String Inflation I

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- Motivation: Generically higher-dim. terms of the effective action can strongly influence the inflationary behaviour $\Longrightarrow$ Study inflation dynamics in a UV-complete theory!

- η-problem: Planck-suppressed operators in the effective action generically lead to corrections of the potential, which spoil the slow-roll conditions:

$$V = \underbrace{V_0}_{\substack{\text{tree-level} \\ \text{flat potential}}} + \Delta V \quad , \quad \Delta V = \langle O_4 \rangle \frac{\phi^2}{M_p^2} \propto V_0 \frac{\phi^2}{M_p^2}$$

$$\Longrightarrow V'' = \underbrace{V_0''}_{\ll 1}\left(1 + \frac{\phi^2}{M_p^2}\right) + \frac{V_0}{M_p^2} \approx \frac{V_0}{M_p^2} \, ,$$

$$\Longrightarrow \eta = M_p^2 \frac{V''}{V} = \frac{V_0}{V_0\left(1+\frac{\phi^2}{M_p^2}\right)} \approx 1 - \frac{\phi^2}{M_p^2} \approx \mathcal{O}(1)$$

in almost the entire field space, except $\phi \approx M_p$.

- Equivalent formulation:

Inflation is spoiled, if there exists a mass-correction of the order of inflation energy:

$$\Delta m_\phi^2 \propto \frac{V_0}{M_p^2} = 3H^2 \quad \Longrightarrow \quad \eta = \mathcal{O}(1) .$$

In string theory the 4D effective action can be computed to arbitrarily high order, but depends on compactification.

Concrete challenge:

i) Inflaton must be identified as one of the (many) scalar fields in string compactifications.

ii) Dynamics of the inflaton and other moduli are generically strongly entangled: For a controlled inflationary evolution (almost) all other scalars have to be fixed by moduli stabilization.

iii) Construct a slow-roll scenario which ends in a proper late-time cosmology, i.e. deSitter space with small cosmological constant.

- Strategy:

+ve cosmological constants with all moduli fixed!

A: Construct the "late-time cosmology model" by a proper flux-compactification.

B: Modify the model to contain inflationary behaviour.

The consistent realization of these two conditions is the main challenge.

Forecast: Fine-tuning seems inevitable!

fine tuning of what? are these parameters susceptible quantum corrections.

• A: Established model: KKLT scenario [1]

Type IIB flux compactification with warping

3 steps: i) Inclusion of fluxes
ii) Inclusion of non-perturbative string effects
iii) Inclusion of D-branes

i) flux field strengths: $G_3 := F_3 - \tau H_3$, $\tau := C_0 + i e^{-\Phi}$

$\tilde{F}_5 = * \tilde{F}_5$ (selfdual)

10D Einstein equation $\Longrightarrow$ Non-zero fluxes are only consistent with warped compactifications:

$$ds^2 = e^{2A(y)} \eta_{\mu\nu} dx^\mu dx^\nu + e^{-2A(y)} g_{mn}(y) dy^m dy^n .$$

$$\Longrightarrow \underbrace{\Delta A}_{\text{total derivative}} \propto |G_3|^2 + |\tilde{F}_5|^2 + \Lambda_{4D}$$

integrate over internal space

$\Longrightarrow$ $\boxed{G_3 = 0,\ \tilde{F}_5 = 0,\ \Lambda = 0}$ oder $\boxed{G_3 \neq 0,\ \tilde{F}_5 \neq 0,\ \Lambda < 0}$

$\boxed{\text{4D deSitter space is not possible}}$ (without D-branes).

- Analysis of the potential:

$$W_0(\tau, z^\alpha) = \int_Y \Omega \wedge G_3$$

$$K(\rho, \tau, z^\alpha) = -3 \ln\left(-i(\rho - \bar{\rho})\right) - \ln\left(-i(\tau - \bar{\tau})\right) - \ln\left(i \int_Y \Omega \wedge \bar{\Omega}\right)$$

(Assumption: Y-volume ρ is the only Kähler modulus)

SUSY-vacuum:

$$D_\rho W_0 = \frac{-3}{\rho - \bar{\rho}} W_0 \overset{!}{=} 0 \implies W_0 = 0, \quad G_3^{(0,3)} = 0,$$

$$D_\tau W_0 = \frac{1}{\tau - \bar{\tau}} \int_Y \Omega \wedge \bar{G}_3 \overset{!}{=} 0 \implies \bar{G}_3^{(0,3)} = G_3^{(3,0)} = 0,$$

$$D_\alpha W_0 = \int_Y \chi_\alpha^{(2,1)} \wedge G_3 \overset{!}{=} 0 \implies G_3^{(1,2)} = 0.$$

$\boxed{G_3 \in H^{2,1}(Y)}$ SUSY flux condition,
$\iff$ G_3 is selfdual.

No-scale potential:

$$G^{\rho\bar{\rho}} = \left(\partial_{\bar{\rho}} \partial_\rho K\right)^{-1} = \left(\partial_{\bar{\rho}} \frac{-3}{\rho - \bar{\rho}}\right)^{-1} = -\frac{1}{3}(\rho - \bar{\rho})^2$$

$$\Rightarrow \quad G^{\rho\bar\rho} D_\rho W_0 D_{\bar\rho} \bar W_0 - 3|W_0|^2$$

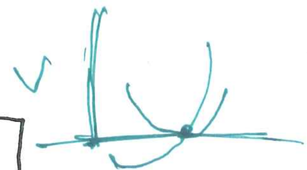
$$= -\tfrac{1}{3}(\rho-\bar\rho)^2 \left(\frac{-3W_0}{\rho-\bar\rho} \right)\left(\frac{3\bar W_0}{\rho - \bar\rho} \right) - 3 W_0 \bar W_0 = 0 .$$

The F-potential is independent of the internal volume ρ, so there exist SUSY-broken vacua with $D_\rho W_0 \neq 0$, which are energetically equal to the SUSY-vacuum.
In all cases we have: $V_{min} = 0 \Rightarrow \Lambda = 0$.

For SUSY, the point $D_\rho W_0 = 0$ is always an extremum of the scalar potential:

$$\begin{aligned} \partial_\rho V_{susy} &= -3\,\partial_\rho \left(e^K |W_0|^2 \right) \\ &= -3\, e^K \left((\partial_\rho K)|W_0|^2 + \partial_\rho |W_0|^2 \right) \\ &= -3\, e^K \underbrace{\left((\partial_\rho K) W_0 + \partial_\rho W_0 \right)}_{=0} \cdot \bar W_0 \end{aligned}$$

$$\Rightarrow \quad V_{susy} = V_{min} .$$



$$\boxed{V_F(\tau, z^\alpha) = e^K \sum_{I,J = \tau,\alpha} G^{I\bar J} D_I W_0 D_{\bar J} \bar W_0 \ \geq 0}$$

For a generic flux-superpotential all scalar moduli, except the volume ρ, are stabilized this way.

Consider effective theory for the last massless modulus ρ!

still vacuum energy $=0$

No inflation

ii) ρ-stabilization:

Include non-perturbative corrections to W_0.

$$\boxed{W(\rho) = \underbrace{W_0}_{\text{tree level!}} + \underbrace{Ae^{ia\rho}}_{\text{Nonperturbative}}} \qquad \text{(origin: instantons)}$$

$$\Rightarrow V_F = e^{K_\rho}\left(G^{\rho\bar\rho} D_\rho W D_{\bar\rho}\overline{W} - 3|W|^2\right)$$

$$= \frac{1}{(-i(\rho-\bar\rho))^3}\left(-\tfrac{1}{3}(\rho-\bar\rho)^2 D_\rho W D_{\bar\rho}\overline{W} - 3|W|^2\right)$$

$$= \frac{i}{3(\rho-\bar\rho)}\left|\partial_\rho Ae^{ia\rho} + (\partial_\rho K)(W_0 + Ae^{ia\rho})\right|^2$$

$$+ \frac{3}{i(\rho-\bar\rho)^3}\left|W_0 + Ae^{ia\rho}\right|^2$$

$$= \frac{i}{3(\rho-\bar\rho)}\left|iaAe^{ia\rho} - \frac{3}{\rho-\bar\rho}(W_0 + Ae^{ia\rho})\right|^2$$

$$\Updownarrow \to 0.$$

$$- \frac{3i}{(\rho-\bar\rho)^3}\left|W_0 + Ae^{ia\rho}\right|^2$$

$$[\ldots] = \frac{a^2A^2}{3i(\rho-\bar\rho)} e^{ia(\rho-\bar\rho)} + \frac{aAW_0}{-\frac{1}{2}(\rho-\bar\rho)^2} e^{\frac{i}{2}a(\rho-\bar\rho)}$$

$$+ \frac{a^2A^2}{-\frac{1}{2}(\rho-\bar\rho)^2} e^{ia(\rho-\bar\rho)}$$

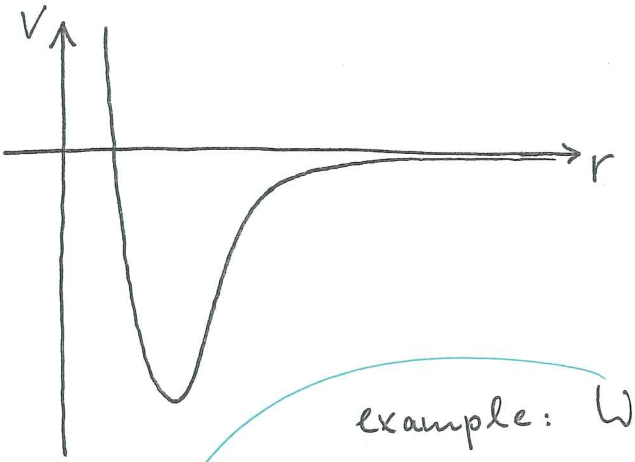
One can show that the axion b in:

$$\rho = b + \underbrace{ir}_{\text{real volume modulus}}$$

is stabilized by a periodic function with $b = 0$ as an allowed minimum. Set $b \mapsto 0$ in the potential.

• potential for r:

$$\boxed{V_F = \frac{a^2A^2}{6r}e^{-2ar} + \frac{aAW_0}{2r^2}e^{-ar} + \frac{aA^2}{2r^2}e^{-2ar}}$$



Stable AdS-vacuum with all moduli fixed, and large volume.
$(W_0 < 0)$

example: $W_0 = -10^{-4}$, $A = 1$, $a = 0{,}1$
$\Rightarrow \langle r \rangle \sim 115\, l_s$.

unnatural

related to SU(N) on gauge group

$a \sim \left(\frac{2\pi}{N}\right)$

treelevel ~ nonperturbative.

$a \sim 0.1$ $\quad N \sim 60$

fine tuning.

OK! Natural

Wo negative to set a minimum

Wo large negative so that ρ min at large!

for small r you need to know α' corrections! ✓

iii) Inclusion of D-branes:

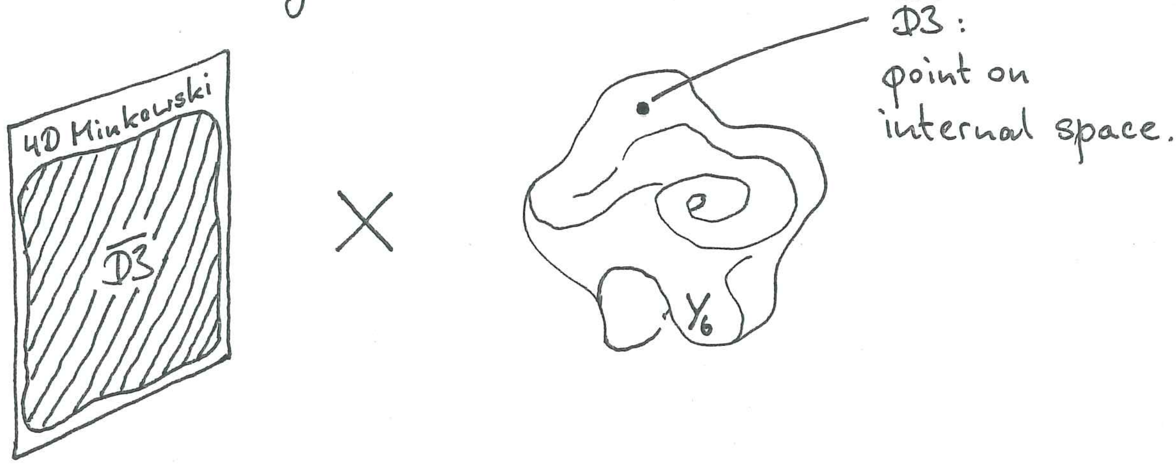
tadpole condition:

$$i\int_Y G_3 \wedge \overline{G}_3 + Q_3 = \underbrace{\int_Y * G_3 \wedge \overline{G}_3}_{\geq 0} + Q_3 = 0$$

$\Longrightarrow$ $Q_3 \leq 0$, add $\overline{D3}$-branes to the model!

(There was already $Q_3 < 0$ before from orientifold planes. Now: More flux and more negative charge.)

• Spacetime-filling $\overline{D3}$-brane:

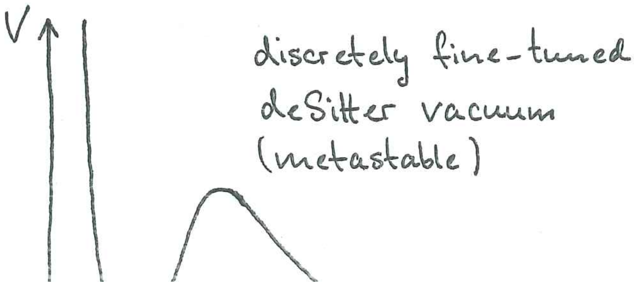


Potential correction (without derivation):

$$\delta V = \frac{D}{r^3}$$

$D = D(N_{\overline{D3}}, \text{fluxes}, \text{warping})$

In special warped geometry D is naturally exponentially small. $\langle r \rangle \sim$ as before.



• B Identify inflaton in the model:

B1 Preferred properties for the inflaton:

- No mass from moduli stabilization
- No mass from higher order corrections
- Not appearing in the Kähler potential*
- Shift symmetry that protects from mass terms

*Generic Kähler η-problem:

$$K = \bar{\phi}\phi + \ldots \quad , \quad V_0 \text{ independent of } \phi .$$

$$\Longrightarrow V_F = e^{K/M_p^2} V_0 \approx \left(1 + \frac{\bar{\phi}\phi}{M_p^2}\right) V_0$$

$$V_F'' = \frac{V_0}{M_p^2} \Longrightarrow \eta = M_p^2 \frac{V''}{V} \approx \frac{V_0}{V_0} \sim \mathcal{O}(1) .$$

Inflaton candidates:

- Kähler moduli in IIB, other than volume
- Real/imaginary parts of complex scalars

  $$K \propto \ln(\tau - \bar{\tau}) \Longrightarrow \phi = \mathrm{Re}(\tau) .$$

- Axions from p-forms over p-cycles.
- Brane position moduli (free massless fields)

B2 Brane - Antibrane inflation:

Modify KKLT scenario by adding a $D3-\overline{D3}$ brane pair in a warped region of the internal space. [2]

- $D3-\overline{D3}$ potential on Calabi-Yau:

$$V(d) \propto 1 - \frac{T_3^3}{M_{10}^8 d^4} \quad , \quad d: \text{distance modulus.}$$

$$\Rightarrow V' = \frac{4T_3^3}{M_{10}^8 d^5} \, , \quad V'' = -\frac{20 T_3^3}{M_{10}^8 d^6} \, , \qquad M_p^2 = \underset{\substack{\uparrow \\ \text{CY-volume}}}{r^6} M_{10}^8 .$$

$$\Rightarrow \eta = M_p^2 \frac{V''}{V}$$

$$= M_p^2 \left(-\frac{20T_3^3}{M_{10}^8 d^6}\right)\left(1 - \frac{T_3^3}{M_{10}^8 d^4}\right)^{-1}$$

$$\approx r^6 M_{10}^8 \left(-\frac{20T_3^3}{M_{10}^8 d^6}\right)\left(1 + \frac{T_3^3}{M_{10}^8 d^4}\right)$$

$$\approx -20\, T_3^3 \frac{r^6}{d^6}$$

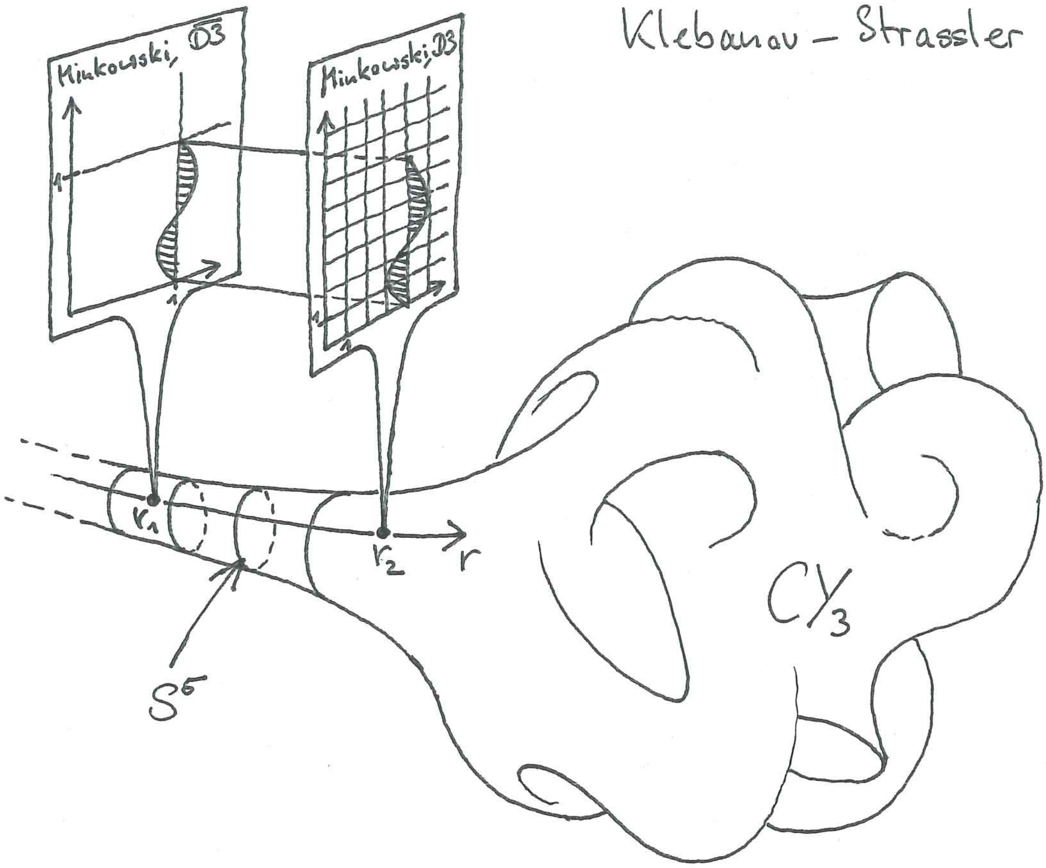
$r^6 \rightarrow$ volume of internal space!

$\eta \ll 1$ is not possible because $d \leq r$ is bounded by the size of the internal space.

In a warped geometry the $D3-\overline{D3}$ potential can be flattened.

• Red-shift effect in a warped throat:

Klebanov – Strassler throat



Local throat geometry:

$$\left(M^{1,3}(r) \tilde{\times} \mathbb{R}_r\right) \times S^5 = \left(AdS_5\right) \times S_5 .$$

$$\eta_{\mu\nu}(r_2) \approx e^{-(r_2-r_1)}\,\eta_{\mu\nu}(r_1)$$

$\Rightarrow$ Wavelengths from "down the throat" are redshifted [3]:
$$\lambda(r_2) \gg \lambda(r_1) .$$

$\Rightarrow$ D3 – $\overline{D3}$ potential gets redshifted.

- Solution for one D3-brane in the throat:

$$ds^2 = h^{-\frac{1}{2}}(r)\left(-dt^2 + d\vec{x}^2\right) + h^{\frac{1}{2}}(r)\, dr^2, \qquad (AdS_5)$$

$$g_{\mu\nu}(r) = h^{-\frac{1}{2}}(r)\, \eta_{\mu\nu},$$

$$C_4 = h^{-1}(r)\, d^4x, \qquad \boxed{h(r) = \frac{R^4}{r^4}}$$

Induced action for the brane:

$$S = -T_3 \int d^4x\, h^{-1} \sqrt{1 - h\,\partial_\mu r\, \partial^\mu r} + T_3 \int C_4$$

$$\approx -T_3 \int d^4x\, h^{-1}\left(1 - \tfrac{1}{2} h\, \partial_\mu r\, \partial^\mu r\right) + T_3 \int h^{-1}\, d^4x$$

$$= \tfrac{1}{2} T_3 \int d^4x\, \partial_\mu r\, \partial^\mu r$$

For small kinetic energies, the single brane is a free scalar field.

For two branes the solution changes to:

$$\boxed{h(r_1, r_2) = \frac{R^4}{r_1^4} + \frac{R^4}{r_2^4}}$$

For the $\overline{D3}$-brane the Chern-Simons term $\propto C_4$ in the action has reversed sign, so the potential does not cancel:

$$S_{\overline{D3}} = -T_3 \int \mathrm{vol}_{\overline{D3}}(r_1, r_2) - T_3 \int C_4 + S_{kin.}$$

$$= -T_3 \int h^{-1} d^4x - T_3 \int h^{-1} d^4x + \dots$$

$$= -2T_3 \int h^{-1} d^4x .$$

$$\Longrightarrow \quad V = 2T_3 h^{-1}$$

$$= 2T_3 \left(\frac{R^4}{r_1^4} + \frac{R^4}{r_2^4} \right)^{-1}$$

$$= 2T_3 \left(\frac{R^4}{r_1^4} \left(1 + \frac{r_1^4}{r_2^4} \right) \right)^{-1}$$

$$\boxed{V \approx 2T_3 \frac{r_1^4}{R^4} \left(1 - \frac{r_1^4}{r_2^4} \right)} \quad \text{for } r_2 \gg r_1 .$$

final result with redshift included

Discussion: $V(r_2) \propto -\frac{r_1^8}{R^4} r_2^{-4}$, this potential for the D3-brane position is very flat for large r_2, because of the exponent, and additionally highly suppressed by a red-shift factor. Generically $r_1 \ll 1$ because the $\overline{D3}$-brane is dynamically stable at the end of the throat.

Inflation ends when the branes come too close and finally annihilate.

Literature:

[1] Kachru, Kallosh, Linde, Trivedi:
de Sitter Vacua in String Theory, 2003

[2] Kachru, Kallosh, Linde, Maldacena, McAllister, Trivedi:
Towards Inflation in String Theory, 2003

[3] Giddings, Kachru, Polchinski:
Hierarchies from Fluxes in String Compactifications, 2002