i Electroweak phase transition [8.11.11] Φ

Sections

- Computation of the effective potential at finite temperature in the SM
- Phase transition from the false to the true vacuum : Bubble nucleation
- EWPT and Baryogenesis

erences:

G. W. Anderson & L. J. Hall PRD 45, p.2685, 1992
M. E. Carrington PRD 45, p. 2933, 1992
S. Coleman PRD 15, p. 2929, 1977
D. Bödeker et al. hep-ph/0412366, 2004
Quiros, Helvetica Physica Acta
eskin & Schroeder, Cap 8, 11 and 19

Boris Panes

· Computation of the effective potential in the SM [1] and [2]

· Electro-weak sector of the SM

$$\mathcal{L}_{EW} = \mathcal{L}_{Higgs} + \mathcal{L}_{Gauge\text{-}field} + \mathcal{L}_{Fermion} + \mathcal{L}_{Yukawa} + \mathcal{L}_{gauge\text{-}fixing}$$

$$\mathcal{L}_{Higgs} = (D_\mu \phi)^\dagger (D^\mu \phi) + c^2(\phi^\dagger\phi) - \lambda(\phi^\dagger\phi)^2 \quad \text{with}$$

$$D_\mu = \partial_\mu + ig\frac{\tau^a}{2}A^a_\mu + ig'\frac{1}{2}B_\mu$$

$$\phi = \frac{1}{\sqrt{2}}\begin{bmatrix} \phi_3 + i\phi_4 \\ \phi_1 + i\phi_2 \end{bmatrix}$$

From the 3-level form of the potential we can assume that the v.e.v of the Higgs field can be taken as constant, and in general different from zero. This feature is conserved when we add 1-loop corrections and also temperature effects (although we can advance that at some temperature the v.e.v is null).

In order to consider a $\mathcal{L}_{EW}$ written in terms of fields with zero v.e.v we can expand the Higgs field around the constant v.e.v.

$$\phi \rightarrow \frac{1}{\sqrt{2}}\begin{bmatrix} \phi_3 + i\phi_4 \\ v + \chi \quad i\phi_2 \end{bmatrix} \quad \text{with} \quad v = \langle\Omega|\phi_1|\Omega\rangle \text{ so that } \langle\Omega|\chi|\Omega\rangle = 0$$

· The Higgs fields acquire mass terms.

$$m^2_\chi(v) = 3\lambda v^2 - c^2$$
$$m^2_{\phi_{2,3,4}}(v) = \lambda v^2 - c^2$$

At the classical level $v^2 = \frac{c^2}{\lambda}$, ϕ_2, ϕ_3, ϕ_4 Goldstone bosons.

· From the covariant derivatives of the Higgs sector, gauge bosons acquire mass terms given by

$$\sim A^\dagger_{\mu A} M^2_{AB}(v) A^\mu_B \quad \text{with} \quad M^2_{AB}(v) = \begin{bmatrix} g^2v^2/4 & 0 & 0 & 0 \\ 0 & g^2v^2/4 & 0 & 0 \\ 0 & 0 & g^2v^2/4 & -gg'v^2/4 \\ 0 & 0 & -gg'v^2/4 & g'^2v^2/4 \end{bmatrix}$$

· Working in the approximation that the only non-zero Yukawa coupling is the t-quark coupling denoted by f, the $\mathcal{L}_{Yukawa}$ term becomes

$$\mathcal{L}_{Yukawa} = f\frac{v}{\sqrt{2}}\bar{t}t + \frac{f}{\sqrt{2}}\bar{t}\chi t - \frac{if}{\sqrt{2}}(\bar{t}\gamma_5\phi_2 t)$$
$$+\frac{f}{2\sqrt{2}}\left[\bar{b}'(1+\gamma_5)(-\phi_3+i\phi_4)t - \bar{t}(1-\gamma_5)(\phi_3+i\phi_4)b'\right]$$

the top quark aquires a mass $m_t^2(v) = \frac{f^2v^2}{2}$

· Working in the R_ξ gauge, where the gauge-fixing part of $\mathcal{L}_{EW}$ is:

$$\mathcal{L}_{gauge-fixing} = -\frac{1}{2\xi}(\partial^\mu A^a_\mu - \tfrac{1}{2}\xi g v\chi^a)^2 - \frac{1}{2\xi}(\partial^\mu B_\mu - \tfrac{1}{2}\xi g' v\chi)^2$$

where $\chi_a = \phi_2, \phi_3, \phi_4$. In the Landau gauge $\xi \to 0$. In this gauge the ghost fields are massless and do not contribute to the v-dependent part of the 1-loop effective potential. Note that the $V_{eff}(v)$ will be gauge dependent, however physical quantities, such as the critique temperature are gauge independent.

- <u>Finite temperature</u> <u>effective potential</u> <u>at 1-loop</u>

$\boxed{v \text{ in this talk}}$

The Effective Potential $V_{eff}(\phi_{cl})$, with $\phi_{cl} = \langle\Omega|\phi|\Omega\rangle$, is defined as

$$\Gamma_T[\phi_{cl}] = \int_0^\beta d\tau \int d^3x\,[-V_{eff}[\phi_{cl}]]\quad, \text{with } \Gamma_T[\phi_{cl}]: \text{ Effective action at finite temperature}$$

Recalling Mathias talk the expression for the $V_{eff}[\phi_{cl}]$ is given by

$$V_{eff}[\phi_{cl}] = V_0(\phi_{cl}) + \sum_B \frac{1}{2}\int\frac{d^4p_E}{(2\pi)^4}\log[p_E^2 + m_B^2(\phi_{cl})] - \sum_F 2\int\frac{d^4p_E}{(2\pi)^4}\ln[p_E^2 + m_F^2(\phi_{cl})] + \dots$$

The tree-level contribution is $V_0(\phi_{cl}) = -\frac{1}{2}c^2\phi_{cl}^2 + \frac{1}{4}\lambda\phi_{cl}^4$. For the 1-loop contribution it is advisable to split the factors for each SM particle

$V_1(v) = V_1^{(0)}(v) + V_1^{(T)}(v)$

$V_1^{(0)}(v) = V_{1,\phi}^{(0)}(v) + V_{1,gb}^{(0)}(v) + V_{1,\psi}^{(0)}(v)$

$V_1^{(T)}(v) = V_{1,\phi}^{(T)}(v) + V_{1,gb}^{(T)}(v) + V_{1,\psi}^{(T)}(v)$

(0) : zero temperature contribution

(T) : finite temperature contribution

Defining the functions $f(m_x(v))$, $g(m_x(v))$ and $h(m_x(v))$ as

$$f(m_x(v)) = \frac{\Lambda^2}{32\pi^2} m_x^2(v) + \frac{m_x^4(v)}{64\pi^2}\left[\ln\left[\frac{m_x^2(v)}{\Lambda^2}\right] - \frac{1}{2}\right] \Rightarrow$$

Λ = cut-off
1-loop contributions (scalars and fermions)

$$g(m_x(v)) = \frac{T}{2\pi^2}\int dk\, k^2 \times \ln\left(1 - e^{-\beta(k^2+m_x^2(v))^{\frac{1}{2}}}\right) \Rightarrow$$

scalar finite T contributions

$$h(m_t(v)) = \frac{T}{2\pi^2}\int dk\, k^2 \ln\left(1 + e^{-\beta(k^2+m_t^2(v))^{\frac{1}{2}}}\right) \Rightarrow$$

Fermion finite T contribution

The contributions of each field are:

$$V_{1,\phi}(v) = f(m_1(v)) + 3f(m_2(v)) + g(m_1(v)) + 3g(m_2(v))$$

$$V_{1,gb}(v) = 3f(m_Z(v)) + 6f(m_W(v)) + 3g(m_Z(v)) + 6g(m_W(v))$$

$$V_{1,\psi}(v) = -12 f(m_t(v)) + 12 h(m_t(v))$$

The different factors in front of each contribution are given by the degrees of freedom for each particle. In order to eliminate the cut-off dependence the normalization condition:

$$\left[\frac{dV^{(0)}(v)}{dv}\right]_{v=\langle v\rangle_0} = 0 \quad \text{with } V^{(0)}(v) = V_{3L}(v) + V_1^{(0)}(v) \text{ is imposed.}$$

If we do not want to restrict to the ultrarelativistic regime $\frac{m(v)}{T} \ll 1$ the contributions of finite temperature have to be computed numerically. However there exist a problem in the Higgs sector because the masses are negative in the region

$$v^2 < \frac{c^2}{3\lambda} \text{ and } v^2 < \frac{c^2}{\lambda} \Rightarrow$$

Complex terms for $k \to 0$
"Infrared problem"

In order to solve the infrared problem the contribution at 1-loop level of the propagators are included. The net effect on the future expressions will be given by a shift on the masses in the m^3 terms (IV)

$$m_1^3(v) \to \left(m_1^2(v) + \Pi_1(0)\right)^{3/2} \quad \text{with} \quad \Pi_1(0) = T^2\left(\frac{g^2}{8} + \frac{(g^2+g'^2)}{16} + \frac{\lambda}{2} + \frac{f^2}{4}\right)$$

$$m_2^3(v) \to \left(m_2^2(v) + \Pi_2(0)\right)^{3/2} \qquad \Pi_2(0) = \Pi_1(0)$$

$$M^3(v) \to \underbrace{\left(M^2(v) + \Pi_{00}(0)\right)^{3/2}}_{\bar{m}_1^3(v),\ \bar{m}_2^3(v),\ \bar{M}^3(v),} \qquad \Pi_{00}(0) = \mathrm{diag}\left(\Pi_{00}^{(2)}(0), \Pi_{00}^{(2)}(0), \Pi_{00}^{(2)}(0), \Pi_{00}^{(1)}(0)\right)$$

$$\Pi_{00}^{(2)}(0) = T^2\left(\frac{2}{3}g^2 + \frac{g^2}{6} + g^2\right)$$

$$\Pi_{00}^{1}(0) = T^2\left(\frac{g'^2}{6} + \frac{5}{3}g'^2\right)$$

Considering this general solution, now we can safely take the limit of high temperature and write the approximated effective potential of the Standard Model:

$$V_{eff}(v,T) = D(T^2 - T_2^2)v^2 - ETv^3 + \frac{\lambda_T}{4}v^4$$

where $$D = \frac{1}{24v^2}\left(m_1^2(v) + 3m_2^2(v) + 6m_W^2(v) + 3m_Z^2(v) + 6m_t^2(v)\right)$$

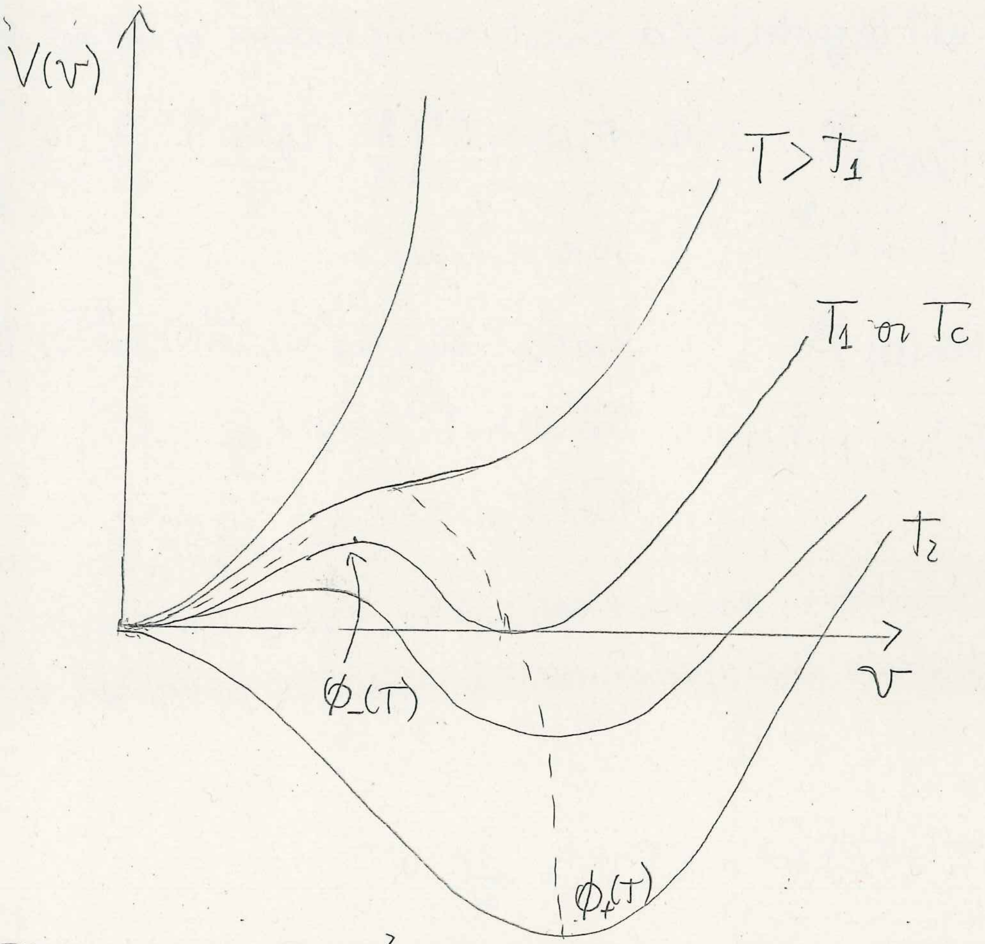
$$10^{-2} \sim E = \frac{1}{12\pi v^3}\left(\bar{m}_1^3(v) + 3\bar{m}_2^3(v) + 6\bar{m}_W^3(v) + 3\bar{m}_Z^3(v) + 6\bar{m}_t^3(v)\right)$$

$$\lambda_T = \lambda + \frac{1}{16v^4\pi^2}\left[6m_W^4(v)\ln\left(\frac{\lambda v^2}{c^2}\frac{T^2 a_b}{m_W^2(v)}\right) + 3m_Z^4(v)\ln\left(\frac{\lambda v^2}{c^2}\frac{T^2 a_b}{m_Z^2(v)}\right)\right.$$
$$+ m_1^4(v)\ln\left(\frac{m_1^2(v)}{2c^2}\frac{T^2 a_b}{m_1^2(v)}\right) + 3m_2^4(v)\ln\left(\frac{m_2^2(v)}{2c^2}\frac{T^2 a_b}{m_2^2(v)}\right)$$
$$\left. - 12m_t^4(v)\ln\left(\frac{\lambda v^2}{c^2}\frac{T^2 a_f}{m_t^2(v)}\right)\right] - \frac{1}{16\pi^2}\left[6\lambda^2 + \frac{3g^4}{16} + \frac{3(g^2+g'^2)^2}{32} - \frac{3f^2}{2}\right]$$

$$T_2^2 = \frac{1}{D}\left[\frac{1}{2}c^2 - \frac{3\lambda c^2}{32\pi^2}\right]$$

The terms D, λ_T and T_2^2 are independent on v. However the term E is not completly independent on v because the shift. For simplicity we take $E \to E(T=0) = E_0$

The Picture of the effective potential



T_1 indicates the degenerate case.

T_2 is found requiring
$V''(v=0) = 0$

$E_0 = E(T=0)$

$$T_1^2 = \frac{1}{1-\frac{E_0^2}{\lambda_T D}} T_2^2, \quad \phi_\pm(T_1) \approx \frac{3E_0T_1}{2\lambda_T} \pm \frac{E_0T_1}{2\lambda_T}, \quad \phi_\pm(T_2) \approx \frac{3E_0T_2}{2\lambda_T} \pm \frac{3E_0T_2}{2\lambda_T}$$

Assuming that $m_H(T=0)$ and $\langle v\rangle_0$ are known values, we can find the values of T_1 and T_2 using the conditions

$$\left(\frac{\partial V^0(v)}{\partial v}\right)_{v=\langle v\rangle_0} = 0 \quad \text{with } V^0(v) = V_{3L}(v) + V^{(0)}_{1-l}(v)$$
(Renormalization condition).

$$\left(\frac{\partial^2 V^0(v)}{\partial v^2}\right)_{v=\langle v\rangle_0} = m_H^2 \qquad \boxed{\langle v\rangle_0 = 246 \text{ GeV}}$$

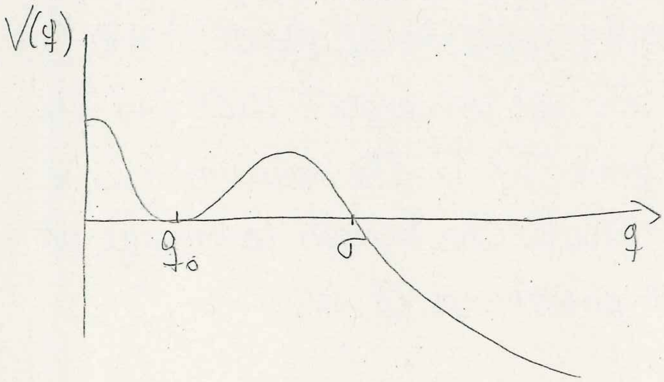
$$\hookrightarrow \quad T_2^2 = \frac{1}{4D}\left(m_H^2 - 3\lambda_T^{1\text{-loop}}\langle v\rangle_0^2\right)$$

$T_2 \sim (1.2 - 1.8)\, m_H$ for $m_H \sim 50 - 175$ GeV

$\leftarrow T_1 \sim T_2$

Bubble nucleations [3] and [1]

One particle Quantum-Mechanics discussion



Starting with a state in the equilibrium point q_0, we can expect that in a future time. The state penetrates the potential barrier to emerge at the point σ with zero kinetic energy. After this point the state evolves classically

- $L = \frac{1}{2}\dot{q}^2 - V(q)$

$$\frac{\Gamma}{V} \sim e^{-B/\hbar}\left[1 + \Theta(\hbar)\right] \quad \text{where } B = 2\int_{q_0}^{\sigma} dq\,(2V)^{1/2},$$

↳ Probability for the decay of the false vacuum per unit of time.

- In n-dimensions

$$L = \frac{1}{2}\dot{\vec{q}}\cdot\dot{\vec{q}} - V(\vec{q}), \quad B = \int_{\vec{q}_0}^{\vec{\sigma}} ds\,(2V)^{1/2} \qquad ds^2 = d\vec{q}\cdot d\vec{q}$$

$\vec{\sigma}$ is some point in the surface Σ of zeros of the potential and the integral is over that path for which B is a minimum.

$$\delta\int_{\vec{q}_0}^{\vec{\sigma}} ds\,(2V)^{1/2} = 0 \quad \Rightarrow \boxed{\frac{1}{2}\frac{d\vec{q}}{d\tau}\cdot\frac{d\vec{q}}{d\tau} - V = 0}$$

← Euler-Lagrange equations for the imaginary-time versions of Hamilton's principle.

$$\delta\int d\tau\, L_E = 0$$

$$L_E = \frac{1}{2}\frac{d\vec{q}}{d\tau}\cdot\frac{d\vec{q}}{d\tau} + V$$

From this equation it is possible to obtain the boundary conditions.

$$\underbrace{\lim_{\tau\to\pm\infty}\vec{q} = \vec{q}_0}_{} \quad \text{and} \quad \underbrace{\left.\frac{d\vec{q}}{d\tau}\right|_0 = 0}_{}$$

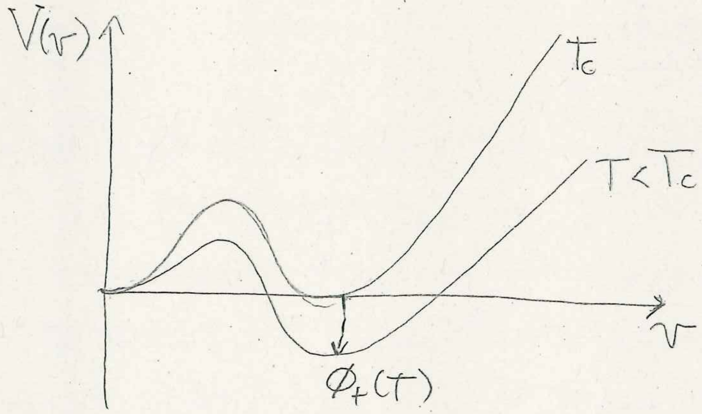
V=0 at $\tau=-\infty$ and symmetry for $\tau = -\tau$

we choose that at $\tau=0$ the kinetic energy is 0 ⇒ we reach the point $\vec{\sigma}$ at $\tau=0$.

→ This condition is independent on the specific point σ, but we fix the value of the energy at the scape point

Thus the parameter $B = \int_{-\infty}^{\infty} d\tau\, L_E = S_E$ + initial condition after crossing the barrier given by $\vec{q}(\tau=0)$

Quantum Field theory case ⊕ Temperature



Starting with a vacuum state in the metastable point $v=0$ for $T>T_c$ we can expect that for the regime $T<T_c$ the vacuum state penetrates the barrier to emerge at the point $v=\phi_+(T)$.

Extending the Quantum mechanics discussion to the realm of QFT⊕T the decay probability for this transition is given by.

$$\frac{\Gamma}{V} = A e^{-S_3/T}\left[1+\Theta(\hbar)\right]$$

O(3) symmetry

where $S_3 = \int d^3x \left[\frac{1}{2}(\vec{\nabla}\phi)^2 + V(\phi,T)\right] = 4\pi\int_0^{\infty} r^2 dr \left[\frac{1}{2}\left(\frac{d\phi}{dr}\right)^2 + V(\phi(r),T)\right]$

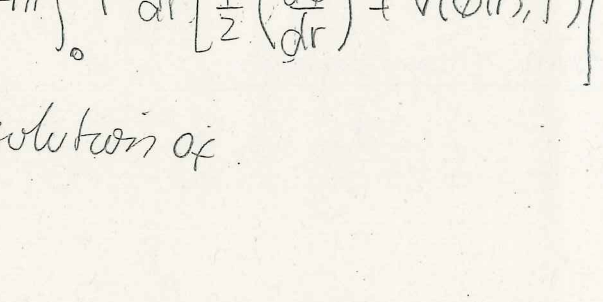
The configuration $\phi(\vec{x})$ is given by the solution of

$$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} = V'(\phi,T)$$

with boundary conditions $\lim_{r\to\infty}\phi(r)=0$ and $\left.\frac{d\phi}{dr}\right|_{r=0}=0$

Considering the limit $\epsilon=\frac{(T_1-T)}{(T_1-T_2)} << 1 \Rightarrow$ The solution for the field configuration at the moment just after the penetration of the potential is:

$$\phi(r) = \frac{v_0}{2}\left(1-\tanh\left[\left[\frac{\lambda_T}{2}\right]^{1/2} v_0(r-R)\right]\right),$$



"Thin-Wall bubble limit"

$$S_3 = \frac{8\pi}{3}\sqrt{2\lambda_T}\, v^3 R^2 - \epsilon\frac{16\pi}{3}\lambda_T v^4 R^3 \;;\quad \frac{\partial S}{\partial R}=0 \Rightarrow R_c = \frac{2}{\sqrt{2\lambda_T}\,(3\epsilon v)}$$

Hence the critical free-energy to temperature ratio is $\frac{S_c}{T} = 2.85\left[\frac{E}{(2\lambda_T)^{3/2}}\right]\frac{1}{\epsilon^2}$

The onset of bubble nucleation

to compute the temperature when the first bubble emerges we need two ingredients:

- $\frac{\Gamma}{\nu} = \Lambda^4(T) e^{-S_3(T)/T}$: probability per unit of volume to produce one transition from the false to the true vacuum. ($\Lambda^4(T) = \omega T^4$)

- $V_H(T) = 8\xi^3 \frac{M_{pl}^3}{T^6}$: Size of causal volume at a temperature T. $\xi \simeq 1/34$.

For probabilities small compared to one, the probability that a bubble was nucleated inside a causal volume during a temperature interval dT is given by:

$$dP = \frac{\Gamma}{\nu} \times V(T)\, dt$$

$$\text{with } t = \frac{\xi M_{pl}}{T^2}$$

$$dP \sim 16\omega\xi^4 \frac{M_{pl}^4}{T^4} e^{-S_3(T)/T} \frac{dT}{T}$$

Integrating this equation and requiring $P \sim 1$, the critical temperature of the first bubble nucleation is given by:

$$\frac{S_3(T_N)}{T_N} \sim 134 \Rightarrow T_c \gtrsim T_N \gtrsim T_2$$

At least one-bubble is created between T_c and T_2. Also is possible to prove that at a temperature $T_R \gtrsim T_2$ the complete space have been converted to the true vacuum.

EWPT and Baryogenesis

The Sakharov Criteria: a small baryon asymmetry η may have been produced in the early universe if three necessary conditions are satisfied:

- Baryon number violation $\Delta B \neq 0$
- C and CP violation
- Departure from thermal equilibrium

$$\eta_B = \frac{n_B - \bar{n}_B}{n_\gamma} \simeq 10^{-10}$$
$\downarrow$
observed

- At sufficiently high temperatures, $T \sim T_c$ for example, the Baryon number violation can be triggered by the production of Sphalerons

Roughly speaking: given $\eta_B \neq 0$ there exist a dilution process.

$$\frac{d\eta_B}{\eta_B} \propto \frac{\Gamma_{sp}}{HT} dT$$

with: Γ_{sp}: rate of sphaleron production

H: Hubble parameter

- The previous process does not produce baryon asymmetry if we start from the equilibrium $n_B^i = \bar{n}_B^i$ and C and CP are conserving. In the SM the amount of CP violation is not enough, thus is necessary to include effective operators like:

$$\mathcal{L} \subset \frac{X_{ij}}{M^2} (\phi^\dagger \phi) \phi u_i^c q_j$$

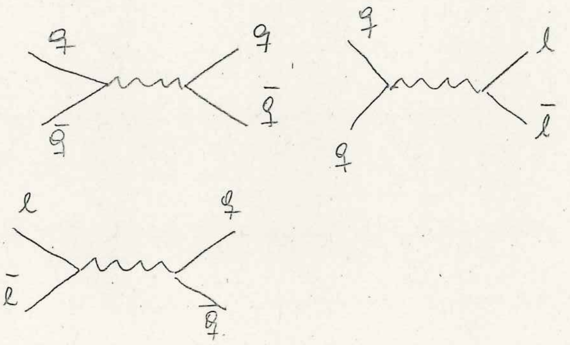
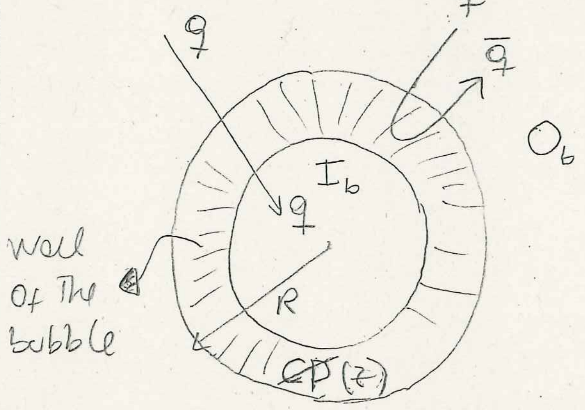
ϕ: Higgs field

$$\Rightarrow m_{ij} = y_{ij} \frac{v}{\sqrt{2}} + \frac{v^3}{2\sqrt{2} M^2} X_{ij}$$

This operator produce the non-trivial effect of different dispersion relations for particles and antiparticles when v is time-position dependent.

- Finally the departure from the equilibrium is given in the EWPT by the production of bubbles. When a bubble is produced the value o v time-position dependent.

A cartoon of the process

$T > T_c$ False vacuum age: $v = 0$, $\not{C}\not{P} = 0$ $n_B^i = \bar{n}_B^i$ Equilibrium system	**$T_N > T > T_2$** Bubble nucleation: $v = v(z)$
	 $n_B^{I_b} > \bar{n}_B^{I_b}$ and $n_B^{O_b} < \bar{n}_B^{O_b}$ $\hookrightarrow n_B^{I_b} > 0$ $\quad$ $\hookrightarrow n_B^{O_b} < 0$
$T_N > T > T_2$ sphaleron process: $v = v(z)$ $\Gamma_{sp}^{O_b} = k(\alpha_W T^4)$ $\quad 0.1 \lesssim k \lesssim 1.0$ $\Gamma_{sp}^{I_b} = C_1 \exp\left[-\left(C_2 \frac{\phi_+(T)}{T}\right)\right]$ $\Delta n_B^{O_b} \approx \Delta T\, n_B^{O_b}\, \Gamma_{sp}^{O_b}(T)/H(T)$ $\Delta n_B^{I_b} \approx \Delta T\, n_B^{I_b}\, \Gamma_{sp}^{I_b}(T)/H(T)$ If $\Gamma_{sp}^{O_b}(T) > H(T)$ Baryon number violation inside the Bubble $\Rightarrow$ dilution $\quad \Delta T < 0$	**$T \sim T_2$** End of transition: $v = \langle v \rangle_0$ $n_B(T_2) \approx n_B^{O_b}(T_2) + n_B^{I_b}$ we need sphaleron production outside the bubble **$T_2 > T \gtrsim 0$** Nowadays $\Delta n_B = \Delta T\, n_B(T_2)\, \Gamma_{sp}^{I_b}(T)/H(T)$ To avoid washout of the asymmetry we need $\Gamma_{sp}/H << 1$ just before T_2

For us the interesting condition is the avoiding of Baryon asymmetry washout. Including some detailed analysis this condition is

$$\left(\frac{\Gamma_{sp}}{H}\right)_{T_2} << 1 \qquad C_1 \sim C_2 \qquad \Rightarrow \qquad \left[\frac{E}{\lambda_T}\right] \gtrsim \frac{1}{2} \qquad \Rightarrow$$

with $\dfrac{\Gamma_{sp}}{HT} = C_1 \exp\left[-C_2 \dfrac{\phi_+(T)}{T}\right]$ $\qquad \dfrac{4Ev^2}{m_H^2} \gtrsim 1$

$$\boxed{\text{In the SM} \oplus \text{EWPT} \quad m_H \lesssim 50\ \text{GeV}}$$