NMSSM Inflation

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1 Notation

Superpotentials W and frame functions Ω are to be regarded as functions of either the superfields or their lowest components, depending on the context. We work in units where $M_{\rm Planck}/\sqrt{8\pi} = 1$.

2 Recap: Standard Model Higgs Inflation

As a reminder we want to go back to the idea of Standard Model Higgs inflation first.
The Lagrangian of the SM non-minimally coupled to gravity reads

$$\mathcal{L} = \sqrt{-g}\ [\mathcal{L}_{SM} - \frac{M^2}{2}R - \frac{1}{2}\xi\Phi^\dagger\Phi R], \qquad (1)$$

where M is a mass parameter, R the scalar curvature, ξ a coupling constant to gravity and Φ the Higgs doublet.
Minimal coupling ($\xi = 0$) is only valid classically and leads also to bad inflation properties, because the self-coupling of the Higgs field is too large and matter fluctuations are many orders of magnitude larger than observed. Therefore one has to consider the non-minimally coupled case.
In unitary gauge the corresponding Jordan frame Lagrangian with the Higgs field ϕ looks like

$$\mathcal{L}_J = \sqrt{-g}\ [-\frac{M_P^2 + \xi\phi^2}{2}R + \frac{\partial_\mu\phi\partial^\mu\phi}{2} - \frac{\lambda}{4}(\phi^2 - v^2)^2]. \qquad (2)$$

After the usual conformal transformation to the Einstein frame

$$g^E_{\mu\nu} = f(\phi)g_{\mu\nu} \quad \text{with} \quad f(\phi) = 1 + \frac{\xi\phi^2}{M_P^2} \qquad (3)$$

and a redefinition of the higgs field to the new scalar field $\tilde{\phi}$

$$\frac{d\tilde{\phi}}{d\phi} = \sqrt{\frac{1}{f(\phi)} + \frac{6\xi^2\phi^2}{f(\phi)^2 M_P}}. \qquad (4)$$

the Lagrangian reads

$$\mathcal{L}_J = \sqrt{-g_E}\ [-\frac{M_P^2}{2}R^E + \frac{\partial_\mu\tilde{\phi}\partial^\mu\tilde{\phi}}{2} - V(\tilde{\phi})]. \qquad (5)$$

with the new Higgs potential

$$V(\tilde{\phi}) = \frac{1}{f(\tilde{\phi})^2}\frac{\lambda}{4}(\phi(\tilde{\phi})^2 - v^2)^2. \qquad (6)$$

For large values of $\phi(\phi >> \frac{M_P}{\sqrt{\xi}})$ the Higgs potential is exponentially flat:

$$V(\tilde{\phi}) = \frac{\lambda M_P^4}{4\xi^2}\left(1 + \exp\left(-\frac{2\tilde{\phi}}{\sqrt{6}M_P}\right)\right)^{-2}. \qquad (7)$$

Especially at $\tilde{\phi} >> M_P$ chaotic inflation is in principal possible and the predicted values for the spectral index $n = 1 - 6\epsilon + 2\eta \simeq 0.97$ for $N = 60$ e-foldings as well as the tensor to scalar perturbation ratio $r = 16\epsilon \simeq 0.0033$ are well within one sigma of the current WMAP measurements.
But nevertheless not everything is such nice as it looks like, when taking the quantum corrections into account. As we saw some weeks ago if the Standard Model Higgs were a gauge singlet, then Higgs inflation would be well behaved in the pure gravity and kinetic sectors, but requires a detailed analysis of the potential sector, where the theory is likely to fail at $\Delta = \frac{M_P}{\xi}$. However, since the Higgs exists as a complex doublet out of 4 real scalars, one has to treat it like a multi-field. But even if one goes to unitary gauge it leads to a cutoff $\Delta = \frac{M_P}{\xi}$ at tree-level.
Whether or not perturbation theory remains viable in the Standard Model Higgs inflation scenario, the Standard Model has other apparent shortcomings such as unsatisfactory grand unification and a lack of a suitable Dark Matter candidate. Therefore we want to have look at the supersymmetric extension of the Standard Model, firstly at an inflationary scenario within the MSSM.

3 Higgs inflation in the MSSM

The best starting point is the Lagrangian of supergravity. The purely bosonic supergravity Lagrangian in component fields looks as the following in the Einstein frame:

$$\mathcal{L}_E = \sqrt{-g_E}\left(-\frac{1}{2}R\left(g_E\right) - G_{i\bar{j}}\left(\phi,\phi^*\right)(D_\mu\phi^i)(D_\nu\phi^{*\bar{j}})g_E^{\mu\nu}\right) - \sqrt{-g_E}V_E\left(\phi,\phi^*\right). \quad (8)$$

The covariant derivative of the scalars is given by

$$D_\mu\phi^i = \partial_\mu\phi^i - A_\mu^a k_a^i, \quad (9)$$

where k_a^i are the Killing vectors and A_μ^a is the bosonic part of the auxiliary field. The positive-definite metric $G_{i\bar{j}}$ is given by

$$G_{i\bar{j}}\left(\phi,\phi^*\right) \equiv \frac{\partial}{\partial\phi^i}\frac{\partial}{\partial\phi^{*\bar{j}}}K\left(\phi,\phi^*\right), \quad (10)$$

where $K\left(\phi,\phi^*\right)$ is the real Kaehler potential. The scalar potential in Eq. (8) is given by

$$\begin{aligned} V_E &= V_E^F + V_D^F \\ &= e^K\left(\nabla_i W G^{i\bar{j}}\nabla_{\bar{j}}W^* - 3WW^*\right) \\ &+ \frac{1}{2}\left(\mathrm{Re}f\right)^{-1\ ab}D^a D^b. \end{aligned} \quad (11)$$

Hereby is $\nabla_i W$ the Kaehler covariant derivative of the superpotential W

$$\nabla_i W = \frac{\partial W}{\partial\phi^i} + \frac{\partial K}{\partial\phi^i}W \quad (12)$$

and D^a the real Killing prepotential

$$D^a = -i\frac{\partial K}{\partial\phi^i}k^{ai}, \quad (13)$$

which leads to

$$D^a = \phi_i^* T_j^{ai}\phi^j, \quad (14)$$

where $T_j^{ai}$is the representation of a gauge group. To sum up the Lagrangian is determined by

- the real Kaehler potential $K\left(\phi,\phi^*\right)$
- the holomorphic superpotential $W(\phi)$
- the holomorphic gauge kinetic function $f_{ab}(\phi)$ defining the action of the vector multiplets
- the frame function $\Omega(\phi,\phi^*)$
- the killing vectors k_a^i.

As usual we take K to have the canonical form

$$K = -3\ln\left(-\frac{\Omega}{3}\right), \quad (15)$$

where Ω stands for the frame function

$$\Omega = \phi_i^*\phi_i - 3. \quad (16)$$

In order to realize MSSM Higgs inflation one introduces a non-minimal coupling in the supergravity Lagrangian (Eq. (8)), such that it couples to a multiplet of chiral superfields.

$$\begin{aligned} \mathcal{L}_E &= \sqrt{-g_E}\left(-\frac{1}{2}\left(1 + X\left(\phi\right)\right)R\left(g_E\right)\right) \\ &- \sqrt{-g_E}G_{i\bar{j}}\left(\phi,\phi^*\right)(D_\mu\phi^i)(D_\nu\phi^{*\bar{j}})g_E^{\mu\nu} \\ &- \sqrt{-g_E}V_E\left(\phi,\phi^*\right). \end{aligned} \quad (17)$$

For that the additional multiplet $X(\phi)$ was introduced. Because of this non minimal coupling the frame function has to be adjusted to

$$\Omega_\chi = \phi_i^*\phi_i - 3 - \frac{3}{2}\left(X(\phi) + h.c.\right). \quad (18)$$

The unique possibility for X in the case of the MSSM is the following choice

$$X = \chi H_1 H_2, \quad (19)$$

where χ is constant (without loss of generality we choose $\chi > 0$). The most general possibility for the superpotential W (by neglecting all fields which do not appear in X) is

$$W = \Lambda + \mu H_1 H_2. \quad (20)$$

Also Λ and μ are constants. H_1 H_2 are the two Higgs doublets in the MSSM, of which we consider only the electromagnetism-preserving direction

$$H_1 = \begin{pmatrix} h_1 \\ 0 \end{pmatrix}, \quad H_2 = \begin{pmatrix} 0 \\ h_2 \end{pmatrix}, \quad (21)$$

with h_1 and h_2 to be real and positive. Defining

$$h_1 = h\cos\beta \qquad h_2 = h\sin\beta \quad (22)$$

we can rewrite Eq. (19) and Eq. (20) in the following way

$$W = \Lambda + \frac{1}{2}\mu h^2\sin(2\beta), \quad X = \frac{1}{2}\chi h^2\sin(2\beta). \quad (23)$$

As we are interested in the case $\chi \gg 1$ and $h^2 \ll 1$, we have to be careful and choose $\chi \sin(2\beta) \geq 0$ to avoid a singularity in the Kaehler potential (cf. Eq. (15)). Since we have chosen $\chi > 0$, the angle β is restricted therefore to $0 \leq \beta \leq \frac{\pi}{2} (\mathrm{mod}\ \pi)$.
Putting everything together one gets the following term for the F-part of the scalar potential V_E^F

$$V_E^F = -12 \cdot \frac{(6\Lambda + \mu h^2 \sin 2\beta)^2 + 12\chi\mu h^2(3\Lambda + \mu h^2 \sin 2\beta)}{(3\chi h^2 \sin 2\beta + 6 - 2h^2)^2(3\chi^2 h^2 - 2\chi h^2 \sin 2\beta + 4)} - \frac{12\mu^2 h^2}{(3\chi h^2 \sin 2\beta + 6 - 2h^2)^2(3\chi^2 h^2 - 2\chi h^2 \sin 2\beta + 4)} \tag{24}$$

According to Eq. (11) and with respect to Eq. (14) the D-term of the scalar potential V_E^D looks like the following

$$V_D = \frac{g^2}{2}\frac{9}{\Omega_\chi^2}\left(\phi^* T^a \phi\right)^2 \tag{25}$$

or especially in the case of MSSM

$$V_D = \frac{9}{\Omega_\chi^2}\frac{g'^2}{8}\left(H_1^\dagger H_1 - H_2^\dagger H_2\right)^2 + \frac{9}{\Omega_\chi^2}\frac{g^2}{8}\left(\sum_{i=1,2} H_i^\dagger \vec{\tau} H_i\right)^2 . \tag{26}$$

Considering Eq. (22) it gives

$$V_D = \frac{9(g^2 + g'^2)h^4 \cos^2 2\beta}{2(3\chi h^2 \sin 2\beta + 6 - 2h^2)^2}. \tag{27}$$

In the next step we want to have a look at the the scalar potential V_E within the regime $\chi h^2 \gg 1 \gg h^2$ for fixed β. One can see that for $\sin 2\beta \gg 2/(\chi h^2)$ (so except the region for very flat V_E^D) the potential V_E is dominated by

$$V_D \approx \frac{g^2 + g'^2}{2\chi^2}\cot^2 2\beta. \tag{28}$$

So it is easy to see that the slow-roll conditions are not fullfilled, because

$$\left|\frac{1}{V_D}\frac{\partial V_D}{\partial \beta}\right| = \frac{8}{|\sin 4\beta|} \tag{29}$$

cannot be made small.
In the D-flat region itself V_E^D vanishs for $\tan\beta = 1$. With the second derivative $\partial^2 V_D/\partial^2\beta > 0$ it is a minimum, which is stable against fluctuations in β. V_E simplifies then to

$$V_E = -\frac{2[3\Lambda^2 + 2\mu\chi h^2(2\mu h^2 + 3\Lambda)]}{3\chi^4 h^6}, \tag{30}$$

which again simplifies to

$$V_E = -8\frac{\mu^2}{(3\chi^3 h^2)} \tag{31}$$

for $\Lambda = 0$. As one can easily see, the scalar potential V_E converges to zero through negative values, which is not suitable for an inflationary scenario! So one can conclude that a Higgs inflation scenario within the MSSM is not possible.

4 Introduction to the NMSSM

In contrast to the MSSM superpotential

$$W_{MSSM} = \mu H_u H_d + \cdots \tag{32}$$

in the NMSSM there is an additional gauge singlet S introduced

$$W_{NMSSM} = \lambda S H_u H_d + \frac{1}{3}\kappa S^3 \cdots . \tag{33}$$

Also the soft SUSY breaking term in the MSSM

$$B\mu H_u H_d + \cdots \tag{34}$$

is adjusted by a S to the following in the NMSSM case

$$W_{NMSSM} = \lambda A_\lambda S H_u H_d + \frac{1}{3}\kappa A_\kappa S^3 \cdots . \tag{35}$$

5 The Einhorn-Jones proposal

Higgs inflation in the MSSM essentially fails because quartic terms in the Higgs potential come only from D-terms, and these vanish in the D-flat direction where β is stabilized. The situation is better in the NMSSM. This is because there is now a superpotential

$$W = \lambda\, S H_1 H_2 + \frac{\rho}{3} S^3 \tag{36}$$

whose first term can give rise to a quartic term in the F-term potential.

For instance, in the rigid SUSY limit, Eq. (36) would give

$$V^F = \left|\frac{\partial W}{\partial S}\right|^2 + \ldots = |\lambda|^2\,|h_1|^2\,|h_2|^2 + \ldots \tag{37}$$

where we have restricted ourselves to the electromagnetism-preserving direction,

$$H_1 = \begin{pmatrix} h_1 \\ 0 \end{pmatrix}, \quad H_2 = \begin{pmatrix} 0 \\ h_2 \end{pmatrix}, \tag{38}$$

with h_1 and h_2 real and positive.

In supergravity, following Einhorn and Jones [1], we choose the frame function

$$\Omega = -3 + S\overline{S} + H_1 H_1^\dagger + H_2 H_2^\dagger + \frac{3}{2}\chi\,(H_1 H_2 + \text{ h.c.})\,, \tag{39}$$

in analogy to the MSSM case. As before, this choice of frame function is of the form

$$\Omega(\phi, \overline{\phi}) = -3 + \sum_{\alpha\beta} \delta_{\alpha\bar{\beta}}\phi^\alpha \overline{\phi}^{\bar{\beta}} + \text{harmonic terms.} \tag{40}$$

Furthermore, we will consider only field configurations with real scalar fields h_1, h_2, and $s \equiv |S|$ (indeed they can be simultaneously chosen real and ≥ 0 provided $\lambda\rho > 0$ and $\chi > 0$, which we will assume to be the case from now on). This implies that the auxiliary vector field of the gravitational multiplet vanishes,

$$\mathcal{A}_\mu = -\frac{i}{2\Omega}\left(\partial_\mu\phi^\alpha\partial_\alpha\Omega - \partial_\mu\overline{\phi}^{\bar{\alpha}}\partial_{\bar{\alpha}}\overline{\Omega}\right) = 0. \tag{41}$$

As was shown in the talk on Jordan frame supergravity, conditions Eq. (40) and Eq. (41) are sufficient conditions to ensure canonical kinetic terms for the scalars in the Jordan frame.

Note that $\chi \neq 0$ explicitly breaks the discrete $\mathbb{Z}_3$ symmetry governing the usual NMSSM superpotential. This is good, because in the presence of an exact $\mathbb{Z}_3$ the NMSSM can have a problem with domain walls. And it is bad, because this is the symmetry which usually forbids a Planck-size mass term $M\,H_1 H_2$ in the NMSSM superpotential, so its absence should now be explained by some other mechanism.

Next we calculate the Einstein-frame scalar potential and see if it can give Higgs inflation for some regime. The general formula was

$$V_E = \frac{9\left(V_J^F + V_J^D\right)}{\Omega^2}, \tag{42}$$

where V_J^F and V_J^D were the F-term and D-term potentials in Jordan frame respectively. Define $h^2 = h_1^2 + h_2^2$ and $\tan\beta = h_2/h_1$, as well as $s = |S|$. In these variables,

$$h_1\,h_2 = \frac{1}{2}h^2\,\sin 2\beta \tag{43}$$

and

$$h_1^2 - h_2^2 = h^2\,\cos 2\beta. \tag{44}$$

For s close to zero we find a surprisingly simple expression for the Jordan frame scalar potential (cf. Eq. (37)),

$$V_J^F = \lambda^2\,|h_1|^2\,|h_2|^2 = \frac{\lambda^2}{4}h^4\,\sin^2 2\beta\,. \tag{45}$$

and

$$V_J^D = \frac{g^2 + g'^2}{8}(h_1^2 - h_2^2)^2 = \frac{g^2 + g'^2}{8}h^4\,\cos^2 2\beta. \tag{46}$$

Eq. (42) finally gives, upon plugging in Eqns. (45), (46) and (39)

$$V_E = \frac{9\left(\frac{\lambda^2}{4}\,h^4\sin^2 2\beta + \frac{g^2+g'^2}{8}h^4\cos^2 2\beta\right)}{\left(-3 + h^2 + \frac{3}{2}\chi\,h^2\sin 2\beta\right)^2}. \tag{47}$$

In the regime $\chi h^2 \gg 1 \gg h^2$ the potential becomes approximately

$$V_E \approx \left(\frac{\lambda}{\chi}\right)^2 + \frac{g^2 + g'^2}{2\chi^2}\cot^2 2\beta \tag{48}$$

The second term is positive definite, so it is minimized for $\cos 2\beta = 0$, i.e. $\beta = \frac{\pi}{4}$, i.e. $\tan\beta = 1$. Since then $\sin 2\beta = 1$, the scalar potential Eq. (47) for h becomes

$$V_E = \frac{9\lambda^2\,h^4}{4(-3 + h^2 + \frac{3}{2}\chi h^2)^2}. \tag{49}$$

In the MSSM case, the potential for β was too steep to give slow-roll inflation, and the potential for h along the D-flat direction was negative definite with $h \to -\infty$ for $h \to 0$. In the NMSSM case, however, we can take β to be stabilized and use h as an inflaton direction: There is now a suitable potential for h, given by Eq. (49). We can obtain slow-roll inflation for λ of order one and χ very large (as in non-supersymmetric Higgs inflation). So all is well?

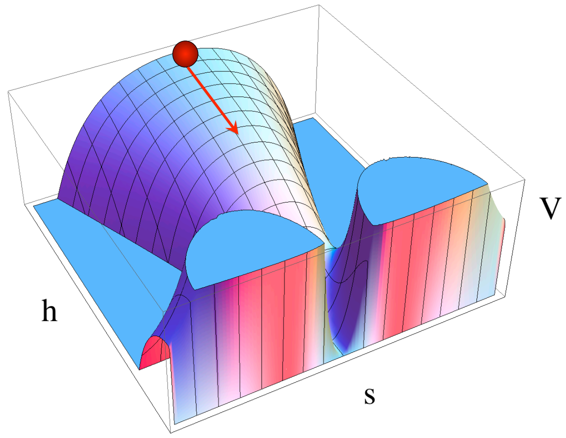

Figure 1: The FKLMV instability (picture stolen from Ref. [2]).

6 The FKLMV instability

Not really. Above Eq. (45) we made the assumption that s can be taken to be close to the origin, and then we dropped s from the analysis. In fact Ferrara et al. [2] found that, in the inflationary regime, the potential has has a saddle point at zero with a large tachyonic mass for s. This can be seen by taking along the full s-dependence of the F-term potential. We replace Eq. (45) by

$$V_J^F = \frac{\lambda^2}{4} h^4 - \lambda\rho\, s^2 h^2 - \frac{2\lambda^2\, s^2 h^2 (\chi h^2 - 2)}{4 + 3\chi^2\, h^2 - 2\chi\, h^2} + \rho^2 s^4 \tag{50}$$

where we have already set $\tan\beta = 1$. For small s this becomes

$$V_J^F \approx \frac{\lambda^2}{4} h^4 - \left(\lambda\rho + \frac{2\lambda^2}{3\chi}\right) s^2\, h^2. \tag{51}$$

The mass-squared of s is negative in Jordan frame, and remains so after transforming to Einstein frame. More precisely, in the inflationary regime $\chi h^2 \gg 1 \gg h^2$ the Einstein-frame scalar potential reads

$$V_E^F = \frac{9}{\Omega^2} V_J^F \approx \frac{\lambda^2}{\chi^2} - \frac{4}{\chi^2 h^2}\left(\lambda\rho + \frac{\lambda^2}{3\chi}\right) s^2 + \mathcal{O}(s^4). \tag{52}$$

While a tachyonic mass is not necessarily a problem (it could still be that the s direction is a viable inflationary direction), it becomes a problem if it is comparable with the Hubble scale, i.e. if the η parameter $\eta \sim |V''/V|$ is not small. The field s has non-canonical kinetic term in Einstein frame; its canonically normalized cousin turns out to be

$$\tilde{s} = \frac{2s}{\sqrt{\chi h}} \tag{53}$$

with a mass

$$m_{\tilde{s}}^2 = -2\left(\frac{\lambda^2}{3\chi^2} + \frac{\lambda\rho}{\chi}\right). \tag{54}$$

Thus the η parameter in the $\tilde{s}$ direction is

$$\eta_{\tilde{s}} = \frac{m_{\tilde{s}}^2}{V} = -\frac{2}{3} - \frac{2\rho\chi}{\lambda} < -\frac{2}{3}. \tag{55}$$

This looks very bad. In summary, taking into account also the s field reveals that the Einhorn-Jones model does give a viable supersymmetric version of Higgs inflation. Instead, in the would-be inflationary regime it suffers from a tachyonic instability along the singlet direction.

7 The Lee correction

It was proposed by Lee [3] to add a quartic term to the frame function,

$$\Delta\Omega = -\zeta\, |S|^4\,. \tag{56}$$

This will result in a positive contribution to the mass of the tachyonic mode. Note that the frame function with this correction violates condition (40), so there will be a non-canonical kinetic term for the S field.

Going through the above procedure with the correction term Eq. (56) included, one finds that the mass for the canonically normalized field $\tilde{s}$ in Einstein frame becomes

$$m_{\tilde{s}}^2 \approx \frac{\lambda^2\zeta h^2}{\chi} - 2\left(\frac{\lambda^2}{3\chi^2} + \frac{\lambda\rho}{\chi}\right) \tag{57}$$

instead of Eq. (54). This expression is valid at $\chi h^2 \gg 1$. Evidently, in this regime the instability can be overcome for sufficiently large positive ζ.

There is no simple analytic analogue of Eq. (57) near the end of inflation, where $\chi h^2 \lesssim 1$.

However, a more detailed study [4] of this regime shows that the inflationary trajectory at $s = 0$ remains stable for all h provided

$$\zeta > \frac{2\rho}{\lambda h^2} + 0.0327 \,. \tag{58}$$

The origin of the higher-order correction could be the following [3]: Suppose that there are two heavy chiral superfields Φ_1 and Φ_2 with superpotential

$$\Delta W = \frac{1}{2}\kappa\, S\Phi_1^2 + M\,\Phi_1\Phi_2 \tag{59}$$

and canonical kinetic terms in Jordan frame. The frame function receives one-loop corrections from Φ_i loops; integrating out the Φ_i, these read

$$\Delta\Omega_{\text{1-loop}} = -\frac{1}{32\pi^2}\Bigg[2M^2 \log\frac{M^2}{\mu^2} + \left(\log\frac{M^2}{\mu^2} + 2\right)|\kappa|^2\,|S|^2 + \frac{|\kappa|^4}{6M^2}|S|^4\Bigg]. \tag{60}$$

The last term is the correction we are looking for; we get $\zeta \approx |\kappa|^4/(192\pi^2 M^2)$.

8 Slow-roll parameters and observable consequences

The slow-roll parameters are

$$\epsilon = \frac{64}{3\chi^2\,h^4}, \quad \eta = -\frac{16}{3\chi\,h^2}. \tag{61}$$

For 60 e-folds we require

$$\chi \approx 10^5\,\lambda. \tag{62}$$

This is interesting regarding the unitarity bound which was previously discussed (especially also in the talk on Higgs inflation). In the NMSSM inflation case, there is no need to have a very large χ coupling, but instead λ can be made small and $N \sim 60$ can still be achieved.

The observational consequences are just as in non-supersymmetric Higgs inflation. For instance, for $\lambda = 10^{-2}$ and $\chi = 10^3$ one obtains

$$n_s \sim 0.97, \quad r \approx 3.3 \cdot 10^{-3} \tag{63}$$

References

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