pp \to G \to \text{jj}\,. \tag{55}$$

The full exchange amplitude turns out to be a function of $M_{(4+n)}$ as well as Λ, the maximal sensible KK mass beyond which quantum gravitational effects cannot be neglected. With $3.1\,\text{pb}^{-1}$ of data correlations between $M_{(4+n)}$ and Λ can be derived. Already now the limits set by LHC data are more stringent than those coming from LEP or the TeVatron.

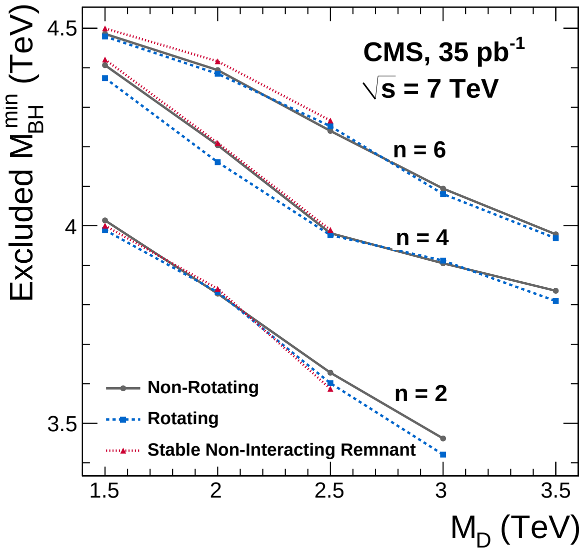

Figure 5: The 95 % confidence level limits on the black hole mass as a function of the higher-dimensional Planck scale $M_D \equiv M_{(4+n)}$ for several benchmark scenarios. The area below each curve is excluded by the search performed in Ref. [19].

5.2 Microscopic black holes

- The possibility of creating black holes at the LHC is a consequence of the low-lying fundamental Planck scale in the ADD scenario. A microscopic black hole is formed when the energy transfer E is larger than the higher-dimensional Planck mass $M_{(4+n)}$ and the impact parameter b is smaller than the Schwarzschild radius r_s of a black hole with mass E,

$$M_{\rm BH} = E > M_{(4+n)}\,, \quad b < r_s\,(M_{\rm BH})\,. \tag{56}$$

where the Schwarzschild radius is given as

$$r_s\,(M_{\rm BH}) = \frac{1}{\sqrt{\pi} M_{(4+n)}} \left[\frac{M_{\rm BH}}{M_{(4+n)}} \frac{8}{n+2} \Gamma\left(\frac{n+3}{2}\right)\right]^{\frac{1}{n+1}}\,. \tag{57}$$

From geometrical considerations the cross section for black hole production at the parton level can be estimated as

$$\sigma_{\rm BH} \sim \pi r_s^2\,. \tag{58}$$

For $n = 6$ and $M_{(10)} = 1\,\text{TeV}$ the cross section for black hole production in pp collisions can be become as large as $100\,\text{pb}$.

- The main signature of a black hole is the democratic and highly isotropic decay into all SM particles via Hawking radiation. The multiplicity of such events typically is very high. Because of color factors most of the energy of a decaying black holes ($\sim 3/4$) goes into quarks and gluons. The Hawking temperature of a black hole in $4+n$ dimensions is given as $T_H = \frac{n+1}{4\pi r_s} \sim 100\,\text{GeV}$. In total, one thus expects that indications for the production of black holes should show up in highly-energetic multijet events. Besides that, black hole decay may be accompanied by gravitational shock waves that carry away energy, momentum and angular momentum.

- The background to black hole signals are primarily QCD multijet events. Events with a photon, a Z-, or a W-boson plus jets and $t\bar{t}$-events also contribute to the background.

- In December 2010, with $35\,\text{pb}^{-1}$ of data at hand, the first CMS study on the production of microscopic black holes at the LHC was published [19]. No signal was found, instead the SM background could be well reproduced. This enabled the CMS collaboration to set upper limits on the production cross section of black holes and to thereby constrain the minimal possible mass of a black hole $M_{\rm BH}$ as a function of $M_{(4+n)}$, cf. Fig. 5.

References

[1] N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, "The Hierarchy problem and new dimensions at a millimeter," Phys. Lett. B **429** (1998) 263 [arXiv:hep-ph/9803315].

[2] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, "New dimensions at a millimeter to a Fermi and superstrings at a TeV," Phys. Lett. B **436** (1998) 257 [arXiv:hep-ph/9804398].

[3] N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, "Phenomenology, astrophysics and cosmology of theories with submillimeter dimensions and TeV scale quantum gravity," Phys. Rev. D **59** (1999) 086004 [arXiv:hep-ph/9807344].

[4] J. D. Lykken and S. Nandi, Phys. Lett. B **485** (2000) 224 [arXiv:hep-ph/9908505].

[5] L. Randall and R. Sundrum, "A Large mass hierarchy from a small extra dimension," Phys. Rev. Lett. **83**, 3370 (1999) [arXiv:hep-ph/9905221].

[6] L. Randall and R. Sundrum, "An Alternative to compactification," Phys. Rev. Lett. **83**, 4690 (1999) [arXiv:hep-th/9906064].

[7] H. Davoudiasl, J. L. Hewett and T. G. Rizzo, "Experimental probes of localized gravity: On and off the wall," Phys. Rev. D **63** (2001) 075004 [arXiv:hep-ph/0006041].

[8] M. E. Peskin and D. V. Schroeder, "An Introduction to quantum field theory," *Reading, USA: Addison-Wesley (1995) 842 p*

[9] E. G. Adelberger, B. R. Heckel and A. E. Nelson, "Tests of the gravitational inverse square law," Ann. Rev. Nucl. Part. Sci. **53** (2003) 77 [arXiv:hep-ph/0307284].

[10] S. A. Hoedl, E. G. Adelberger, J. Gundlach, B. R. Heckel and S. Schlamminger, "Gravity scale particle physics with torsion pendulums," *Prepared for 24th International Symposium on Lepton-Photon Interactions at High Energy (LP09), Hamburg, Germany, 17-22 Aug 2009*

[11] D. J. Kapner, T. S. Cook, E. G. Adelberger, J. H. Gundlach, B. R. Heckel, C. D. Hoyle and H. E. Swanson, "Tests of the gravitational inverse-square law below the dark-energy length scale," Phys. Rev. Lett. **98**, 021101 (2007) [arXiv:hep-ph/0611184].

[12] J. Chiaverini, S. J. Smullin, A. A. Geraci, D. M. Weld and A. Kapitulnik, "New experimental constraints on nonNewtonian forces below 100 microns," Phys. Rev. Lett. **90**, 151101 (2003) [arXiv:hep-ph/0209325].

[13] S. K. Lamoreaux, "Demonstration of the Casimir force in the 0.6 to 6 micrometers range," Phys. Rev. Lett. **78**, 5 (1997) [Erratum-ibid. **81**, 5475 (1998)].

[14] L. J. Hall and D. Tucker-Smith, "Cosmological constraints on theories with large extra dimensions," Phys. Rev. D **60**, 085008 (1999) [arXiv:hep-ph/9904267].

[15] S. Hannestad and G. Raffelt, "New supernova limit on large extra dimensions," Phys. Rev. Lett. **87**, 051301 (2001) [arXiv:hep-ph/0103201].

[16] S. Hannestad and G. G. Raffelt, "Stringent neutron star limits on large extra dimensions," Phys. Rev. Lett. **88**, 071301 (2002) [arXiv:hep-ph/0110067].

[17] V. Krutelyov [CDF and D0 Collaboration], "Searches for Large Extra Dimensions at the Tevatron," arXiv:0807.0645 [hep-ex].

[18] R. Franceschini, G. F. Giudice, P. P. Giardino, P. Lodone and A. Strumia, "LHC bounds on large extra dimensions," arXiv:1101.4919 [hep-ph].

[19] V. Khachatryan *et al.* [CMS Collaboration], "Search for Microscopic Black Hole Signatures at the Large Hadron Collider," Phys. Lett. B **697**, 434 (2011) [arXiv:1012.3375 [hep-ex]].