the solution:

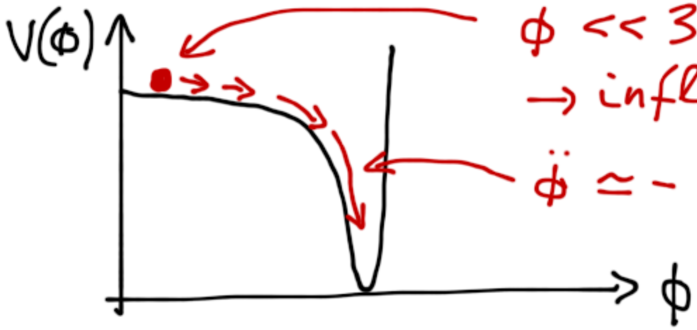
avoid trapping in a metastable minimum of the potential at $V>0$ and the subsequent tunneling ...

$\downarrow$

(I) Slow-roll inflation

(Albrecht & Steinhardt; Linde '82)

$\rightarrow$ slow-roll in potential $V(\phi)$:



$\ddot{\phi} \ll 3H\dot{\phi} = -\frac{\partial V}{\partial \phi}$

$\rightarrow$ inflation

$\ddot{\phi} \simeq -\frac{\partial V}{\partial \phi}$ $\rightarrow$ no inflation

consider: $\vec{\nabla}\phi = 0$, only $\dot{\phi}$

redshifts fast, if $a \sim e^{H\cdot t}$

then if: $\boxed{\dot{\phi}^2 \ll V}$ $\Rightarrow p = -\rho$

$\Downarrow$

motion dominated by potential energy

$$\begin{cases} a \sim e^{H\cdot t} \\ H \simeq \text{const.} \end{cases}$$

need this for $N_e \simeq H\cdot t \overset{!}{\simeq} 60$ efolds to solve the horizon etc. problems ...

can ensure this, if slow-roll:

e.o.m. for ϕ:

$$\ddot{\phi} + 3H\dot{\phi} = -\frac{\partial V}{\partial \phi} \equiv -V'$$

slow-roll: $|\ddot{\phi}| << |3H\dot{\phi}|, |V'|$

$$\Rightarrow 3H\dot{\phi} = -V' \quad \text{slow-roll e.o.m.} \quad (*)$$

then i): $p \simeq -\rho$

$$\Rightarrow 1 >> \frac{\dot{\phi}^2}{V} = \frac{V'^2}{9VH^2}, \; H^2 \simeq \frac{V}{3}$$

$$= \frac{1}{3}\left(\frac{V'}{V}\right)^2 \equiv \frac{2}{3}\epsilon, \; \boxed{\epsilon = \frac{1}{2}\left(\frac{V'}{V}\right)^2}$$

$\Rightarrow$ $\boxed{\epsilon << 1 \text{ ensures } p \simeq -\rho.}$

1st slow-roll condition

ensure slow-roll for long time:

↳ maintain: $\ddot{\varphi} << 3H\dot{\varphi}$

from $(*)$ $\Rightarrow \dot{\varphi}^2 = \frac{V'^2}{3V}$

$$\Rightarrow \dot{\varphi}\ddot{\varphi} = \frac{1}{2}\left(\frac{V'^2}{3V}\right)' \cdot \dot{\varphi}$$

$$\Rightarrow \ddot{\varphi} = V' \cdot \left(\frac{1}{3}\frac{V''}{V} - \frac{V'^2}{V^2}\right)$$

define: $\boxed{\eta \equiv \frac{V''}{V}}$

$$\Rightarrow \frac{\ddot{\varphi}}{3H\dot{\varphi}} = 2\epsilon - \frac{1}{3}\eta << 1$$

implies: $\boxed{\eta << 1}$ if $\epsilon << 1$

2nd slow-roll condition

if:

$\epsilon, \eta \ll 1$

then:

$$\epsilon \simeq \epsilon_H = -\frac{\dot{H}}{H^2} = 1 - \frac{1}{H^2}\frac{\ddot{a}}{a}$$

$$\eta \simeq \eta_H = \frac{\dot{\epsilon}_H}{\epsilon_H H}$$

$\Rightarrow$ $\epsilon \ll 1$ implies $\epsilon_H \ll 1$

$\Rightarrow$ $\ddot{a} > 0$

consistent.

ϵ_H, η_H 'physical' Hubble slow-roll parameters

can do much more:

- multiple fields
- higher derivatives
- non-trivial kinetic terms

but here simple single-field slow-roll will suffice.

General story here:

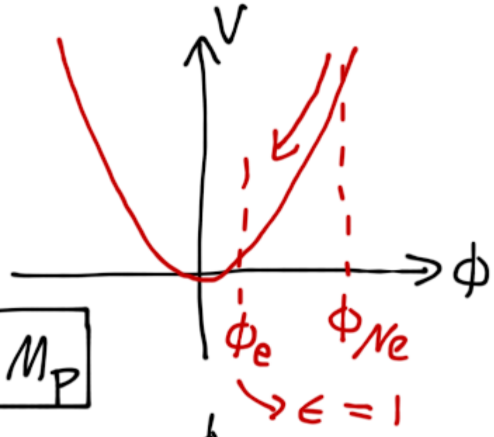
- need a $V(\phi)$ that satisfies $\epsilon, \eta \ll 1$ at some ϕ_{N_e},
- $\epsilon, \eta < 1$ for $N_e \simeq 60$ efolds at least, then $\epsilon > 1$ must be reached at some ϕ_e $\to$ slow-roll ends, ϕ oscillates $\to$ reheating, FRW

2 classes of $V(\phi)$:

i) large-field models

examples: $V(\phi) \sim \phi^p$, $p \geq 2$

e.g. $V(\phi) = \frac{m^2}{2}\phi^2$



ϕ_e: $\epsilon(\phi_e) = 1$

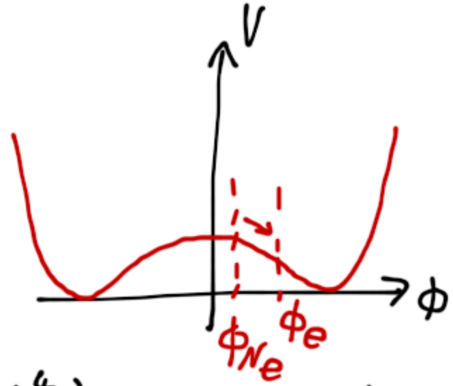
$$\epsilon = \frac{p^2}{2\phi^2} \Rightarrow \boxed{\phi_e \sim M_P}$$

$$N_e = \int_{t_e}^{t_{N_e}} H\,dt = \int_{\phi_e}^{\phi_{N_e}} \frac{H}{\dot{\phi}}\,d\phi = \int_{\phi_e}^{\phi_{N_e}} \frac{d\phi}{\sqrt{2\epsilon}}$$

$$\simeq \frac{\phi_{N_e}^2}{2p} \Rightarrow \boxed{\phi_{N_e} = \sqrt{2p\,N_e} \gg M_P}$$

ii) small-field models

example: hill-top



in $V(\phi) = \lambda(\phi^2 - v^2)^2$

and $\phi_{N_e} \ll v$

↓

$$V(\phi) = V_0\left(1 - \frac{2}{v^2}\phi^2 + \frac{\phi^4}{v^4}\right), \quad V_0 = \lambda v^4$$

$$\Rightarrow \epsilon = \frac{8\phi^2}{v^4}\left(1 - \frac{\phi^2}{v^2}\right), \quad \phi \ll v$$

$$\Rightarrow \phi_e \simeq \frac{1}{2\sqrt{2}}\left(\frac{v}{M_P}\right)^2 \cdot M_P$$

if $v \lesssim M_P$ → small-field

↳ if a 2nd field ends slow-roll by 'waterfall' into a minimum

→ 'hybrid' inflation ...

(II) How to test inflation – density fluctuations from inflation

i) The gift of inflation – (near) scale-invariant quantum fluct- uations in (near) de Sitter space !

$$\downarrow$$

How do we see that ?

Newtonian gravity in GR :

analogy: 4-velocity u in SR

$$u = (u^0, u^i) = (\sqrt{1-\vec{v}^2}, \vec{v}) \xrightarrow[\vec{v} \to 0]{} (1, 0, 0, 0)$$

"u^0 dominates in non-relativistic limit"

thus :

gravitational potential

Newton's $\Delta \zeta = 4\pi G \rho$

must come from

$$R_{00} - \frac{1}{2} g_{00} R = 8\pi G \cdot T_{00} = 8\pi G \cdot \rho$$

metric that does that :

$$ds^2 = (1 - 2\zeta)\, dt^2 - (1 + 2\zeta)\left[dr^2 + r^2 d\Omega_2^2\right]$$

with: $\gamma = \frac{2GM}{r}$ for mass M at $r = 0$

look at metric

$\rightsquigarrow$ at $r = R_S = 2GM$ time stops, dilated infinitely – like $v \to c$ in SR

$\longrightarrow$ <u>event horizon</u> of a black hole of mass M at:

$$R_S = 2GM$$

<u>'Schwarzschild radius'</u>

$$R_S = \frac{2M}{M_p^2} \quad \ldots$$

$$\Rightarrow A = 4\pi R_S^2 = 8\pi \cdot \frac{M^2}{M_p^4}$$

$$\Leftrightarrow M = \frac{M_p^2}{\sqrt{8\pi}} \sqrt{A}$$

$$\Rightarrow dM = \frac{M_p^2}{\sqrt{8\pi}} \frac{dA}{2\sqrt{A}}$$

$$= \frac{M_p^4}{16\pi} \cdot \frac{dA}{M}$$

$$= \frac{M_p^2}{8\pi} \cdot \frac{M_p^2}{2M} \cdot dA$$

$$\Rightarrow \quad \frac{dM}{M_P^2} = \hbar \frac{M_P^2}{2M} \cdot \frac{dA}{8\pi\hbar}$$

$$\hat{=} \quad dE = T \cdot dS$$

$\Rightarrow$ identify:

$$S = \frac{A}{8\pi\hbar}$$

$$T = \frac{\hbar}{2GM} \sim \frac{1}{R_S}$$

$\rightsquigarrow$ general rule:
a system with an event horizon of size R_H produces long-wave length quanta with temperature T:

$$T \sim \frac{1}{R_H}$$

now back to dS:

$\rightsquigarrow$ has an event horizon of size $R_{dS} \sim H^{-1}$

$\Rightarrow$ massless field quanta, e.g.:

- gravitons δg_{ij}
- inflaton field quanta $\delta\phi$

  $\vdots$

are produced with temperature:

$$T_{dS} \sim H$$

$\rightsquigarrow$ thermal fluctuations:

$$\langle \delta g_{ij} \rangle \sim \langle \delta\phi \rangle \sim T_{dS} \sim H$$

Compute power spectrum of 2-point function of fluctuations:

$$\langle \delta g_{ij}^2 \rangle \sim \langle \delta\phi^2 \rangle \sim H^2$$