STRING INFLATION II   M. CICOLI   1

· MOTIVATION

i) Test String Theory (mainly cosmology more promising than particle phenomenology)

ii) Inflation is UV-sensitive (η-problem) $\Rightarrow$ need to know the UV completion

iii) String Theory has many non-trivial constraints to model building

iv) Restrict the number of field-theoretic models

v) Find where we are in the Landscape

vi) Understand reheating (right degrees of freedom)

UV-SENSITIVITY

$$V = \left(1 + \frac{\varphi^2}{M_P^2} + \frac{\varphi^4}{2M_P^4}\right) V_0(\varphi)$$

[for example from expanding e^K in the $N=1$ F-TERM SUGRA SCALAR POTENTIAL]

$$V' = \left(1 + \frac{\varphi^2}{M_P^2} + \frac{\varphi^4}{2M_P^4}\right) V_0'(\varphi) + \left(\frac{2\varphi}{M_P^2} + \frac{2\varphi^3}{M_P^4}\right) V_0(\varphi)$$

$$V'' = \left(1 + \frac{\varphi^2}{M_P^2} + \frac{\varphi^4}{2M_P^4}\right) V_0''(\varphi) + \left(\frac{2}{M_P^2} + \frac{6\varphi^2}{M_P^4}\right) V_0(\varphi) + 2\left(\frac{2\varphi}{M_P^2} + \frac{2\varphi^3}{M_P^4}\right) V_0'(\varphi)$$

· Assume

$$\varepsilon_0 \equiv \frac{M_P^2}{2}\left(\frac{V_0'}{V_0}\right)^2 \ll 1 \qquad \eta_0 \equiv M_P^2 \frac{V_0''}{V_0} \ll 1 \qquad x \equiv \frac{\varphi}{M_P}$$

$$\Rightarrow \eta \equiv M_P^2 \frac{V''}{V} = \eta_0 + \frac{4x(1+x^2)\sqrt{2\varepsilon_0}}{1+x^2+\frac{x^4}{2}} + 2\,\frac{(1+3x^2)}{1+x^2+\frac{x^4}{2}}$$

· Small-field inflationary models $\varphi \ll M_P \iff x \ll 1$

$$\Rightarrow \eta \simeq \underbrace{\eta_0 + 4x\sqrt{2\varepsilon_0}}_{\ll 1} + \underset{\mathcal{O}(1)}{\underset{2}{2}} \simeq \mathcal{O}(1)$$

· Large-field inflationary models $\varphi \gg M_P \iff x \gg 1$

$$\Rightarrow \eta \simeq \eta_0 + \frac{8}{x}\sqrt{2\varepsilon_0} + \frac{12}{x^2} \ll 1$$

How?

Closed string
η problem some solutions.

BUT I CANNOT TRUST THE EFT!!

$\Rightarrow$ INFLATION is even more UV-sensitive if I look at $\Delta\varphi \gg M_P$

Open string – η problem fine tuning (last talk)

Why are we interested in $\Delta\varphi \gg M_p$? IN ORDER TO GET GRAVITY WAVES!

LYTH BOUND

$$N_e = \frac{1}{M_p^2}\int \frac{V}{V'}\,d\varphi \qquad r = \frac{T}{S} = 16\,\varepsilon = 8 M_p^2\left(\frac{V'}{V}\right)^2$$

$$\Rightarrow \quad \Delta N_e = \frac{1}{M_p^2}\,\frac{V}{V'}\,\Delta\varphi$$

$$\Leftrightarrow \quad \frac{\Delta\varphi}{M_p} = M_p\frac{V'}{V}\,\Delta N_e = \Delta N_e\sqrt{\frac{r}{8}}$$

$\Delta N_e \simeq 5$ for the scales relevant for the measure of r

$$\Rightarrow \quad \frac{\Delta\varphi}{M_p} \simeq \left(\frac{r}{0.1}\right)^{1/2}$$

NB This $\Delta\varphi$ corresponds to $\Delta N_e \simeq 5$ $\Rightarrow$ $\Delta\varphi$ corresponding to $\Delta N_e = 50-60$ is even bigger!

PRESENT LIMIT: $r < 0.2$

FORECASTS: ESA PLANCK $r \sim 0.1$ $\longrightarrow$ $r \sim 0.05$ OK!

BALOON SPIDER, ETC, BICEP $r \sim 0.01$

NASA CMBPol $r \sim 0.001$

If $r \sim 0.15$ $\Rightarrow$ $\Delta\varphi \simeq 2M_p$ to get $\Delta N_e \simeq 5$

$\Rightarrow$ $\Delta\varphi \gg M_p$ to get $\Delta N_e \simeq 50-60$!!

NB $V^{1/4} = M_{inf} \simeq M_{GUT}\, r^{1/4}$ $\Rightarrow$ SEE GUT-scale PHYSICS!

SOLUTION TO THE η-PROBLEM

Need 2 mechanisms

1) Obtain $\eta_0 \ll 1$
2) Avoid higher order operators!!!

$$\frac{\Delta\phi}{M_p} = \Delta N\sqrt{\frac{r}{8}} \sim 5\sqrt{\frac{r}{8}} \sim \frac{5}{2\sqrt{2}}\sqrt{r}$$

TWO CLASSES OF MODELS IN STRING THEORY

- Focus on single-field slow-roll inflation

1) INFLATON is an open string mode (see STRING INFLATION I WERKSTATTSEMINAR)

2) INFLATON is a closed string mode → I WILL FOCUS ON THIS CASE

1) Inflaton is a brane-position modulus: $D3/\overline{D3}$; $D3/D7$

$\eta_0 \sim O(1)$ without warping BUT then $\eta_0 \ll 1$ with warping

BUT what about dim-6 M_p-suppressed operators?

Kahler potential for inflaton φ and volume modulus T

$$K = -3 \ln\left[(T+\overline{T}) - \overline{\varphi}\varphi\right]$$

The volume mode is fixed a la KKLT $\Rightarrow$ $T+\overline{T} = \langle(T+\overline{T})\rangle$

$$\Rightarrow K = -3 \ln\left[\langle(T+\overline{T})\rangle \left(1 - \frac{\overline{\varphi}\varphi}{\langle(T+\overline{T})\rangle}\right)\right] =$$

$$= \underbrace{-3 \ln \langle(T+\overline{T})\rangle}_{\equiv k_0} + 3\,\frac{\overline{\varphi}\varphi}{\langle(T+\overline{T})\rangle}$$

CANONICAL NORMALISATION:

$$\varphi_c = \frac{\sqrt{3}\,\varphi}{\sqrt{\langle(T+\overline{T})\rangle}} \qquad \Rightarrow \qquad K = k_0 + \overline{\varphi}_c\varphi_c$$

$$\Rightarrow \quad V_F = e^{k_0}\,U(\varphi_c)\,e^{\overline{\varphi}_c\varphi_c} \simeq e^{k_0}\,U(\varphi_c)\left(1 + \frac{\overline{\varphi}_c\varphi_c}{M_p^2}\right)$$

$\delta\eta \sim O(1)$!!

$\Rightarrow$ need FINE-TUNING!!!

- In addition cannot get LARGE TENSOR MODES due to BOUNDS on field RANGES
- Simple argument:

x - radial position of the brane $\qquad$ L - characteristic size of the CY: $\text{Vol} = L^6$

$\Rightarrow \Delta x < L$ $\qquad$ GEOMETRIC BOUND

Canonically normalised field $\quad \varphi = M_s^2 x \quad \Rightarrow \quad \dfrac{\Delta\varphi}{M_p} = \Delta x\,\dfrac{M_s^2}{M_p} < \dfrac{L M_s^2}{M_p}$

BUT $\quad M_s = \dfrac{M_p}{\sqrt{\text{Vol}\, M_s^6}} = \dfrac{M_p}{L^3 M_s^3} \iff M_p = M_s^4 L^3$ from DIMENSIONAL REDUCTION

$$\Rightarrow \frac{\Delta\varphi}{M_p} < \frac{L M_s^2}{L^3 M_s^4} = \left(\frac{1}{L M_s}\right)^2$$

BUT in order to trust the EFT need $L \gg l_s = M_s^{-1} \Rightarrow L M_s \gg 1$

$\Rightarrow \Delta\varphi \ll M_p$ $\qquad$ THIS KEEPS HOLDING ALSO FOR WARPED CASES!

2) CLOSED STRING MODES   Kähler moduli inflation ✓

TYPE IIB EFFECTIVE ACTION for CY ORIENTIFOLD COMPACTIFICATIONS WITH D3/D7 AND O3/O7

$h_{11} = h_{11}^{(+)} + h_{11}^{(-)}$ under the orientifold projection

Moduli:

$T_i = \tau_i + i\, b_i^{(+)}$ $\quad \tau_i = \text{Vol}(D_i)$ $\quad b_i^{(+)} = \int_{D_i} C_4$ (R-R 4-FORM) $\quad i = 1, \dots, h_{11}^+$

$G_j = c_j - i S\, b_j^{(-)}$ $\quad c_j = \int_{\hat{D}_j} C_2$ (R-R 2-FORM) $\quad b_j^{(-)} = \int_{\hat{D}_j} B_2$ (NS-NS 2-FORM) $\quad j = 1, \dots, h_{11}^-$

$S = e^{-\phi} + i C_0$ (ϕ: DILATON; C_0: R-R 0-FORM)

a) Real part of T-moduli: $\mathcal{V}$ (CY VOLUME), $\tau_{\text{blow-up}}$, $\tau_{\text{fibr.}}$

Get $\eta_0 \ll 1$ due to the NO-SCALE structure of K if $h_{11}^+ \geq 1$ and keeping $\mathcal{V}$ FIXED during inflation

$\Rightarrow$ the inflaton is a combination of the Kähler moduli orthogonal to $\mathcal{V}$

$\Rightarrow$ get NO dim-6 M_P-suppr. operators!

MORE PRECISELY:

$$K = K_{\text{tree}} + \delta K_{(\alpha')} = -2\ln\left(\mathcal{V} + \frac{\xi}{2 g_s^{3/2}}\right) \simeq -2\ln\mathcal{V} - \frac{\xi}{g_s^{3/2}\mathcal{V}}$$

$W = W_0$, with $\xi \propto \frac{(h_{12} - h_{11})}{(2\pi)^3} \sim \mathcal{O}(1)$

K_{tree} $\to$ it does NOT depend on the T-moduli!

K_{tree} is of NO-SCALE TYPE, i.e.

$$V = e^K \left(\sum_{U,S} K^{\alpha\bar{\beta}} D_\alpha W D_{\bar{\beta}} \overline{W} + \underbrace{\sum_T K_{\text{tree}}^{i\bar{j}} D_i W D_{\bar{j}} \overline{W} - 3|W|^2}_{= (K_{\text{tree}}^{i\bar{j}} K_i K_{\bar{j}} - 3) W_0^2 = 0} \right)$$

$\Rightarrow$ all the τ-directions are flat!

$\Rightarrow$ $\delta K_{\alpha'} \simeq -\frac{\xi}{g_s^{3/2}\mathcal{V}}$ breaks the NO-SCALE STRUCTURE BUT LIFTING ONLY THE VOLUME DIRECTION

$\Rightarrow$ $(h_{11}^+ - 1)$-directions are still flat!

$\Rightarrow$ they are natural inflaton CANDIDATES

On top of that, $K_{\text{tree}} = -2\ln\mathcal{V}$ also depends ONLY on a particular combination which is the OVERALL VOLUME

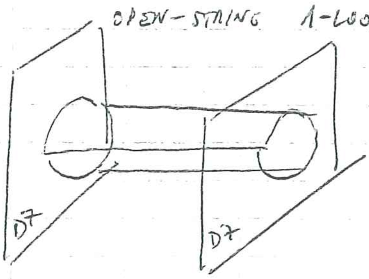
$\Rightarrow$ no problems from expanding e^K

Only $\mathcal{V}$ direction lifted
all other directions flat!

Are there other perturbative CORRECTIONS? YES, g_s corrections.

- $K = K_{\text{tree}} + \delta K_{(\alpha')} + \underbrace{\delta K_{(g_s)}}$

OPEN-STRING 1-LOOP CORRECTIONS



can be interpreted as the tree-level exchange of a closed-string carrying KK momentum

$$\Rightarrow \quad \delta K_{(g_s)} \sim \frac{m_{KK}^{-2}}{\mathcal{V}}$$

m_{KK}^{-2} — 2-pt function in string frame

$\mathcal{V}$ — Weyl-rescaling from string to Einstein frame

$$m_{KK} \sim \frac{M_s}{\ell}$$

ℓ — size of 2-cycle transverse to D7

$$\Rightarrow \quad \ell = t^{1/2}$$

$$\Rightarrow \quad \delta K_{(g_s)} \sim \sum_i \frac{t_i}{\mathcal{V}} \; > \; \delta K_{(\alpha')} \sim \frac{1}{\mathcal{V}} \qquad \text{since } t_i \gg 1 \text{ to trust EFT}$$

$\Rightarrow$ all directions might be lifted instead of just $\mathcal{V}$

BUT THERE IS A GENERIC CANCELLATION IN THE SCALAR POTENTIAL such that

$$\delta V_{(\alpha')} > \delta V_{(g_s)} \qquad \text{even if} \quad \delta K_{(\alpha')} < \delta K_{(g_s)}$$

$\Rightarrow$ EXTENDED NO-SCALE STRUCTURE $\Rightarrow$ useful to solve the η-problem!!

In fact can show that if

$$\begin{cases} K = K_0 + \delta K \\ W = W_0 \end{cases}$$

$\Rightarrow$ if δK is a homogeneous function in the t's of degree $n = -2$ $\Rightarrow$

$\Rightarrow$ at leading order $\delta V = 0$

<u>Proof</u> Expand $K^{-1} = (K_0 + \delta K)^{-1}$ and use homogeneity

$$\Rightarrow \quad V = \underset{\substack{\| \\ 0 \\ \text{NO-SCALE}}}{V_0} + \underset{\substack{\| \\ 0 \\ \text{EXTENDED NO-SCALE}}}{V_1} + \underset{\neq 0}{V_2}$$

$$V_1 = -\frac{W_0^2}{\mathcal{V}^2}\,\frac{1}{4}\,n(n+2)\,\delta K \quad = 0 \qquad \text{for } n = -2$$

$$V_2 = \frac{W_0^2}{\mathcal{V}^2} \sum_i K_{ii}$$

1- modulus example: $\mathcal{V} = \tau^{3/2} = t^3 \Leftrightarrow t = \sqrt{\tau}$

$$\begin{cases} K = -2\ln\mathcal{V} - \frac{\hat{\xi}}{\mathcal{V}} + \frac{\sqrt{\tau}}{\mathcal{V}} \\ W = W_0 \end{cases} \qquad \left[\hat{\xi} = \xi/g_s^{3/2}\right]$$

$$V = \underbrace{0}_{\text{NO-SCALE}} + \underbrace{\frac{\hat{\xi} W_0^2}{\mathcal{V}^3}}_{\delta V_{(\alpha')}} + \underbrace{0 \cdot \frac{W_0^2\sqrt{\tau}}{\mathcal{V}^3}}_{\text{EXTENDED NO-SCALE}} + \underbrace{\frac{W_0^2}{\sqrt{\tau}\,\mathcal{V}^3}}_{\delta V_{(g_s)}} + \underbrace{\frac{W_0^2}{\tau^{3/2}\mathcal{V}^3}}_{\text{subleading } g_s \text{ corr.}}$$

$$\Rightarrow \frac{\delta V_{(g_s)}}{\delta V_{(\alpha')}} \sim \frac{1}{\sqrt{\tau}} \ll 1$$

INTERPRETATION of the CANCELLATION with the Coleman-Weinberg potential

$$\delta V_{1\text{-loop}} \simeq 0 \cdot \Lambda^4 + \Lambda^2\, \mathrm{STr}(M^2) + \mathrm{STr}\left(M^4 \ln\left(\frac{M^2}{\Lambda^2}\right)\right)$$

$$\Lambda = M_{KK} = \frac{M_s}{\tau^{1/4}} = \frac{M_P}{\tau^{1/4}\mathcal{V}^{1/2}} = \frac{M_P}{\mathcal{V}^{2/3}}$$

$$\Rightarrow \delta V_{1\text{-loop}} \simeq 0 \cdot \frac{1}{\mathcal{V}^{8/3}} + \frac{1}{\mathcal{V}^{10/3}} + \frac{1}{\mathcal{V}^4}$$

for SUSY theory Λ^4 term drops!

OUR CASE

$$\delta V_{(g_s)} \simeq 0 \cdot \frac{\sqrt{\tau}}{\mathcal{V}^3} + \frac{1}{\sqrt{\tau}\,\mathcal{V}^3} + \frac{1}{\tau^{3/2}\mathcal{V}^3}$$

$$\underset{\tau \simeq \mathcal{V}^{2/3}}{\simeq} 0 \cdot \frac{1}{\mathcal{V}^{8/3}} + \frac{1}{\mathcal{V}^{10/3}} + \frac{1}{\mathcal{V}^4} \qquad \text{PERFECT MATCHING!}$$

$\Rightarrow$ can generate the INFLATIONARY POTENTIAL IN TWO WAYS:

1) STRING LOOPS $\longrightarrow$ "FIBRE INFLATION"
$\longrightarrow$ "BLOW-UP INFLATION"

2) TINY NON-PERTURBATIVE EFFECTS $\left(W = W_0 + \sum_i A_i e^{-a_i T_i}\right)$ ONLY IF there are NO LOOPS since

$$\delta V_{(g_s)} \gg \delta V_{(\text{NON-PERT})} \quad \text{in the inflationary region}$$

$\Rightarrow$ generate the INFLATIONARY POTENTIAL via ED3s and NOT GAUGINO CONDENSATION!

b) Axions: like C_4-, C_2- or B_2- AXIONS

SHIFT SYMMETRY: $a \to a + c$ broken only NON-PERTURBATIVELY

$\Rightarrow$ K_{tree} does NOT depend on a

$\Rightarrow$ NO η-problem from dim 6 M_p-supp. operators!

BUT still have to explain why $\eta_0 = M_p^2 \frac{V_0''}{V_0} \ll 1$

i) 1 C_4-AXION: NOT FLAT enough!

ii) N C_4-AXIONS: "N-FLATION" can get $\eta_0 \ll 1$ and r large

↙ problem moduli stabilization

$\epsilon \sim \left(\frac{M_p}{f}\right)\frac{1}{N^2} \ll 1$

BUT hard to realize since needs to fix the real parts of T-moduli at an ~~mass~~ energy much larger than the AXION POTENTIAL

iii) C_2-AXION: "AXION MONODROMY"

└ can get $\eta_0 \ll 1$ plus $\Delta\varphi \gg M_p$ from MONODROMY

FOLLOW MARTIN'S STRATEGY

A) Find a dS VACUUM with all closed string moduli stabilised

$\Rightarrow$ EFT and whole potential under control

B) Look for inflationary directions

A) KKLT-SCENARIO

$$\begin{cases} K = K_{tree} = -2 \ln \mathcal{V} \\ W = W_0 + \sum_{i=1}^{h^{1,1}} A_i e^{-a_i T_i} \end{cases}$$

MANY PROBLEMS:

1) Need to fine-tune W_0 such that $W_0 \sim W_{np} \sim e^{-a\tau}$

This guarantees: i) get a MINIMUM

ii) can safely neglect PERTURBATIVE CORRECTIONS TO K since

$V_{np} \sim e^K W_0 W_{np}$ $\quad$ $V_p \sim e^K W_0^2 K_p$

$$\Rightarrow \frac{V_{np}}{V_p} \sim \frac{W_{np}}{W_0 K_p} \underset{W_0 \sim W_{np}}{\sim} \frac{1}{K_p} \gg 1 \quad \text{since } K_p = K_{\alpha'} \sim \frac{1}{\mathcal{V}} \ll 1 \quad \text{for } \mathcal{V} \gg 1$$

BUT for natural values of $W_0 \sim O(1) \gg W_{np}$ this is NOT true anymore

2) Need NON-PERT. effects for each 4-cycle: VERY HARD to get

W_{np} definitely generated ONLY for RIGID cycles!

3) Need to add $\overline{D3}$ to get dS $\Rightarrow$

$\Rightarrow$ break SUSY EXPLICITLY $\Rightarrow$ loose control

SOLUTION to these problems: LARGE VOLUME SCENARIOS

- Let $W_0 \sim O(1)$ and consider also perturbative corrections to K
- Simplest case with $h_{1,1} = 2$ $\quad \mathbb{C}P^4_{[1,1,1,6,9]}$

$$\mathcal{V} = \tau_B^{3/2} - \tau_s^{3/2}$$

$$\begin{cases} K = -2 \ln\left(\tau_B^{3/2} - \tau_s^{3/2}\right) - \dfrac{\xi}{g_s^{3/2}\mathcal{V}} \\ W = W_0 + \underbrace{A_s e^{-a_s \tau_s}} \end{cases}$$

Need NP-corrections ONLY in $\underbrace{\tau_s}_{\text{BLOW-UP mode}}$ which is a RIGID CYCLE!

$\Rightarrow$ scalar potential for $\mathcal{V} \gg 1$

from AXION-MINIMISATION $\left(\cos(a_s b_s)\right)$

$$V = \underbrace{\frac{\sqrt{\tau_s}\, a_s^2 A_s^2 e^{-2a_s\tau_s}}{\mathcal{V}} - \frac{a_s A_s W_0 \tau_s e^{-a_s\tau_s}}{\mathcal{V}^2}} + \frac{\xi W_0^2}{g_s^{3/2}\mathcal{V}^3}$$

$$\frac{\partial V}{\partial \tau_s} = 0 \Rightarrow a_s \tau_s \simeq \ln \mathcal{V}$$

$$V = -\frac{W_0^2 (\ln \mathcal{V})^{3/2}}{\mathcal{V}^3} + \frac{\xi W_0^2}{g_s^{3/2}\mathcal{V}^3}$$

$$\frac{\partial V}{\partial \mathcal{V}} = 0 \Rightarrow \ln \mathcal{V} \simeq \frac{\xi^{2/3}}{g_s}$$

$\Rightarrow$ MINIMUM at

$$\tau_s \sim \frac{1}{g_s} \sim O(10) \quad \text{for } g_s \simeq 0.1$$

$$\mathcal{V} \sim W_0\, e^{a_s \tau_s} \sim W_0\, e^{a_s/g_s} \gg 1$$

NB $\begin{cases} a_s = 2\pi & \text{for ED3} \\ a_s = \dfrac{2\pi}{N} & \text{for GAUGINO CONDENSATION in an } SU(N)\text{-theory} \end{cases}$

MINIMUM is AdS and ~~SUSY~~ since $V \sim O\left(\frac{1}{\mathcal{V}^3}\right)$

BUT $V = e^K \left[K^{i\bar{j}} D_i W D_{\bar{j}} \overline{W} - 3|W|^2\right]$

$$-3 e^K |W|^2 \sim O(1/\mathcal{V}^2) \Rightarrow D_i W \neq 0 \Rightarrow \text{~~SUSY~~}$$

$F_{T_B} \neq 0 \Rightarrow \tau_B$ breaks SUSY

$\Rightarrow$ the Goldstino is the τ_B-modulino which is eaten-up by the gravitino

NB SUSY is BROKEN SPONTANEOUSLY

- can get dS VACUA using D-terms
- in-fact wrap a D7 around τ_B and turn-on a gauge flux on an internal 2-cycle

$$\Rightarrow \text{generate} \quad V_D = g^{+2}\left(\xi_B - \sum_i q_i K_i \varphi_i\right)^2$$

$$\xi_B = \underbrace{q_B}_{\text{gauge-flux}} \frac{\partial K}{\partial T_B} = \frac{1}{2} q_B \frac{\partial K}{\partial \tau_B} = -\frac{3}{2} q_B \frac{1}{\tau_B}$$

can show that $\langle|\varphi_i|\rangle = 0$ using V_F

$$\Rightarrow \quad g^{-2} = \frac{\tau_B}{4\pi}$$

$$\Rightarrow \quad V_D \simeq \frac{p}{\tau_B^3} \simeq \frac{p}{\mathcal{V}^2} \quad \Rightarrow \text{ fine-tune } p \sim O(1/\mathcal{V}) \text{ and get } V_D \sim O(1/\mathcal{V}^3)$$

NB In the limit $W_0 \ll 1$ this VACUUM coincides with the KKLT-one.

$\Rightarrow$ more general treatment.

- can generalize to $\mathcal{V} = \left(\tau_B^{3/2} - \sum_j \lambda_j \tau_j^{3/2}\right)$ — all BLOW-UP modes with NP-effects
- STRING LOOPS are negligible due to the EXTENDED NO-SCALE STRUCTURE

B)

BLOW-UP INFLATION $\quad h_{11} = 3$

$$\mathcal{V} = \alpha\left(\tau_1^{3/2} - \gamma_2 \tau_2^{3/2} - \gamma_3 \tau_3^{3/2}\right)$$

$$\begin{cases} K = -2\ln\left(\mathcal{V} + \dfrac{\xi}{2 g_s^{3/2}}\right) \\ W = W_0 + A_2 e^{-a_2 T_2} + A_3 e^{-a_3 T_3} \end{cases}$$

$$V = \sum_{i=2}^{3} a_i^2 A_i^2 \frac{\sqrt{\tau_i}}{\mathcal{V}} e^{-2a_i\tau_i} - \sum_{i=2}^{3} a_i A_i W_0 \frac{\tau_i}{\mathcal{V}^2} e^{-a_i\tau_i} + \frac{\xi W_0^2}{g_s^{3/2}\mathcal{V}^3}$$

MINIMUM at:

$$a_2\tau_2 \simeq a_3\tau_3 \simeq 1/g_s \qquad \mathcal{V} \simeq W_0 \sqrt{\tau_2}\, e^{a_2\tau_2} \simeq W_0\sqrt{\tau_3}\, e^{a_3\tau_3}$$

- displace τ_2 far from its minimum and let it drive inflation
- T_3 keeps the volume stable during inflation

$$\frac{\varphi}{M_P} \simeq \frac{\tau_2^{3/4}}{\mathcal{V}^{1/2}} \qquad \text{RIGHT PREDICTION: } 0.960 < n_s < 0.967$$

$$V \simeq V_0 - \beta\left(\frac{\varphi}{M_P \mathcal{V}}\right)^{4/3} e^{-a\mathcal{V}^{2/3}\left(\frac{\varphi}{M_P}\right)^{4/3}}$$

$\Rightarrow$ SMALL-FIELD INFLATION: $r \ll 1$

to get inflation $\varepsilon \ll 1$, $\eta \ll 1$, need $\quad \mathcal{V}^{2/3}\varphi^{4/3} \gg M_P^{4/3} \iff \varphi \gg \dfrac{M_P}{\mathcal{V}^{1/2}} = M_S$

$$\varphi \gg \frac{M_P}{\mathcal{V}^{1/2}}$$

$$\overset{\shortparallel}{\frac{\tau^{3/4}}{\mathcal{V}^{1/2}} M_P} \gg \frac{M_P}{\mathcal{V}^{1/2}} \iff \tau^{3/4} \gg 1 \text{ in string units}$$

regime where you can trust the EFT!!

- fix the value of $\mathcal{V}$ by the requirement of generating enough density perturbations

$$\frac{\delta\rho}{\rho} \sim 10^{-5} \implies \mathcal{V} \sim (10^6 \div 10^7)\, \ell_s^6$$

- Loop-corrections:

$$\delta V_{(g_s)} \sim \frac{1}{\sqrt{\tau_2}\,\mathcal{V}^3} \sim \frac{1}{\varphi^{2/3}\,\mathcal{V}^{10/3}}$$

$$\implies \delta\eta \sim M_P^2 \frac{\delta V''_{(g_s)}}{V_0} \sim \frac{1}{\varphi^{8/3}\,\mathcal{V}^{1/3}\,\hat{\xi}} \qquad \text{where } V_0 \sim \frac{\hat{\xi}}{\mathcal{V}^3}$$

for $\varphi \sim \frac{\tau_2^{3/4}}{\mathcal{V}^{1/2}} \implies \delta\eta \sim \frac{\mathcal{V}}{\tau_2^2\,\hat{\xi}} \gg 1$ for τ_2 small

$\delta\eta$ can become $\delta\eta \ll 1$ for τ_2 large BUT then $\delta V_{(g_s)} \gg \delta V_{(np)}$ and the inflationary potential is completely different

PROBLEM May hit the walls of the Kähler cone!

Consider a different scenario with $V_{inf} = \delta V_{(g_s)}$

FIBRE INFLATION

$$\mathcal{V} = \alpha\left(\sqrt{\tau_1}\,\tau_2 - \gamma_3\,\tau_3^{3/2}\right)$$

$$\begin{cases} K = -2\ln\left(\mathcal{V} + \dfrac{\xi}{2 g_s^{3/2}}\right) \\ W = W_0 + A_3\, e^{-a_3 T_3} \end{cases}$$

$$\implies V = a_3^2 A_3^2 \frac{\sqrt{\tau_3}}{\mathcal{V}} e^{-2a_3\tau_3} - a_3 A_3 W_0 \frac{\tau_3}{\mathcal{V}^2} e^{-a_3\tau_3} + \frac{\xi W_0^2}{g_s^{3/2}\,\mathcal{V}^3}$$

$V = V(\tau_3, \mathcal{V})$ ONLY TWO MODULI : $\langle\tau_3\rangle \sim 1/g_s \quad \langle\mathcal{V}\rangle \sim W_0 \sqrt{\langle\tau_3\rangle}\, e^{a_3\langle\tau_3\rangle}$

$\implies$ direction in (τ_1, τ_2)-plane $\perp \mathcal{V}$ still flat $\implies$ perfect INFLATON CANDIDATE!

- Include subleading loop-correction

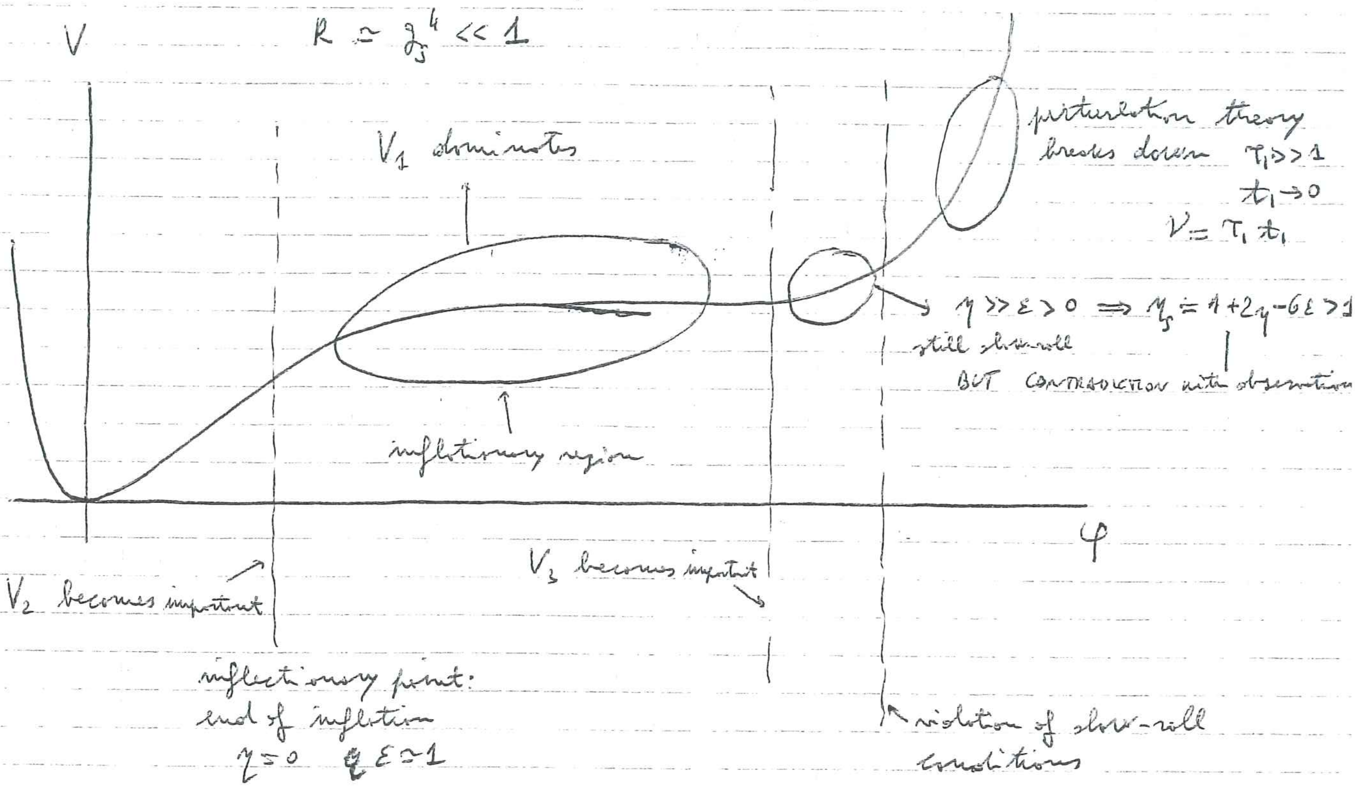
$$\delta V_{(g_s)} = \left(\frac{A}{\tau_1^2} - \frac{B}{\mathcal{V}\sqrt{\tau_1}} + \frac{C\,\tau_1}{\mathcal{V}^2}\right)\frac{W_0^2}{\mathcal{V}^2}$$

fix $\langle\tau_1\rangle = c\,\mathcal{V}^{2/3} \qquad c = \frac{(g_s C_1)^{4/3}}{C_{12}^{2/3}}$

- Work in the $(\mathcal{V}, \tau_1)$-plane, displace τ_1 far from its minimum and let it drive inflation
- $$\frac{\varphi}{M_p} = \frac{\sqrt{3}}{2} \ln \tau_1$$

$$\Rightarrow \quad V = \frac{\beta}{\mathcal{V}^{10/3}} \Big(3 - \underbrace{4\, e^{-\varphi/(\sqrt{3}M_p)}}_{V_1} + \underbrace{e^{-4\varphi/(\sqrt{3}M_p)}}_{V_2} + \underbrace{R\, e^{2\varphi/(\sqrt{3}M_p)}}_{V_3} \Big)$$

$$R \simeq g_s^4 \ll 1$$



$\Rightarrow$ in the inflationary region

$$V \simeq \frac{\beta}{\mathcal{V}^{10/3}} \left(3 - 4\, e^{-\varphi/(\sqrt{3}M_p)} \right)$$

- Large field potential: $\Delta\varphi \gg M_p$
- All adjustable parameters enter ONLY in the prefactor $\Rightarrow$ VERY PREDICTIVE SCENARIO

$$\boxed{r \simeq 6\,(n_s - 1)^2}$$

$$\begin{cases} n_s \simeq 0.97 \\ r \simeq 0.005 \end{cases}$$

- Fix the inflationary scale by matching COBE: $\mathcal{V} \sim 10^4\, l_s^6$

$$\Rightarrow \quad M_s \simeq M_{GUT}$$

AXIONS a

SHIFT SYMMETRY (perturbative) : $a \to a + c$

$\Rightarrow$ NO dim-6 M_p-suppr. operators $\cancel{\frac{V}{M_p^2} a^2}$

- typical AXION potential

$$V(a) = \Lambda^4 \left(1 - \cos\left(a/f_a\right)\right) \qquad (*)$$

f_a: AXION decay constant — $\mathcal{L} \sim \frac{1}{2}\partial_\mu a\, \partial^\mu a + \frac{a}{f_a}\int F \wedge F$

like for example for C_4-AXIONS $\quad T_i = \tau_i + i\, b_i$, $\quad \tau_i = \text{Vol}(\Sigma_i)$, $\quad b_i = \int_{\Sigma_i} C_4$

$V(a)$ might be generated by a stringy instanton wrapping Σ_i:

e.g. LVS : $W = W_0 + A_s e^{-a_s T_s} \qquad T_s = \tau_s + i b_s$

$$V = \frac{a_s^2 A_s^2 \sqrt{\tau_s}\, e^{-2a_s\tau_s}}{\mathcal{V}} M_p^4 + a_s A_s W_0 \frac{\tau_s e^{-a_s \tau_s}}{\mathcal{V}^2} \cos(a_s b_s) M_p^4 + \frac{\xi W_0^2}{\mathcal{V}^3} M_p^4$$

$$\Rightarrow \quad \Lambda^4 \sim \frac{e^{-a_s\tau_s} M_p^4}{\mathcal{V}^2} \sim \frac{M_p^4}{\mathcal{V}^3} \quad \Rightarrow \quad \Lambda \sim \frac{M_p}{\mathcal{V}^{3/4}}$$

from (*)

$$V' = \frac{\Lambda^4}{f_a} \operatorname{sen}\left(\frac{a}{f_a}\right) \qquad V'' = \frac{\Lambda^4}{f_a^2} \cos\left(\frac{a}{f_a}\right)$$

$$\Rightarrow \begin{cases} \varepsilon = \dfrac{M_p^2}{2} \dfrac{1}{f_a^2} \dfrac{\operatorname{sen}^2(a/f_a)}{\left(1 - \cos(a/f_a)\right)^2} \simeq \left(\dfrac{M_p}{f_a}\right)^2 \\ \eta = \dfrac{M_p^2}{f_a^2} \dfrac{\cos(a/f_a)}{1 - \cos(a/f_a)} \simeq \left(\dfrac{M_p}{f_a}\right)^2 \end{cases}$$

$\varepsilon \sim \eta \sim \left(\frac{M_p}{f_a}\right)^2 \ll 1 \iff f_a \gg M_p$ NOT THE CASE!!

$\Rightarrow$ the potential is NOT flat enough!

WAY-OUT: consider N-AXIONS : "N-FLATION"

$$\mathcal{L} \sim \sum_{i=1}^{N} \left[\frac{1}{2} (\partial a_i)^2 - \Lambda^4 \left(1 - \cos\left(\frac{a_i}{f_a}\right)\right) \right]$$

eqs. of motion

$$\ddot{a}_i + 3H\dot{a}_i = -\frac{\partial V}{\partial a_i}$$ each axion evolves indipendently

$\Rightarrow$ collective motion

$$\varepsilon \simeq \left(\frac{M_p}{f_a}\right)^2 \frac{1}{N^2} \qquad \eta \sim \left(\frac{M_p}{f_a}\right) \frac{1}{N}$$

$N \gg 1 \Rightarrow \varepsilon \ll 1 \quad \eta \ll 1$

- can even get large tensor modes BUT each single axion has $\Delta\varphi_a \ll M_p$

- BUT HARD TO REALISE – Need $V(\tau) \gg V(a)$, i.e. $m_\tau \ll m_a$
- BUT in general NP-effects give $m_\tau \sim m_a$

e.g. LVS as before $\quad m_{\tau_s} \sim m_{b_s} \sim \frac{M_p}{V} \sim m_{3/2}$

One should try to fix all the τ's PERTURBATIVELY – hard to get WITHOUT FINE-TUNING.

- Other issue: check that these AXIONS do not get eaten-up by any GREEN–SCHWARZ MECHANISM.

"AXION MONODROMY"

- Just 1-AXION with flat potential and $\Delta\varphi \gg M_p$

How can you get it? since the AXION is periodic $\varphi_a \to \varphi_a + \underbrace{f_a}$?

PERIOD given by the AXION decay const.

→ Use MONODROMY
└ it studies how "objects" behave when they turn around a singularity

D5-brane wrapping a 2-cycle Σ_2 of size $\mathrm{Vol}(\Sigma_2) = L_2^2$

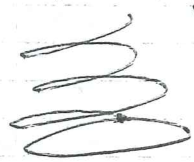
$$\Rightarrow \int_{\Sigma_2} B_2 = b \quad \text{AXION}$$

$$S_{DBI}^{D5} \sim -\frac{1}{g_s} \int_{M_4 \times \Sigma_2} d^6\xi \sqrt{\det(G+B)} = -\frac{1}{g_s}\int d^4x \sqrt{-g}\sqrt{L_2^4 + b^2}$$

at large $b \quad \Rightarrow \quad V \sim \mu^3 b f = \mu^3 \varphi \quad$ linear in φ

$\Rightarrow$ the D5 breaks the SHIFT–SYMMETRY and gives a NON–PERIODIC CONTRIBUTION

as $b \to b + f \quad \Rightarrow \quad V$ undergoes a MONODROMY that "unwraps" the axion circle



BUT b appears also in K_{tree} $\Rightarrow$ NOT a good candidate! due to η-problem

$$\Rightarrow \text{use} \quad c = \int_{\Sigma_2} C_2 \quad \text{since } K_{tree} \text{ does NOT depend on it!}$$

- Wrap an NS5-brane (5-dim object which couples magnetically to B_2) around Σ_2

$$\Rightarrow S_{NS5} \sim -\frac{1}{g_s}\int d^4x \sqrt{-g}\sqrt{L_2^4 + g_s^2 c^2}$$

at large $c \quad \Rightarrow \quad V(c) \sim c \quad$ linear $\left(+ \text{NP-effects}\right)$: $V \sim \mu^3\varphi + \Lambda^4 \cos\left(\frac{\varphi}{f}\right)$ — from ED1 wrapping Σ_2 / dark

- Need to fix the real part of T-moduli at higher-scales ⇒ use WARPING to AVOID DESTABILISATION
- Predictions $\quad \Delta\varphi \sim 11\, M_p \quad \Rightarrow \begin{cases} n_s \simeq 0.975 \\ r \simeq 0.07 \end{cases}$ NB Still MISSING a COMPACT EXAMPLE!!
- Cosine–MODULATION gives RIPPLES in the power-spectrum and RESONANT NON-GAUSSIANITIES

ISSUES BEYOND ~~INFLATION~~ GETTING JUST INFLATION IN STRING COSMOLOGY

- (P)REHEATING – available for D3/$\overline{D3}$ inflation and Kähler moduli inflation

- FINITE-TEMPERATURE CORRECTIONS from the THERMAL BATH

$$\Rightarrow \text{FIND } \underbrace{T_{max}}_{\text{DECOMPACTIFICATION temperature}} \Rightarrow T_{RH} < T_{max}$$

$$\begin{cases} \text{KKLT:} & T_{max} \sim \sqrt{m_{3/2} M_P} \sim 10^{10}\ \text{GeV} \\ \text{LVS:} & T_{max} \sim \left(m_{3/2}^3 M_P\right)^{1/4} \sim 10^{7}\ \text{GeV} \end{cases} \quad \text{for } m_{\chi_2} \sim 1\ \text{TeV}$$

- POST-INFLATIONARY (NON) THERMAL HISTORY of the MODULI

  ↳ NON THERMAL DARK MATTER from MODULI DECAY?

- TENSION between COSMOLOGY and PARTICLE PHENOMENOLOGY:

  - can I get inflation and TeV-scale SUSY from the same compactification?

$$V \simeq 3 H^2 M_P^2 \qquad m_{3/2} \simeq \frac{W_0 M_P}{\mathcal{V}} = \left(e^{-K} W_0^2\right)^{1/2}$$

$$\Rightarrow \begin{cases} V_{KKLT} \simeq m_{3/2}^2 M_P^2 \\ V_{LVS} \simeq m_{3/2}^3 M_P \end{cases} \qquad M_{inf} = V^{1/4} \sim (H M_P)^{1/2}$$

$$\Rightarrow \underline{\text{KKLT:}} \quad m_{3/2} \sim H \sim \frac{M_{inf}^2}{M_P}$$

$$\Rightarrow \underline{\text{LVS:}} \quad m_{3/2} \sim H^{2/3} M_P^{1/3} \sim \frac{M_{inf}^{4/3}}{M_P^{1/3}}$$

$\frac{\delta\rho}{\rho} \sim 10^{-5}$ generically sets $M_{inf} \sim M_{GUT}$ $\Rightarrow$ $m_{3/2} \gg 1$ TeV

KNOWN SOLUTIONS

1) Use $\mathcal{V}$ as the inflaton – FINE TUNED

   - $\mathcal{V}$ small DURING INFLATION $\Rightarrow$ get HIGH inflationary scales
   - $\mathcal{V}$ large at the end of INFLATION $\Rightarrow$ get LOW $m_{3/2}$

2) SEQUESTERING: MSSM at quiver locus (orbifold singularities)

   $\Rightarrow$ hierarchy between M_{SOFT} and $m_{3/2}$

   e.g. $M_{SOFT} \sim \frac{m_{3/2}}{\mathcal{V}}$ $\Rightarrow$ can get $M_{SOFT} \sim 1$ TeV for large $m_{3/2}$

- STUDY MULTI-FIELD INFLATIONARY DYNAMICS

i) CURVATON $\longrightarrow$ KNOWN: FIBRE INFL. + BLOW-UP INFL. $\Rightarrow$ get LARGE NON-GAUSSIANITIES
ii) MODULATION MECHANISM $\longrightarrow$ NOT KNOWN