

4.7.]

IV Inflation

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recall of the horizon problem: 145

light propagates as $ds^2 = 0$ at t_0
 $\Rightarrow dr = \frac{dt}{a(t)}$ can see $d = a_0 \int_0^{t_0} \frac{dt'}{a(t')}$

today: "Hubble radius" $\sim t_0 \sim \frac{1}{H_0}$
 $\sim 4 \cdot 10^{10}$ ly, CMB
 matter or radiation

CMB: photon decoupling at
 $3000 \text{ K} \approx 0.3 \text{ eV} \rightarrow Z = 1100$
 $\rightarrow 3 \text{ K today}$

$t_{\text{CMB}} \approx 400,000 \text{ yr}$

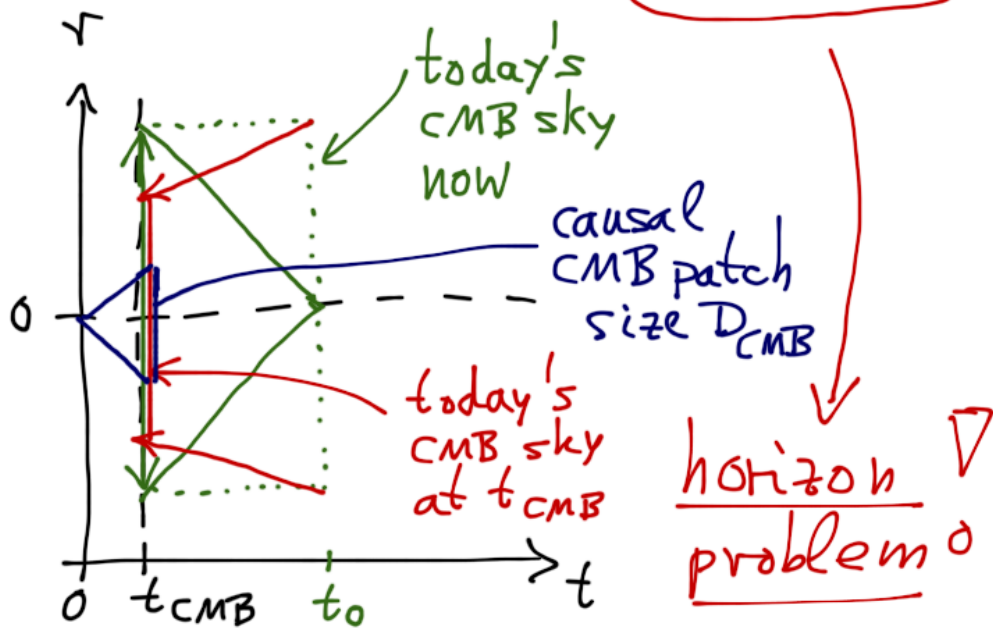
today's CMB sky was smaller
at t_{CMB} by:

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$$\frac{a(t_{\text{CMB}})}{a_0} = \left(\frac{t_{\text{CMB}}}{t_0}\right)^{2/3} = \frac{1}{1+z} = 10^{-3}$$

← matter domination since recombination

so $4 \cdot 10^7 \text{ ly}$ then. But causal patch at t_{CMB} only $D_{\text{CMB}} \approx 4 \cdot 10^5 \text{ ly}$.



another problem: spatial flatness 147

recall: $\Omega_k \sim \frac{k}{a^2}$, $\Omega_m \sim \frac{1}{a^3}$, $\Omega_{\text{rad.}} \sim \frac{1}{a^4}$

$\Rightarrow \Omega_k / \Omega_{\text{rad.}} \sim a^2$ GROWS

today: $\langle E_{\text{CMB}} \rangle \approx T_{\text{CMB}} \approx 3\text{K}$
 $\approx 10^{-3} \text{ eV}$

at nucleosynthesis: $T_{\text{Nuc}} \sim 10^6 \text{ eV}$

after quantum gravity epoch: $T_{\text{P}} \sim M_{\text{P}} \sim 10^{27} \text{ eV}$

we have: $T \sim \frac{1}{a}$

and: $|\Omega_k(\text{today})| \lesssim 10^{-2}$

$\Rightarrow \begin{cases} |\Omega_k(T_{\text{Nuc}})| \lesssim 10^{-18} \\ |\Omega_k(T_{\text{P}})| \lesssim 10^{-60} \end{cases}$ HOW?

an idea: try getting $\ddot{a} > 0$ ¹⁴⁸

$\Rightarrow a(t)$ grows faster than t

\Rightarrow today's CMB sky was smaller than causal patch at some early time

solves horizon problem

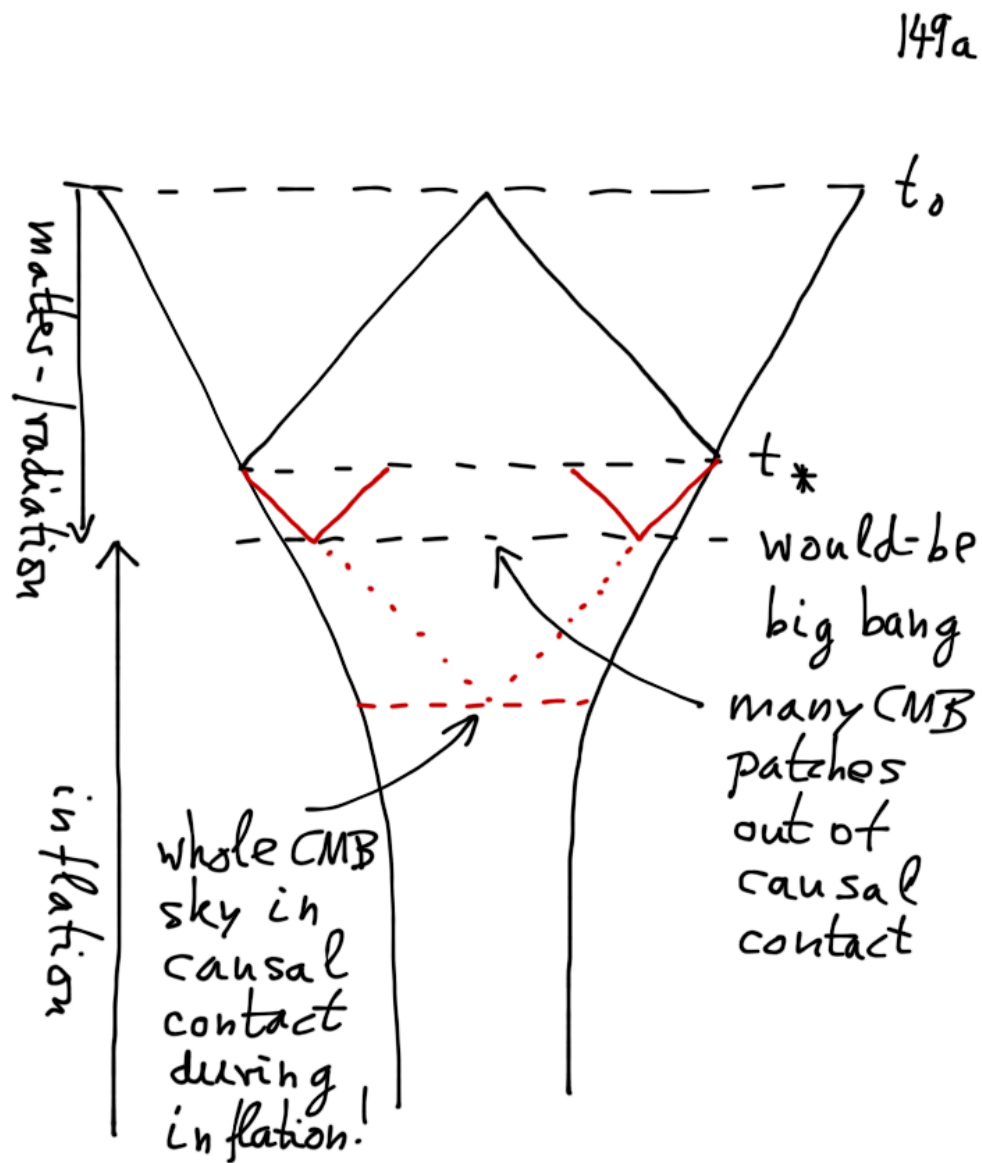


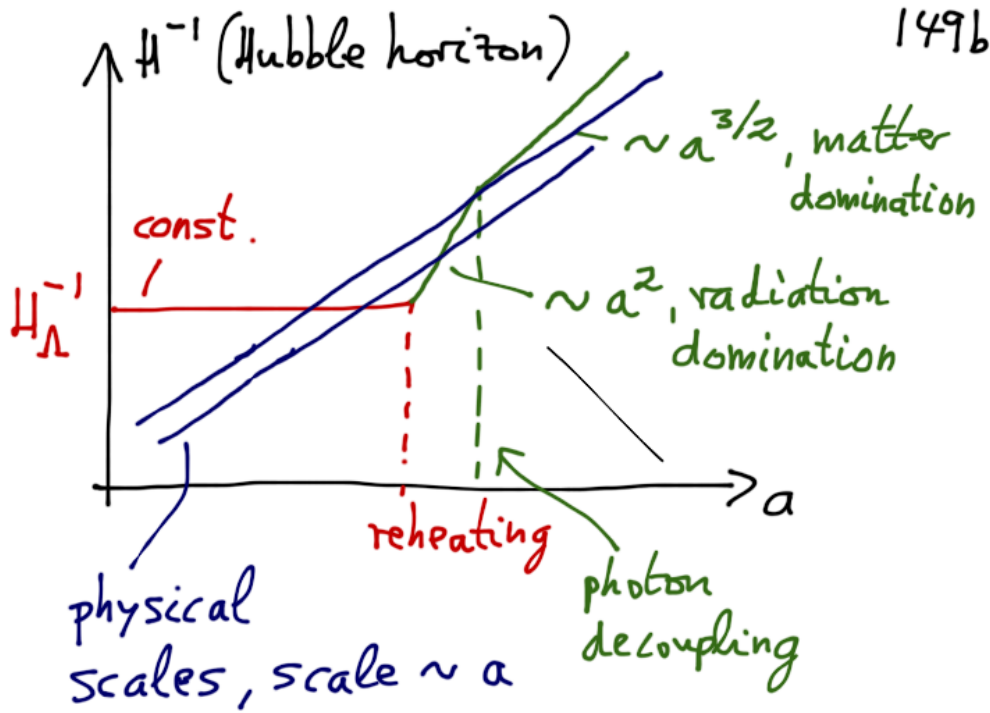
try a finite epoch of a quasi-cosmological constant - a large one:

$$\Rightarrow P_{\Lambda} = \text{const.} = -P_{\Lambda}$$

$$\Rightarrow H_{\Lambda}^2 = \frac{P_{\Lambda}}{3M_P^2} = \text{const.} = \frac{\dot{a}^2}{a^2}$$

$$\Rightarrow a \sim e^{H_{\Lambda} \cdot t} \text{ inflation}$$





\rightarrow modes left the horizon during inflation & re-entered afterwards \rightarrow were in causal contact early on
 \rightarrow solves horizon problem & some other (flatness, ...)

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horizon during inflation:

\sim calculate distance, that light can reach at time $t_e > 0$ after emission at $t_i < t_e$:

$$\begin{aligned}
 d_{\text{inf.}} &= a(t_i) \cdot \int_{t_i}^{t_e} \frac{dt}{a(t)} \\
 &= e^{H_{\Lambda} t_i} \frac{e^{-H_{\Lambda} t_e} - e^{-H_{\Lambda} t_i}}{-H_{\Lambda}} \\
 &= \frac{1}{H_{\Lambda}} \left(1 - e^{-H_{\Lambda} (t_e - t_i)} \right) \\
 &\rightarrow \frac{1}{H_{\Lambda}} \text{ for } t_e - t_i \rightarrow \infty
 \end{aligned}$$

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 while during matter- or radiation
 domination:

$$d_{\text{hor.}} \sim t e^{-t_i} \rightarrow \infty$$

for $t e^{-t_i} \rightarrow \infty$

↪ totally different
 behaviour!

→ during inflation there
 is a true event horizon
 at H_{Λ}^{-1} distance around us,
 similar to a black hole event horizon,
 but turned 'inside-out'...

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 solves flatness problem:

$$\Omega_{\Lambda} \approx \text{const.}$$

$$|\Omega_k| \sim \left| \frac{k}{a^2} \right| \lesssim 10^{-60}$$

if $H_{\Lambda} \cdot \Delta t_{\text{inf.}} \gtrsim 60$

"e-folds"

$$\equiv N_e$$

also:

$$\frac{a(t + \Delta t_{\text{inf.}})}{a(t)} = e^{H_{\Lambda} \cdot \Delta t_{\text{inf.}}} \equiv e^{N_e} \gtrsim e^{60}$$

would have all visible CMB scales
 today originating from a tiny causal
 region early on...

how to do this?

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@historically 1st: Guth '80

→ supercool Universe into a metastable state of positive vacuum energy → problems with exit & reheating

↓

use:

1 scalar field ϕ ...

action:

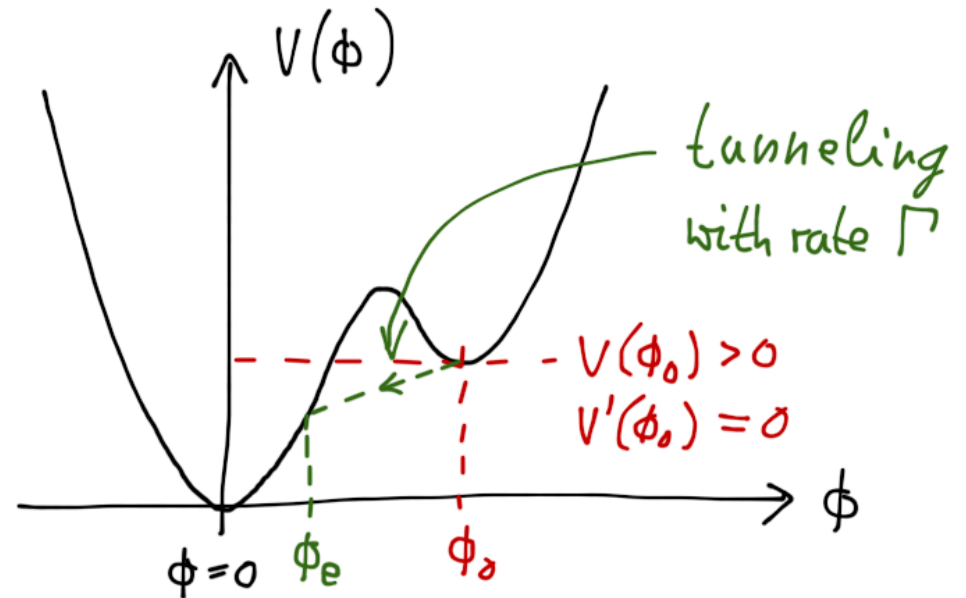
$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right]$$

↑
scalar potential

↓
 $\int \delta S / \delta g^{\mu\nu}$

$$T_{\mu\nu} : \rho = \frac{1}{2} (\partial\phi)^2 + V, \quad p = \frac{1}{2} (\partial\phi)^2 - V$$

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metastable vacuum of potential $V(\phi)$ emulates $\Lambda > 0$:



at $\phi = \phi_0$: local minimum, $\dot{\phi} = 0$
⇒ $\rho = V(\phi_0) = \text{const.} > 0$, $p = -V(\phi_0) = \underline{\underline{-\rho}}$

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 \Rightarrow while ϕ at ϕ_0 : inflation
 $\Leftrightarrow a \sim e^{\sqrt{V(\phi_0)} \cdot t}$

\Rightarrow once ϕ tunnels :

$\sim \phi$ exits at $\phi_e < \phi_0$,
then rolls quickly to
 $\phi = 0$ & oscillates
around $\phi = 0$:

non-relativistic matter
(ϕ has mass m)

$\Leftrightarrow a \sim t^{2/3} \Leftrightarrow \ddot{a} < 0$
inflation has ended

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 \Rightarrow if ϕ coupled to
other fields (SM fields) :

\sim decays to SM fields

while oscillating
around $\phi = 0$, once
inflation ended via
tunneling to $\phi_e \dots$

decay converts part
of $V(\phi_0)$ into new
radiation & heat —
"reheating"

problem:

≈ if tunneling slow, life-time
 $\tau = \frac{1}{\Gamma}$ of ϕ at ϕ_0 long

⇒ many e-folds $N_e = H(\phi_0) \cdot \tau$

but too few regions with
 exit by tunneling — one
 such region not big enough
 for our universe: need many
 that merge ...

≈ need fast tunneling, short
 lifetime → not enough e-folds ↓