Higgs Inflation

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1. Motivation

- SM should be valid up to Planck scale
- νMSM : SM + 3 additional light singlet fermions explain neutrino osc., dark matter, baryon asymmetry, and inflation
- Inflaton $\equiv$ Higgs boson of SM
- Extension in order to explain dark energy possible

SM + Gravity:
$$S_1 = \int d^4x \sqrt{-g}\left[\mathcal{L}_{SM} - \frac{M_P^2}{2} R\right]$$

$M_P = \frac{1}{\sqrt{8\pi G}}$; $\mathcal{L}_{SM}$: Lagrangian density of SM ; R: Ricci scalar ; $\sqrt{-g} = \sqrt{-\det(g_{\mu\nu})}$

$$\mathcal{L}_{SM} \supset V_H(\phi) \qquad V_H(\phi) = -\mu^2\phi^2 + \lambda_H \phi^4$$

ϕ: Higgs field : a scalar minimally coupled to gravity.
(No direct coupling to R)

- Can ϕ be responsible for inflation?

Not with S_1 since λ_H is too large!
If ϕ were inflaton matter fluctuations would be many orders of magnitude larger than observed!

$$V_H \simeq \lambda_H \phi^4 \text{ for large } \phi \ ; \ \boxed{\lambda_H = \frac{m_H^2}{v^2} \approx \mathcal{O}(1)!}$$

In Inflation with $\lambda\phi'^4$ potential
$\boxed{\lambda \simeq 10^{-15}}$ in order to obtain acceptable density pert.

SM + Gravity non-minimal coupled : Solution change S_1 to
$$S_2 = \int d^4x \sqrt{-g}\left[\mathcal{L}_{SM} - \frac{M^2}{2} R - \frac{1}{2}\xi\phi^2 R\right]$$

- Non minimal coupling of ϕ to gravity!

Such theories are already known in the literature

If one considers QFT on classical curved spacetime introduction of ξ is not an option, but necessity!

2. Non minimal coupling

$$\mathcal{L} \supset -\frac{1}{2}M_p^2 R - \frac{1}{2}\xi\phi^2 R - V(\phi)$$

- **Why $\xi \neq 0$?**
  - ξ is generated by first loop corrections even if absent. Renormalization shifts a theory with $\xi = 0$ to $\xi \neq 0$ (Ex $\xi = 0 \to \xi \simeq 10^{-1}$ for general $V = \lambda\phi^4$)
  - Pragmatic view: ξ usually crucial for success or failure of inflation $\to$ take it into account and decide a posteriori whether or not its effects are negligible.
- **What value of ξ ?**
  - Value of ξ should be set by physics of ϕ field and should not be another free parameter of the theory.
  - This is not always possible.

    Some choices:
    - $\xi = 0$ minimal coupling
    - $|\xi| >> 1$ strong coupling
    - $\xi = 1/6$ conformal coupling (KG eq. conformally inv. for $V = 0$ or $V = \lambda\phi^4$)

Additionally $\xi = 1/6$ is the only allowed value in classical field theory. Otherwise massive field ϕ could propagate on the lightcone in flat limit $\Rightarrow$ Violation of EINSTEINS EQUIVALENCE PRINCIPLE

- 1985: Hosotani Y. Phys. Rev. D 32 For SM Higgs $\xi \leq 0$ or $\xi \geq 1/6$

  but only 2008 Higgs boson as inflaton.

3. Classical Analysis

- Treat Higgs field as a single scalar: all transverse modes will be frozen due to Hubble friction

Jordan Frame ACTION

$$\boxed{S_{INF} = \int d^4x \sqrt{-g}\left[-\frac{1}{2}M_p^2 f(\phi)R + \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - V(\phi)\right]}$$

$$f(\phi) = 1 + \frac{\xi\phi^2}{M_p^2}$$

3.1 Conformal transformation

EINSTEIN FRAME: $g^E_{\mu\nu} = f(\phi) g_{\mu\nu}$; $g^E = f(\phi)^4 g$; $\sqrt{-g} = f(\phi)^{-2}\sqrt{-g^E}$

in 4D: $R^E = f(\phi)^{-1}\left[R + \frac{6}{\sqrt{f(\phi)}}\Box\sqrt{f(\phi)}\right]$

- $\Box\sqrt{f(\phi)} = g^{ab}\partial_b\left(\frac{\partial\sqrt{f}}{\partial\phi}\partial_a\phi\right) = \frac{\xi\phi}{\sqrt{f}M_p^2} g^{ab}\partial_b\partial_a\phi + \frac{\xi}{\sqrt{f}\,f M_p^2} g^{ab}\partial_b\phi\,\partial_a\phi$

$\rightarrow R^E = \frac{1}{f(\phi)}R + \frac{6}{f^2(\phi)M_p^2}\xi\phi g^{ab}\partial_b\partial_a\phi + \frac{6}{f^3 M_p^2} g^{ab}\partial_b\phi\,\partial_a\phi$

USE $\partial^a\left(\frac{3\xi\phi}{f}\partial_a\phi\right) = \frac{3\xi\phi}{f(\phi)}\Box\phi + \frac{3\xi}{f^2(\phi)} - \frac{3\xi^2\phi^2}{f^2(\phi)M_p^2}$

AND DROP TOTAL DERIVATIVE

$$\rightarrow S_{INF} = \int d^4x\sqrt{-g^E}\left[-\frac{1}{2}M_p^2 R^E + \frac{1}{2}\Big(f^{-1}(\phi) + \underbrace{\frac{6\xi^2\phi^2}{f(\phi)^2 M_p^2}}_{\text{from } R\to R^E}\Big)\partial_\mu\phi\,\partial^\mu\phi - \frac{V}{f(\phi)^2}\right]$$

- Define $\tilde{\phi}$ by $\boxed{\frac{\partial\tilde{\phi}}{\partial\phi} = \sqrt{f^{-1}(\phi) + \frac{6\xi^2\phi^2}{f(\phi)^2 M_p^2}}}$

EINSTEIN FRAME ACTION

$$\boxed{S_{INF} = \int d^4x\sqrt{-g^E}\left[-\frac{1}{2}M_p^2 R^E + \frac{1}{2}\partial_\mu\tilde{\phi}\,\partial^\mu\tilde{\phi} - \frac{V}{f(\tilde{\phi})^2}\right]}$$

3.2. HIGGS Potential

$$V(\tilde{\phi}) = \frac{1}{f(\tilde{\phi})^2}\frac{\lambda_H}{4}\left(\phi(\tilde{\phi})^2 - v^2\right)^2$$

A) ϕ small $\phi \ll M_p$

- $\frac{\xi\phi^2}{M_p^2} \simeq 0$; $f(\phi) = 1$

$\frac{\partial\tilde{\phi}}{\partial\phi} \simeq 1 \rightarrow \boxed{\tilde{\phi}\simeq\phi}$

$V(\tilde{\phi}) = V_{SM}(\tilde{\phi}) = \frac{\lambda_H}{4}(\tilde{\phi}^2 - v^2)^2$

B) ϕ large $\phi \gg \frac{M_p}{\sqrt{\xi}}$

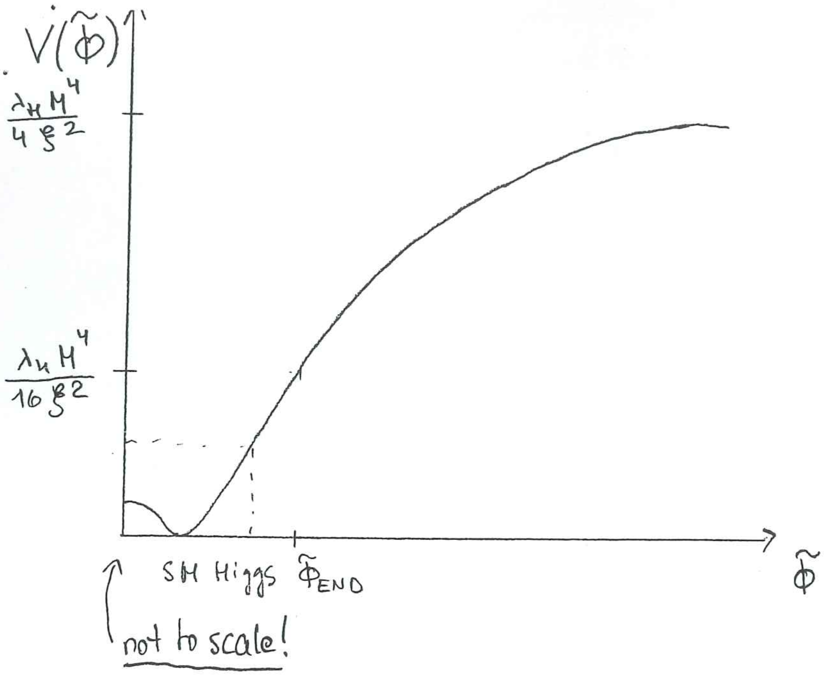
- $f(\phi) = \frac{\xi\phi^2}{M_p^2}$; $f^{-1}(\phi)\simeq 0$

$\frac{\partial\tilde{\phi}}{\partial\phi} = \frac{M_p\sqrt{6}}{\phi} \rightarrow \boxed{\phi \simeq \frac{M_p}{\sqrt{\xi}} e^{\frac{\tilde{\phi}}{M_p\sqrt{6}}}}$

- $f^2(\tilde{\phi}) = \left(1 + e^{\frac{2\tilde{\phi}}{M_p\sqrt{6}}}\right)^2$
- $\phi(\tilde{\phi})^4 = \frac{M_p^4}{\xi^2} e^{\frac{4\tilde{\phi}}{M_p\sqrt{6}}}$

$$\boxed{V(\tilde{\phi}) = \frac{\lambda_H M_p^4}{4\xi^2}\frac{1}{\left(1 + e^{-\frac{2\tilde{\phi}}{\sqrt{6}M_p}}\right)^2}}$$

FLAT AT LARGE $\tilde{\phi}$ VALUES!
Potential similar to the one from $f(R)$-model.



3.3 Slow-roll

- $\epsilon = \frac{M_p^2}{2}\left(\frac{dV/d\tilde{\phi}}{V}\right)^2$

$$\frac{dV}{d\tilde{\phi}} = \frac{\lambda_H M_p^3}{\xi^2\sqrt{6}} e^{-\frac{2\tilde{\phi}}{\sqrt{6}M_p}}\left(1+e^{-\frac{2\tilde{\phi}}{\sqrt{6}M_p}}\right)^{-3} \rightsquigarrow \frac{dV/d\tilde{\phi}}{V} = \frac{4}{\sqrt{6}M_p}\frac{e^{-\frac{2\tilde{\phi}}{\sqrt{6}M_p}}}{1+e^{-\frac{2\tilde{\phi}}{\sqrt{6}M_p}}}$$

$$\frac{x}{1+x} \simeq x - \dots$$

$$= \frac{4}{\sqrt{6}M_p} e^{-\frac{2\tilde{\phi}}{\sqrt{6}M_p}} ; \quad e^{-\frac{4\tilde{\phi}}{\sqrt{6}M_p}} = \left(\frac{\sqrt{\xi}}{M_p}\right)^{-4}\phi^{-4}$$

- $\boxed{\epsilon = \frac{4M_p^4}{3\xi^2}\phi^{-4}}$

- $\eta = M_p^2 \frac{d^2V/d\tilde{\phi}^2}{V}$

$$\frac{d^2V}{d\tilde{\phi}^2} \simeq \frac{\lambda_H M_p^2}{\xi^2 3} e^{-\frac{2\tilde{\phi}}{\sqrt{6}M_p}}\left(1+e^{-\frac{2\tilde{\phi}}{\sqrt{6}M_p}}\right)^{-3} \text{ drop higher order}$$

$$\boxed{\eta \simeq -\frac{4}{3}\frac{M_p^2}{\xi}\phi^{-2}}$$

END OF SLOW ROLL: $\epsilon \simeq 1 \rightsquigarrow \boxed{\phi_{end} \simeq \left(\frac{4}{3}\right)^{1/4}\frac{M_p}{\sqrt{\xi}} \simeq 1.07\frac{M_p}{\sqrt{\xi}}}$

# of e-foldings for the change from ϕ_0 to ϕ_{end}:

$$N = \int_{\tilde{\phi}_E}^{\tilde{\phi}_0}\frac{1}{M_p^2}\frac{V}{dV/d\tilde{\phi}}\,d\tilde{\phi} = \int_{\phi_E}^{\phi}\frac{1}{M_p^2}\frac{V}{dV/d\phi}\cdot\left(\frac{\partial\tilde{\phi}}{\partial\phi}\right)^2 d\phi \simeq \underline{\frac{3}{4}\frac{\xi}{M_p^2}\left(\phi_0^2-\phi_{END}^2\right)}$$

since $V(\phi) = \frac{\lambda M_p^4}{4\xi^2}\left(1+\frac{M_p^2}{\xi}\phi^{-2}\right)^{-2}$; $V/\partial V/\partial\phi = \frac{1}{4}\frac{\xi}{M_p^2}\phi^3\left(1+\frac{M_p^2}{\xi}\phi^{-2}\right)$

(N) $\boxed{N \simeq \frac{3}{4}\frac{\phi_0^2-\phi_{END}^2}{M_p^2/\xi}}$ $\rightsquigarrow$ as long $\boxed{\sqrt{\xi} <<< 10^{17} \Leftrightarrow \frac{M_p^2}{\xi} >>> v^2}$ Scale of SM does not enter!

3.3 Reheating

$$\phi_{END} = \left(\frac{4}{3}\right)^{1/4} \frac{M_P}{\sqrt{\xi}} = \frac{M_P}{\sqrt{\xi}} e^{\frac{\tilde{\Phi}_{END}}{\sqrt{6} M_P}} \rightsquigarrow \tilde{\Phi}_{END} = \sqrt{6} M_P \ln\left[\left(\frac{4}{3}\right)^{\frac{1}{4}}\right]$$

$$\rightsquigarrow \boxed{V(\tilde{\Phi}_{END}) = \frac{\lambda M_P^4}{16 \xi^2}}$$

- Interactions of the Higgs-boson with SM strong after end of inflation $\rightsquigarrow$ reheating right after slow-roll (instantaneous)

$$\rho_R = V(\tilde{\Phi}_{END}) \rightsquigarrow \boxed{T_R \simeq \left(\frac{2\lambda}{\pi^2 g^*}\right)^{1/4} \frac{M_P}{\sqrt{\xi}} \simeq 2 \cdot 10^{15}\,\mathrm{GeV}}$$

$$\rho_R = \frac{\pi^2 g^*}{30} T^4, \quad g^* = 106.75$$

3.4 Connection with measured quantities

- # of e-foldings between: scale λ_{COBE} (which enters the horizon right now k is measured by COBE) leaves the Hubble horizon during inflation and end of inflation?

$$N_{COBE} = \ln\left(\frac{a_{END}}{a_{COBE}}\right) = \ln\left(\frac{a_{END} H_{END}}{\underbrace{a_{COBE} H_{COBE}}_{a_0 H_0}} \cdot \frac{H_0}{H_{END}}\right) = \ln\left(\frac{a_{END} H_{END}}{a_0 H_0}\right) - \underbrace{\ln\left(\frac{H_{END}}{H_0}\right)}_{\text{usually} \ll 1}$$

$$N_{COBE} = \ln\left(\underbrace{\frac{a_{END}}{a_{Reh}}}_{\text{Matter dom.}} \cdot \underbrace{\frac{a_{Reh}}{a_0}}_{\text{Rad dom.}} \cdot \frac{V_E^{\frac{1}{2}}}{M_P} \cdot \frac{1}{H_0}\right) = \ln\left(\left(\frac{\rho_{END}}{\rho_{Reh}}\right)^{-\frac{1}{3}} \underbrace{\left(\frac{\rho_{Reh}}{\rho_0}\right)^{-\frac{1}{4}}}_{\rho_{Reh} \simeq V_E} \frac{V_E^{\frac{1}{2}}}{M_P} \frac{1}{H_0}\right)$$

$$\rho_0^{\frac{1}{4}} \propto T_0 \simeq 2 \cdot 10^{-13}\,\mathrm{GeV} \qquad \frac{1}{H_0} \simeq 3000 h^{-1}\,\mathrm{Mpc}$$

$$\boxed{N_{COBE} = 62 - \frac{1}{3}\ln\left(\left(\frac{V_{END}}{\rho_{Reh}}\right)^{\frac{1}{4}}\right) - \ln\left(\frac{10^{16}\,\mathrm{GeV}}{V_{END}^{\frac{1}{4}}}\right)}$$

in our case $V_{END} \simeq \rho_{Reh}$

$\Rightarrow \underline{N_{COBE} = 62}$

$$\rightsquigarrow \boxed{\phi_{COBE} \overset{(N)}{\simeq} 9.4 \frac{M_P}{\sqrt{\xi}}}$$

- Density perturbations: $\left(\frac{\delta\rho}{\rho}\right)^2 \propto \frac{V^3}{V'^2} \propto \frac{V}{\varepsilon}$

COBE Normalisation: $\boxed{\frac{V}{\varepsilon} = (0.027 M_P)^4}$

$$\frac{V}{\varepsilon} \simeq \frac{3}{16} \lambda \Phi^4 \simeq \frac{3}{16} \lambda \frac{M_P^4}{\xi^2} \left(\frac{8}{6} N_c\right)^2 \rightsquigarrow \boxed{\xi \simeq \sqrt{\frac{\lambda}{3}} \frac{N_{COBE}}{0.027^2} \simeq 49000\sqrt{\lambda} = 49000 \frac{m_H}{v}}$$

- Deduce ξ from fundamental theory $\Rightarrow$ Higgs mass connected to Perturbations!

- Spectral index $n = 1 - 6\epsilon + 2\eta \simeq \underline{0.97}$ $(N = 60)$

- Tensor to scalar ratio $r = 16\epsilon \simeq \underline{0.0033}$

4. Quantum Corrections

4.1. for a single scalar field without Potenti

a) Jordan Frame

Recall the new Lagrangian term of this theory

$$\mathcal{L} = \frac{1}{2} \xi \varphi^2 R \qquad\qquad S = \int d^4x \sqrt{-g}\, \mathcal{L}$$

Expanding the metric around flat space

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{M_p} h_{\mu\nu}$$

results for the Ricci scalar in

$$R \sim \frac{1}{M_p} \Box h$$

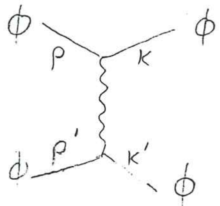
This expansion leads to the Lagrangian

$$\mathcal{L} \sim \frac{\xi}{M_p} \varphi^2 \Box h$$

The scattering process of two scalars via a graviton

t-channel:

$t = (k-p)^2$

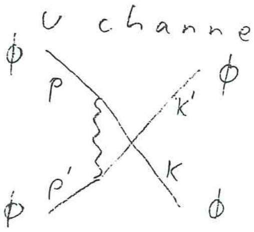


$$\sim \frac{(p-k)^2 (p'-k')^2}{(p-k)^2} \frac{\xi^2}{M_p^2} \sim t \frac{\xi^2}{M_p^2} \sim E^2 \frac{\xi^2}{M_p^2}$$

seems to lead to a cutoff scale $\Lambda = \frac{M_P}{\xi}$.

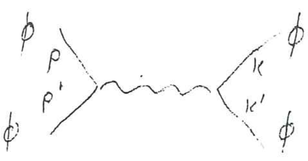
This result changes, however, if one takes u and s channel into account.

u channel



$$\sim u \frac{\xi^2}{M_p^2}$$

s-channel



$$\sim s \frac{\xi^2}{M_p^2}$$

As the sum over these terms cancel to $m^2(\varphi) \frac{\xi^2}{M_p^2}$, the usual gravity Lagrangian term $\mathcal{L} = \frac{1}{2} M_p^2 R$ stays the leading divergence term with cutoff $\boxed{\Lambda = M_P}$

b) Einstein Frame

Expansion of the non-canonical kinetic Lagrangian around small φ values

$$K(\varphi) = -\frac{1}{2} f^{-1}(\varphi)\,(\partial\varphi)^2 - 3\,\frac{\xi^2}{M_P^2}\, f^{-2}(\varphi)\,(\varphi\,\partial\varphi)^2$$

$$\overset{\varphi\to 0}{\approx} -\frac{1}{2}(\partial\varphi)^2 - 3\,\frac{\xi^2}{M_P^2}(\partial\varphi)^2\varphi^2 + \frac{1}{2}\,\frac{\xi}{M_P^2}(\partial\varphi)^2\varphi^2 + \dots$$

leads to a 4-point vertex for the $2\varphi \to 2\varphi$ scattering:

$$\frac{\xi^2}{M_P^2}(\partial\varphi)^2\varphi^2 \sim \frac{\xi^2}{M_P^2}\left[\partial(\varphi^3\partial\varphi) + \varphi^3\Box\varphi\right] \sim$$  $$\sim \frac{\xi^2}{M_P^2}E^2$$

Putting the external legs on shell, however, makes this term non-divergent.

This is true to all orders in every scattering process, as can be seen by using the canonica redefined field $\tilde{\varphi}$ as it obeys a free field equation.

4.2. The Potential in the Einstein Frame

Remember the potential in the Einstein frame in the limit for small $\tilde{\varphi}$

$$V_E(\tilde{\varphi}) = f^{-2}(\tilde{\varphi})\, V_J(\tilde{\varphi}) = \left(1 + \frac{\xi}{M_P^2}\varphi(\tilde{\varphi})\right)^{-2} \frac{\lambda}{4}\varphi^4(\tilde{\varphi})$$

$$\overset{\tilde{\varphi}\to 0}{\approx} \frac{\lambda}{4}\tilde{\varphi}^4 - \lambda\frac{\xi^2}{M_P^2}\tilde{\varphi}^6 - \frac{\lambda}{3}\frac{\xi}{M_P^2}\tilde{\varphi}^6 + \dots$$

In this case the scattering process $2\varphi \to (2+n)\varphi$ leads to an divergence proportional to

$$\frac{\lambda^2}{E^2}\frac{\xi^n}{M_P^n}E^n$$

which suggest a cut off

$$\Lambda \sim \lambda^{-2/n}\frac{M_P}{\xi} \xrightarrow{n\to\infty} \frac{M_P}{\xi}$$

This result, however, is controversial, as the original potential is non-polynomial. Therefore some authors claim that the flatness of the potential for large $\tilde{\varphi}$ leads to a good behaviour.

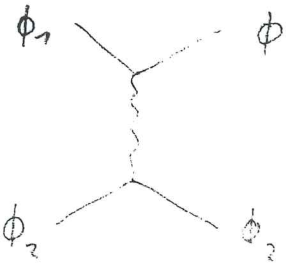
4.3. multiple fields

a) Jordan frame

As the SM Higgs mechanism includes more than on field, one has to extend the analysis to multiple fields. Going one step back and neglecting the potential, one has to analyse the consequences of the Lagrangian

$$\mathcal{L} = \frac{\xi}{M_p} \varphi_i \varphi_i \Box h$$

As before the t-channel of the $\varphi_1 \varphi_2 \to \varphi_1 \varphi_2$ scattering process leads to



$$\sim \frac{\xi^2}{M_p^2} E^2$$

But in this case there is no u or s channel

And therefore no cancellation, the cutoff scale is $\Lambda = \frac{M_p}{\xi}$!

b) Einstein frame

Including multiple fields in the Einstein frame

$$K = (\partial \varphi_i)^2 - 3 \frac{\xi^2}{M_p^2} (\varphi_i \partial \varphi_i)^2 + \ldots$$

introduces cross terms like

$$\frac{\xi^2}{M_p^2} \varphi_1 \partial \varphi_1 \varphi_2 \partial \varphi_2$$

It is not possible to performe a field redefinition which makes K canonical. Hence the argument we used before to eliminate the low cutoff does not apply any more.

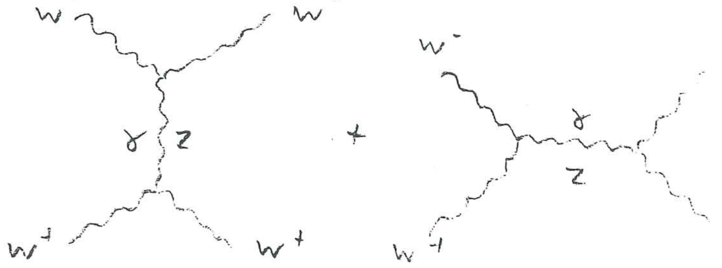
c) Unitary Gauge

Some authors claim that one does not have to deal with multiple fields, since one can use unitary gauge

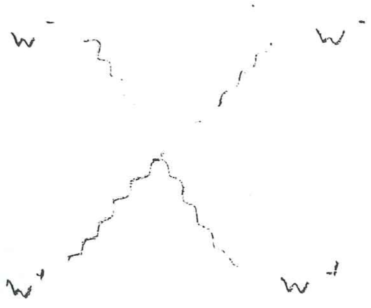
$$\varphi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

In this case however, one has to calculate the contributions of the would-be Goldston bosons in the scattering of the longitudinal components of the gauge bosons.

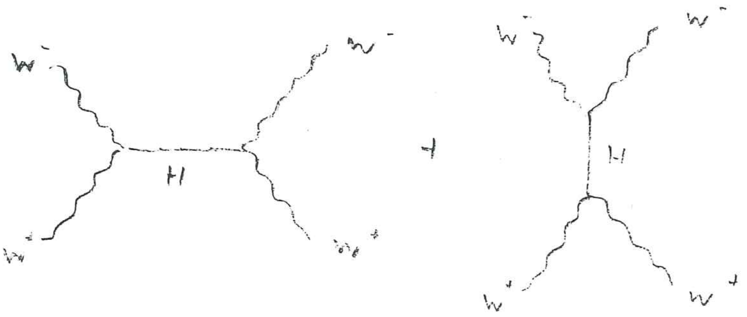
For instance the scattering of two W bosons gets additional contributions:



$$\sim \alpha \frac{p^4}{m_W^4} + \beta \frac{p^2}{m_W^2} + \dots$$



$$\sim -\alpha \frac{p^4}{m_W^4} - \frac{8}{9} \beta \frac{p^2}{m_W^2} + \dots$$



$$\sim -\frac{1}{9} \beta \frac{p^2}{m_W^2} \left(1 + \gamma \frac{\xi^2}{m_P^2}\right) + \dots$$

leading to a cutoff $\Lambda = \frac{m_P}{\xi}$!

5 Conclusion

Higgs Inflation might have been a promising theory on classical level. On quantum level, however, it seems impossible to explain inflation just with minimally coupled Higgs bosons. Even though discussion goes on.