Reheating and Preheating after Inflation : an Introduction

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These notes are intended to give a global picture of the general mechanism responsible for the creation of the hot Big Bang Universe as we are observing it. They are mainly based on Mukhanov's book [1], as well as on the Kofman et al. articles [2, 3], from which the plots have also been taken.

I. REHEATING

A. Evolution of the inflaton field

The dynamics of the inflaton field is described by a Klein-Gordon equation coupled to a Friedmann equation

$$0 = \ddot{\phi} + 3H\dot{\phi} + m^2\phi \,, \tag{1}$$

$$H^2 = \frac{8\pi}{3M_P^2}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right) . \tag{2}$$

For chaotic inflation $V(\phi) = \frac{1}{2}m\phi^2$, Eq. (2) can be conveniently parametrized using the Hubble parameter H and the angular variable θ defined via

$$\dot{\phi} = \sqrt{\frac{3}{4\pi}}HM_P\sin\theta \,, \qquad m\phi = \sqrt{\frac{3}{4\pi}}HM_P\cos\theta \,. \tag{3}$$

Using Eq. (1), the dynamics of these two independent variables is described by

$$\dot{H} = -3H^2\sin^2\theta \,, \tag{4}$$

$$\dot{\theta} = -m - \tfrac{3}{2}H\sin 2\theta \,. \tag{5}$$

For $mt \gg 1$, the second term in the r.h.s of Eq. (5) can be neglected ; the scalar field ϕ thus oscillates with a frequency $\omega \simeq m$. The Hubble rate can then be extracted from Eq. (4)

$$H(t) \equiv \left(\frac{\dot{a}}{a}\right) = \frac{2}{3t}\left(1 - \frac{\sin(2mt)}{2mt}\right)^{-1} , \tag{6}$$

where the oscillating term is small compared to unity. The behavior of $\phi(t)$ is obtained by solving Eq. (3) using an expansion in $(mt)^{-1}$

$$\boxed{\phi(t) \simeq \Phi(t)\cos(mt)\left(1 + \frac{\sin(2mt)}{2mt}\right)} , \tag{7}$$

where

$$\boxed{\Phi(t) = \frac{M_P}{\sqrt{3\pi}\,mt}} . \tag{8}$$

This solution has a simple interpretation which is illustrated in Fig. 1: at the end of the inflationary era, the friction term $3H\dot{\phi}$ becomes subdominant and Eq. (1) describes an oscillator, the amplitude of which gets damped due to the universe expansion.

The behavior of the scale factor can be extracted from Eq. (6) :

$$a(t) \propto t^{2/3} . \tag{9}$$

In consequence, the energy density of the field ϕ decreases in the same way as the energy density of non relativistic particles of mass m:

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}m^2\phi^2 \sim a^{-3} . \tag{10}$$

The inflaton oscillations can be interpreted as a collection of scalar particles, independent from each other, oscillating coherently at the same frequency m.

For time intervals larger than the oscillating period, the energy and number densities are related to the amplitude Φ in a simple way

$$\rho_\phi = \frac{1}{2}m^2\Phi^2 \,, \qquad n_\phi = \frac{1}{2}m\Phi^2 . \tag{11}$$

B. Decay of the scalar field

The reheating process occurs when the inflaton energy density is transferred to the energy density of other fields, leading to a decrease of the oscillating amplitude much faster than the one described in Eq. (7).

Consider the interaction of the inflaton field ϕ with a scalar field χ and a fermion field ψ

$$\begin{aligned} L &= \tfrac{1}{2}\phi_{,i}\phi^{,i} - V(\phi) + \tfrac{1}{2}\chi_{,i}\chi^{,i} - \tfrac{1}{2}m_\chi^2(0)\chi^2 \\ &+ \bar{\psi}\big(i\gamma^i\partial_i - m_\psi(0)\big)\psi - \tfrac{1}{2}g^2\phi^2\chi^2 - h\bar{\psi}\psi\phi \,. \end{aligned} \quad (12)$$

To be general, we suppose that the effective potential possess a minimum at $\phi = \sigma$, and is quadratic in ϕ

$$V(\phi) \sim \tfrac{1}{2}m^2\,(\phi-\sigma)^2 \;. \quad (13)$$

Doing the usual shift $\phi-\sigma \to \phi$ after symmetry breaking, the potential gets interaction terms linear in ϕ

$$\Delta\mathcal{L} = -g^2\sigma\phi\chi^2 - h\bar{\psi}\psi\phi\,, \quad (14)$$

which eventually lead to inflaton tree-body decays

$$\Gamma(\phi\to\chi\chi) = \frac{g^4\sigma^2}{8\pi m}\,, \quad \Gamma(\phi\to\bar{\psi}\psi) = \frac{h^2 m}{8\pi}\,. \quad (15)$$

In order to consider the effects related to particle production, let us consider the following Klein Gordon equation describing the homogeneous scalar field

$$\ddot{\phi} + 3H(t)\,\dot{\phi} + \big(m^2 + \Pi(\omega)\big)\,\phi = 0 \;. \quad (16)$$

where $\Pi(\omega)$ is the inflaton polarisation operator with 4-momentum $k = (\omega,0,0,0)$. Neglecting for simplicity the time dependance of H and $\mathrm{Im}\,\Pi$, Eq. (16) has the following solution

$$\boxed{\phi(t) \approx \Phi(t)\,\sin(mt)}\,, \quad (17)$$

where

$$\Phi(t) \approx \Phi_0\,\exp\left[-\frac{1}{2}\left(3H + \frac{\mathrm{Im}\,\Pi(m)}{m}\right)t\right] \;. \quad (18)$$

For $m \gg \min(2\,m_\chi, 2\,m_\psi)$, $\Pi(\omega)$ has an imaginary part which can be identify with the decay width thanks to the optical theorem through unitarity:

$$\mathrm{Im}\,\Pi(\omega) = m\,\Gamma_\phi\,. \quad (19)$$

The amplitude of the oscillations of the field ϕ decreases as

$$\boxed{\Phi(t) = \Phi_0\exp\left[-\frac{1}{2}(3H+\Gamma)t\right]} \quad (20)$$

due to the universe expansion as well as due to particle production. The field $\Phi(t)$ obeys the equation

$$\frac{1}{a^3}\frac{d}{dt}(a^3\Phi^2) = -\Gamma\Phi^2\,. \quad (21)$$

Multiplying the latter by m and using Eq. (11), one obtains the following equation for the number density

$$\boxed{\frac{d}{dt}(a^3 n_\phi) = -a^3\,n_\phi\,\Gamma}\,. \quad (22)$$

This is nothing else than a so-called Boltzmann equation, showing that the comoving number density of ϕ particles exponentially decreases with the decay rate Γ.

C. The reheating temperature

During the oscillating phase the Universe behaves in the same way as if it was dominated by non-relativistic particles of mass m : $H(t) \sim 2/(3t)$. The inflaton energy density is then transferred to the relativistic decay products, the energy density of which decreasing much faster than the energy of the oscillating field ϕ. The reheating process eventually ends when $H < \Gamma_\phi$. The *reheating time* is defined as the time at which the transition between these two regimes occurs : $t_{\mathrm{reh}} \simeq 2/(3\Gamma_\phi)$. By equating the inflaton energy density

$$\rho_\phi = \frac{M_P^2}{6\pi\,t_{\mathrm{reh}}^2}\,, \quad (23)$$

with the one of a thermal bath

$$\rho_{\mathrm{rad}} = \frac{\pi^2}{30}\,g_\star\,T_R^4\,, \quad (24)$$

containing $g_\star$ DOFs, one gets the *reheating temperature*

$$\boxed{T_R \simeq 0.2\;(200/g_\star)^{1/4}\,\sqrt{\Gamma_\phi\,M_P}} \quad (25)$$

It is remarquable that T_R does only depend on the particle theory parameters and not on the initial value of ϕ.

For $h \sim 10^{-2}$, $\Gamma_\phi \sim 10^8$ GeV and $T_R \sim 10^8$ GeV. For $g^2 \sim 10^{-2}$ and $v \sim 10^{11}$ GeV, $\Gamma \sim 10^3$ GeV and $T_R \sim 10^{10}$ GeV.

Imposing $\Gamma_\phi \ll m \sim 10^{-6}M_{\mathrm{P}}$ and $g_\star \sim \mathcal{O}(100)$, $T_R \ll 10^{15}$ GeV, thus rendering GUT baryogenesis impossible in such kind of scenarios. Also, $T_R \leq 10^9 - 10^{10}$ GeV in order to avoid the overproduction of gravitinos, an upper bound which turns out to be in tension with thermal leptogenesis scenarios [5].

D. Remarks

1. If one would have considered the interaction $\Delta\mathcal{L} = -\frac{g^2}{2}\phi^2\chi^2$, then the rate in that case is $\Gamma(\phi\phi \to \chi\chi) \sim \frac{g^2\phi^2}{8\pi m}$, which is a function of ϕ. Since $\phi \sim t^{-1}$ and $H \sim t^{-1}$, this interaction rate never overcomes the Hubble expansion rate, thus rendering reheating through such an interaction impossible.
2. We have perturbative reheating if and only if Γ_ϕ can decrease more slowly than t^{-1}.
3. Typically, for reheating to complete, one either has to have symmetry breaking or coupling to fermions.

Reheating thus implies constraints on the structure of the theory and on the couplings of ϕ to the other fields.

II. PREHEATING

A. Intuitive Picture

Up to know, particle production has been addressed disregarding the previously created χ particles. It turns out that Γ_{eff}, the effective ϕ decay width, can be much larger than Γ_ϕ due to *Bose enhancement* effects.

If $m \gg m_\chi$, most of the χ particles produced in ϕ decays carry momentum $k \simeq m/2$, and lead to a Bose enhancement of the ϕ decay. This can be seen at first if we consider the matrix element of such a process:

$$|\langle n_\phi - 1, n_{\mathbf{k}} + 1, n_{-\mathbf{k}} + 1|\hat{\mathbf{a}}^+_{\mathbf{k}}\hat{\mathbf{a}}^+_{-\mathbf{k}}\hat{\mathbf{a}}^-_\phi|n_\phi, n_{\mathbf{k}}, n_{-\mathbf{k}}\rangle|^2 = (n_{\mathbf{k}} + 1)(n_{-\mathbf{k}} + 1)n_\phi \quad (26)$$

where $n_{\pm\mathbf{k}}$ are the occupation numbers of the χ particles. One can thus define an *effective* decay width

$$\boxed{\Gamma_{\text{eff}} \simeq \Gamma_{\phi\to\chi\chi}\,(1 + 2\,n_k)} \quad (27)$$

The ϕ particles decay at rest into two χ particles, and if $g\phi \ll m^2/8$, each of them carries a momentum k located in the thin shell Δk around k_0

$$k = k_0 \pm \Delta k\,, \quad (28)$$

where we assumed $m^2 \gg m_\chi^2 + 2g\phi$ and where

$$k_0 = \frac{m}{2}\,, \qquad \Delta k = \frac{2g\Phi}{m}\,. \quad (29)$$

The occupation number n_k can then be expressed

$$n_{k=m/2} = \frac{n_\chi}{(4\pi k_0^2\Delta k)/(2\pi^3)} \simeq \frac{\pi^2\Phi}{g}\,\frac{n_\chi}{n_\phi}\,. \quad (30)$$

In consequence, we have Bose enhancement as soon as

$$\boxed{n_\chi > \frac{\pi^2\Phi}{g}\,n_\phi}\,, \quad (31)$$

i.e. as soon as a fraction $\sim g$ of ϕ particles has been converted into χ ones. *The elementary theory of reheating fails quickly after the beginning of reheating.*

The Boltzmann equation Eq. (15) can then be rewritten

$$\frac{1}{a^3}\frac{d(a^3 n_\chi)}{dN} = \frac{g^2}{2m^2}\left(1 + \frac{2\pi^2\Phi}{g}\,\frac{n_\chi}{n_\phi}\right) n_\phi\,, \quad (32)$$

where $N \equiv mt/2\pi$ is the number of inflation oscillations up to t. Neglecting the expansion, and assuming Φ to be constant disregarding the particle production, one gets

$$\boxed{n\chi \propto \exp\left(4\pi\,\mu\,N\right)}\,, \quad (33)$$

where $\mu \equiv \pi g\Phi M_P/(4m^2)$ is the *parameter of instability.*

B. EOM for quantum fluctuations

Collective effects such as Bose enhancement limit the range of applicability of the elementary theory of reheating. Therefore, a rigorous description of the first stage of reheating, commonly referred to as *preheating*, can only be based on non-perturbative techniques. In particular, we will show in this second part of the talk how the phenomenon of *parametric resonance* may result in explosive particle production. The physical picture one should keep in mind is the following: Due to their coupling to the coherently oscillating classical inflaton field ϕ quantum fluctuations of the scalar field χ experience a resonant amplication. This causes an exponential growth of the corresponding occupation numbers.

The details of reheating in combination with spontaneous symmetry breaking will be the subject of the next seminar talk. For now, we thus restrict ourselves to a toy model-like scenario of chaotic inflation in which the scalar potential is given as:

$$V(\phi,\chi) = \frac{1}{2}m^2\phi^2 + \frac{1}{2}m_\chi^2(0)\chi^2 + \frac{1}{2}g^2\phi^2\chi^2 \tag{34}$$

The time evolution of the quantum fluctuations of the field χ is ruled by the classical equation of motion – the Klein-Gordan equation in an expanding flat Friedmann-Lemaître-Robertson-Walker Universe:

$$\ddot{\chi} + 3H\dot{\chi} - \frac{1}{a^2}\nabla_x^2\chi + V_{,\chi} = 0 \tag{35}$$

Writing χ in the Heisenberg representation allows us to proceed to Fourier space. With $\vec{x}$ and $\vec{k}$ denoting the comoving position and momentum vectors:

$$\chi(t,\vec{x}) = \int \frac{d^3k}{(2\pi)^{3/2}}\left[a_k\chi_k(t)e^{-i\vec{k}\cdot\vec{x}} + a_k^\dagger\chi_k^*(t)e^{i\vec{k}\cdot\vec{x}}\right] \tag{36}$$

The temporal part of the momentum eigenfunction with momentum $k = |\vec{k}|$ (field mode) χ_k hence satisfies:

$$\ddot{\chi}_k + 3H\dot{\chi}_k + \left(\frac{k^2}{a^2} + m_\chi^2(0) + g^2\phi^2(t)\right)\chi_k = 0 \tag{37}$$

where $\phi(t) = \Phi(t)\sin(mt)$. Eq. (37) describes a harmonic oscillator with variable frequency (parametric oscillator) that is damped by the expansion of the Universe, viz. the friction term $3H\dot{\chi}_k$. As is well known from classical mechanics a concerted choice of parameters may cause parametric oscillators to resonantly excite themselves, a feature which goes by the name of parametric resonance. In our context this means that depending on k we expect some of the modes χ_k to get parametrically excited.

To alleviate our further calculations we neglect the expansion of the Universe, set the bare χ mass $m_\chi(0)$ to zero and assume a slow variation in $\Phi(t)$ compared to the oscillation frequencies of the fields ϕ and χ:

$$H = 0\,;\; a = 1\,;\; m_\chi(0) = 0\,;\; \Phi(t) \approx \text{const.} \tag{38}$$

This leaves us with an effective χ mass $m_\chi^{\text{eff.}} = g\phi$. The scenario $m_\chi(0) \gg m$ would allow for the production of particles above the scale of inflation which could rescue GUT baryogenesis models.

Eq. (37) now turns into:

$$\ddot{\chi}_k + \omega_k^2(t)\chi_k = 0\,;\; \omega_k^2(t) = k^2 + g^2\Phi^2\sin^2(mt) \tag{39}$$

Substituting $z = mt$ and using $\sin^2(z) = \frac{1}{2}(1-\cos(2z))$ we may rewrite Eq. (39) as Mathieu's differential equation:

$$\boxed{\chi_k'' + (A_k - 2q\cos(2z))\chi_k = 0} \tag{40}$$

with the parameters A_k and q being defined as:

$$\boxed{A_k = \frac{k^2}{m^2} + 2q}\;;\;\boxed{q = \frac{g^2\Phi^2}{4m^2}} \tag{41}$$

In solid state physics when describing charge carriers in certain periodic crystalline solids the stationary Schrödinger equation may be cast as an Mathieu equation as well. That the unique solution of Eq. (40) for vacuum initial conditions closely resembles Bloch waves:

$$\psi_{n\vec{k}}(\vec{r}) = e^{i\vec{k}\cdot\vec{r}}P(\vec{k},n,\vec{r}) \tag{42}$$

where P is invariant under translations by a lattice vector. Standard references now tell us [4]:

$$\chi_k(z) = e^{m_k z}P(A_k,q,z) \tag{43}$$

where P is periodic in z with period π. The real part μ_k of the Mathieu exponent m_k is always non-negative:

$$\mu_k(q) = 0 \quad\Rightarrow\quad |\chi_k| \text{ is stable.} \tag{44}$$

$$\mu_k(q) > 0 \quad\Rightarrow\quad |\chi_k| \text{ grows exponentially.} \tag{45}$$

A primitive contour plot of μ_k as function of q and A_k, the so-called stability-instability chart of the Mathieu equation, is shown in Fig. 2. According to Eq. (41) we identify the different ranges of physical momenta that experience parametric resonance as the white regions above the line $A_0 = 2q$. The width of the resonance bands $\Delta A_k^{(l)}$, $l \in \mathbb{N}$, and the preheating effeciency are solely controlled by the parameter q which is related to the inflaton amplitude Φ:

$$q < 1 \quad\Rightarrow\quad \text{Narrow resonance;}\quad 2\pi\mu_k \ll 1 \tag{46}$$

$$q > 1 \quad\Rightarrow\quad \text{Broad resonance;}\quad 2\pi\mu_k \sim \mathcal{O}(1) \tag{47}$$

The occupation numbers n_k count by how many quanta the respective modes χ_k are populated. Matching the quantum with the classical expression for the energy eigenvalue of a harmonic oscillator we find:

$$n_k = \frac{\varepsilon_k}{\omega_k} - \frac{1}{2} = \frac{1}{\omega_k}\left(\frac{1}{2}\left|\dot{\chi}_k\right|^2 + \frac{1}{2}\omega_k^2\left|\chi_k\right|^2\right) - \frac{1}{2} \quad (48)$$

Instable modes $\chi_k \sim \exp\left(\mu_k z\right)$ hence entail exponentially growing occupation numbers n_k:

$$n_k \sim \exp\left(2\mu_k z\right) = \exp\left(2\mu_k m t\right) \quad (49)$$

That is, the growth rate of those n_k is proportional to the present occupation number:

$$\boxed{\dot{n}_k \sim \Gamma_{\rm PR} n_k} \; ; \; \boxed{\Gamma_{\rm PR} = 2\mu_k m} \quad (50)$$

whereby we have rediscovered the effect of Bose enhancement in our non-perturbative calculation. Finally notice that Eq. (49) implies that also the number density of χ particles n_k increases explosively:

$$n_\chi(t) = \int \frac{d^3p}{(2\pi)^3} n_k(t) \quad ; \quad p = k/a \quad (51)$$

Having outlined the generic features of preheating we will now turn to a more detailed discussion of the two regimes of narrow and broad resonance.

C. Narrow resonance regime ($q < 1$)

Fig. 3 displays numerical solutions for $\chi_k(t)$ and $n_k(t)$ with $k \simeq m$ in the narrow resonance regime ($q = 0.1$). As $\omega_k \simeq m$ for $k \simeq m$ and $q \ll 1$ the mode χ_k oscillates approximately with the same frequency as the inflaton field ϕ. The occupation number n_k increases ally in agreement with Eq. (49).

The structure of the instability bands is dictated by the theory of Mathieu's equation:

$$A_k^{(l)} \simeq l^2 \; ; \; \Delta A_k^{(l)} \simeq q^l \;\Rightarrow\; k^2 \simeq m^2\left(l^2 - 2q \pm q^l\right) \quad (52)$$

We recognize the first band as the widest and most important one. It is centered around $k \simeq m$, has a width of mq and exhibits an instability parameter $\mu_k^{(1)}$ of:

$$\mu_k^{(1)} \simeq \sqrt{(q/2)^2 - (k/m - 1)^2} \simeq q/2 \quad (53)$$

Combining Eqs. (50) and (53) we find $\Gamma_{\rm PR} = mq$. Given $k \sim lm$ in the $l^{\rm th}$ instability band preheating has a natural interpretation in the particle picture as the collective process $2l \times \phi \to \chi\chi$.

There are three main reasons why preheating is so much more efficient than ordinary inflaton decays: First, in preheating the growth rate of n_k is proportional to n_k itself, not to the number density of inflatons n_ϕ as in decays. Second, the perturbative decay rate of inflatons Γ_ϕ is suppressed by g^4/m whereas $\Gamma_{\rm PR}$ can be quite sizable if q is not too small (see below). Third, preheating can also produce χ particles off the mass shell which opens new channels for the energy transfer from the ϕ to the χ field.

$$\text{Decays:} \quad \omega_k^2 - k^2 = 0 \quad (54)$$

$$\text{Preheating:} \quad \omega_k^2 - k^2 = g^2\Phi^2\sin^2(mt) \quad (55)$$

A necessary condition for successful preheating is that it proceeds at a faster rate than decays:

$$\boxed{\Gamma_{\rm PR} = qm = \frac{g^2\Phi^2}{4m} \gtrsim \Gamma_\phi} \quad (56)$$

Once decays take over, $\Gamma_\phi > \Gamma_{\rm PR}$, the inflaton amplitude decreases exponentially and any resonance disappears. In more visual terms a violation of condition (56) also means that the intrinsic width of the inflaton mass eigenstate has become broader than the first resonance band around $k \simeq m$.

A realistic treatment of preheating beyond our toy model faces further complications. The expansion of the Universe, for instance, augments the inflaton decay rate with the friction term $3H$ and, within a time $\Delta t \sim qH^{-1}$, redshifts the χ modes out of the resonance layers. This provides us with two conditions for $\Gamma_{\rm PR} = qm$ the stronger of which is the second:

$$qm \gtrsim \Gamma_\phi + 3H \; ; \; qm \gtrsim \Delta t^{-1} \;\Rightarrow\; \boxed{q^2 m \gtrsim H} \quad (57)$$

The χ bosons may also be removed from the resonance bands as they change their momenta or decay into other particles due to secondary interactions (rescatterings). Finally, one must not neglect the backreactions of the χ particles on the inflaton field. Not only does the inflaton amplitude decrease due to χ production, also its effective mass receives ever-growing and eventually dominating contributions from the χ fluctuations:

$$\left(m_\phi^{\text{eff.}}\right)^2 = m^2 + g^2 \left\langle \chi^2 \right\rangle \;\Rightarrow\; q \to \frac{\Phi^2}{4\left\langle \chi^2 \right\rangle} \tag{58}$$

Preheating typically ends at time t_{PR} after N_{PR} inflaton oscillations when condition (57) becomes violated. This usually happens for:

$$q \sim \mathcal{O}\left(10^{-1}\right) \;\Rightarrow\; \Phi \sim m/g \tag{59}$$

$$\Rightarrow\; t_{\text{PR}} \sim \frac{gM_p}{3m^2}\;;\; \boxed{N_{\text{PR}} \sim \frac{gM_p}{6\pi m} \sim \text{few} \times 10} \tag{60}$$

Subsequently, reheating is described by the elementary theory of perturbative inflaton decays. As T_R is mainly sensitive to the last stages of reheating it should be calculated within the elementary theory and not at the end of preheating.

D. Broad resonance regime ($q > 1$)

Although we now know *everything* about preheating, there is still *more* to say about it, viz. about its sensitivity to initial conditions. In particular in chaotic inflation the initial amplitude of inflaton oscillations may be very large, $\Phi_0 \gtrsim M_p$, resulting in a very broad ($q \gg 1$) and extremly efficient parametric resonance. It can be shown that typically $\mu_k \simeq 0.18$. From Eq. (49) we then deduce:

$$n_k \sim \exp\left(2\mu_k m t\right) = \exp\left(4\pi\mu_k N\right) \sim 10^N \tag{61}$$

Fig. 4 displays numerical solutions for $\chi_k(t)$ and $n_k(t)$ with $k \simeq m$ in the broad resonance regime ($q \simeq 200$). As we see, the χ_k mode oscillates much faster than the inflaton field and particle production only occurs for very small values of $\phi(t)$. The former observation is due to the mostly large effective χ mass:

$$m_\chi^{\text{eff.}} = g\phi \simeq g\Phi \gg m \quad ; \quad \text{at most times} \tag{62}$$

explaining a ratio of order $q^{1/2}$ between the χ and ϕ oscillation frequencies. Turning to our second observation we note that in view of the comparably slow ϕ oscillation the frequency ω_k mostly only experiences an adiabatic variation. Hence the quantum number n_k of the parametric oscillator with variable frequency ω_k is almost an adiabatic invariant:

$$n_k \sim \frac{1}{2}\omega_k^2 \left|\chi_k\right|^2 / \omega_k \simeq \text{const.} \;\Rightarrow\; \left|\chi_k\right| \sim \omega_k^{-1/2} \tag{63}$$

It only varies when the adiabaticity condition imposed on the change of ω_k is violated:

$$\text{Particle production when:} \quad \boxed{\dot{\omega} \gtrsim \omega^2} \tag{64}$$

which, as we will show, only happens around the time when the inflaton passes through zero, $|\phi| \lesssim \phi_* \ll \Phi$. In these *acts of creation* the effective χ mass vanishes, ω_k becomes very small and the amplitude of the χ_k mode blows up, cf. Eq. (64).

For small ϕ we may approximate $\dot{\phi} = m\Phi\cos(mt) \simeq m\Phi$. With $\omega_k^2 = k^2 + g^2\phi^2$ and $\dot{\omega}_k \simeq \omega^{-1} g^2 \phi m \Phi$ we then deduce the range of excited momenta k from Eq. (64):

$$0 \leq k^2 \lesssim \left(g^2 \phi m \Phi\right)^{2/3} - g^2\phi^2 \tag{65}$$

which forces ϕ to be smaller than $(m\Phi/g)^{1/2}$ and becomes maximal for:

$$\phi = \phi_* \simeq \frac{1}{2}\left(m\Phi/g\right)^{1/2} \simeq \frac{1}{3}\Phi q^{-1/4} \tag{66}$$

The typical momenta k of particles that are produced in the broad resonance regime may then be estimated as:

$$0 \leq 2k \lesssim k_* = \left(gm\Phi\right)^{1/2} = \sqrt{2} m q^{1/4} \tag{67}$$

where k_* is a measure for the maximal momentum scale that can be reached during preheating. Eq.(67) tells us that typically $k \gg m$ which indicates the collective interaction of many ϕ quanta in the production of χ particles. Finally, we compute the duration of each act of creation:

$$\Delta t_* \simeq \frac{2\phi_*}{\dot{\phi}} \simeq \frac{\left(m\Phi/g\right)^{1/2}}{m\Phi} = k_*^{-1} \sim \omega_*^{-1} \tag{68}$$

Particle production occurs within one period of oscillation $\sim \omega_*^{-1} = \left(k_*^2 + g^2\phi_*^2\right)^{-1/2}$ of the mode χ_k which is in agreement with the uncertainty principle:

$$\boxed{\Delta t_* \omega_* \sim 1} \tag{69}$$

Due to the decrease in the inflaton amplitude the broad resonance eventually becomes narrow. Ref. [2] estimates that this will happen after a time $t_{\rm BR}$:

$$t_{\rm BR} \sim \frac{1}{m} \ln \left(\frac{m}{g^5 M_p} \right) \Rightarrow N_{\rm BR} \sim \mathcal{O}(10) \qquad (70)$$

At this time we approximately have:

$$q \simeq 1 \,;\, \Phi^2 \simeq \left\langle \chi^2 \right\rangle \,;\, \rho_\phi \simeq \rho_\chi \,;\, p \simeq \rho/3 \qquad (71)$$

That is, preheating facilitates an almost instantaneous transition from the epoch with vacuum-like equation of state to the epoch of radiation domination. Contrary to earlier expectations a prolonged intermediate stage of matter domination does not exist.

Bringing back into play the expansion of the Universe, the complexity in describing preheating in the broad resonance regime increases significantly. As a virtue of the expansion we note that all excited modes, $0 \leq k \lesssim k_*/2$, are redshifted away from k_*. This stabilizes preheating and makes it less sensitive to rescattering and backreaction effects. On the other hand, as a consequence of the momentum redshift and the rapid decrease in the inflaton amplitude, the resonance turns into what has been named a *stochastic resonance*.

$$\omega_k^2 = \frac{k^2}{a^2} + g^2 \Phi^2(t) \sin^2(mt) \qquad (72)$$

Due to the non-periodical variation of the frequency ω_k the phases of the mode χ_k at successive zero-crossings of the inflaton field, $\phi(t_i) = 0$, are completely uncorrelated thereby leading to only stochastic changes in the occupation number n_k. The number density of χ particles then only grows on average exponentially. Intermediately it may either increase or decrease. Such a process could never be explained in the classical particle picture. We thus realize that the creation of almost all particles that populate our present Universe was a purely quantum mechanical effect.

[1] V. Mukhanov, *Cambridge, UK: Univ. Pr. (2005) 421 p*

[2] L. Kofman, A. D. Linde, A. A. Starobinsky, Phys. Rev. Lett. **73**, 3195-3198 (1994). [hep-th/9405187].

[3] L. Kofman, A. D. Linde, A. A. Starobinsky, Phys. Rev. **D56**, 3258-3295 (1997). [hep-ph/9704452].

[4] N.W. Mac Lachlan, "Theory and Application of Mathieu functions," (Dover, New York, 1961).

[5] An elegant solution to circumvent the gravitino problem is to consider right handed neutrinos as being responsible for the reheating of the Universe. This scenario present the peculiar property of being able to produce matter and dark matter from a single mechanism (see Kai's talk this afternoon !)

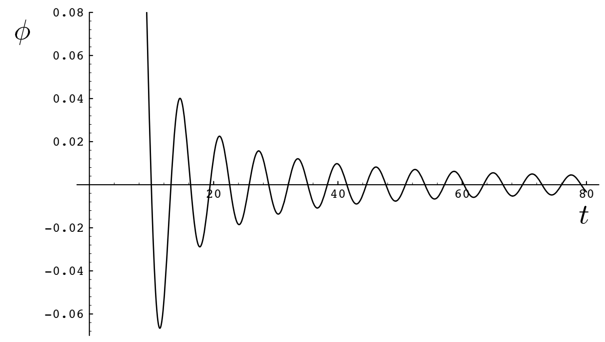

FIG. 1: Oscillations of the field ϕ after inflation in the theory $\frac{1}{2}m^2\phi^2$. The value of the scalar field is measured in units of M_p, time is measured in units of m^{-1}. Figure taken from Ref. [3].

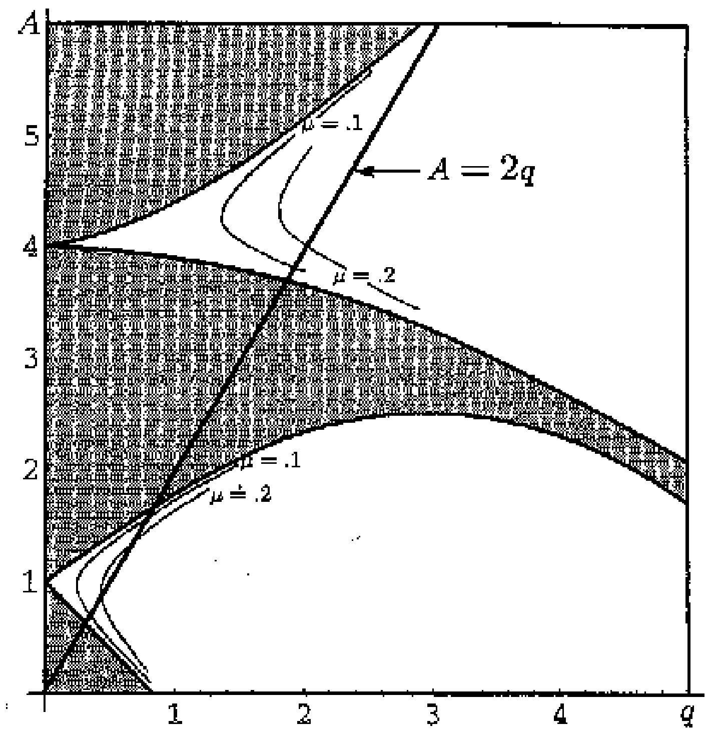

FIG. 2: Sketch of the stability-instability chart of the canonical Mathieu equation (40) taken from Ref. [2]. Gray bands indicate regions of stability ($\mu_k = 0$), white bands regions of instability ($\mu_k > 0$). The line $A_0 = 2q$ shows the values of A_k and q for $k = 0$. For $k \neq 0$ the corresponding function graphs $A_k(q)$ are obtained by parallely shifting upwards the line A_0 by k^2/m^2.

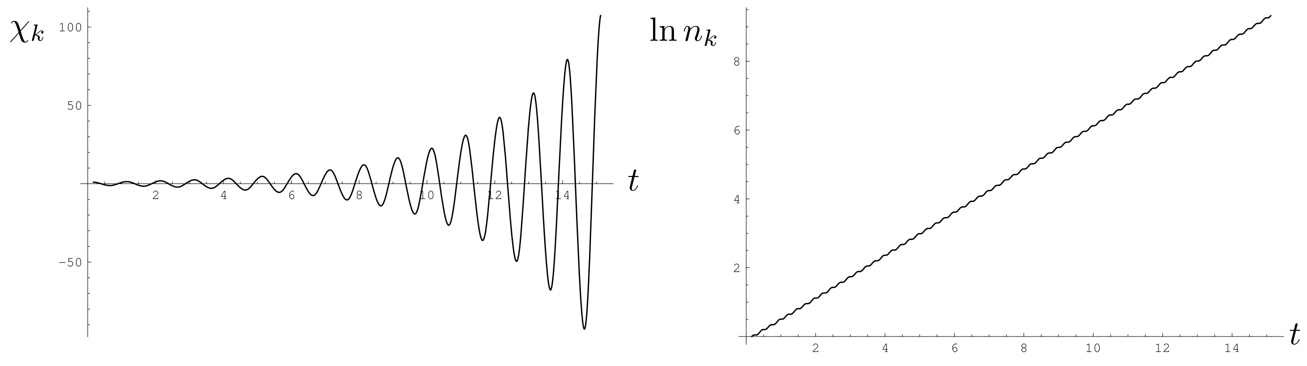

FIG. 3: Narrow parametric resonance for the momentum k corresponding to the maximal speed of growth, $k \simeq m$. Time is shown in units of $m/2\pi$, which is equal to the number of oscillations of the inflaton field ϕ. For each oscillation of the field ϕ the mode of the field χ oscillates one time. Left: growth of the mode χ_k, Right: logarithm of the occupation number n_k of particles in this mode. In this particular example $q = 0.1$ and hence $\mu_k \simeq q/2 = 0.05$. Figures taken from Ref. [3].

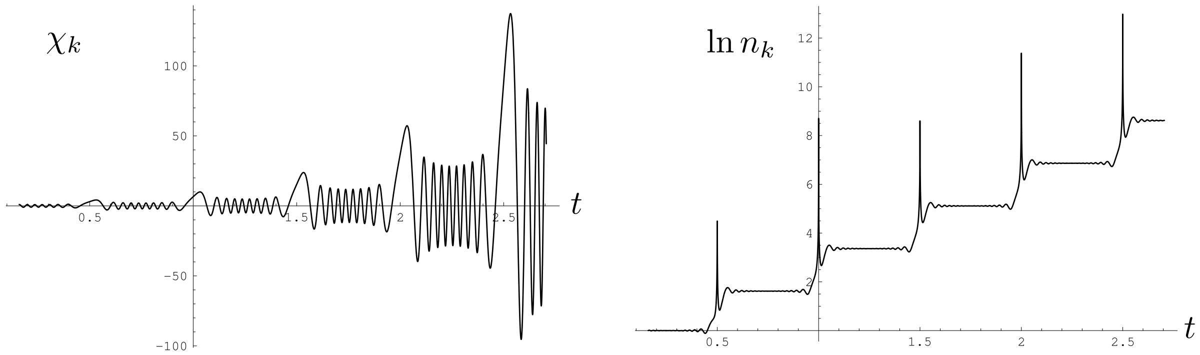

FIG. 4: Broad parametric resonance for $k \simeq m$. For each oscillation of the field ϕ the mode of the field χ oscillates many times. The peaks in the χ_k oscillations corresponds to the time intervals when $|\phi| \lesssim \phi_*$. In this particular example $q \simeq 200$ and the average rate of growth of n_k is close to maximal in the context of our model, $\mu_k \simeq 0.3$. Figures taken from Ref. [3].