25.6.

$\lambda_0 \simeq 1.633$ phase space factor

& life time $\tau_n = \Gamma^{-1}_{n\to pe\bar{\nu}}$ (56)

$$\Rightarrow \frac{G_F^2}{2\pi^3}(1+3g_A^2)\, m_e^5 = (\tau_n \lambda_0)^{-1}$$

$$\Rightarrow \Gamma_{ep\to n\nu} = (\tau_n \lambda_0)^{-1} \cdot \int_q^\infty d\epsilon \, \frac{\epsilon(\epsilon-q)^2\sqrt{\epsilon^2-1}}{\left(1+e^{\epsilon z}\right)\left(1+e^{(q-\epsilon)z_\nu}\right)} \quad (57)$$

prefactor can be fixed by neutron life time measurement

$\Downarrow$

(58) $g_A \simeq 1.26$ axial vector coupling of the nucleon

relativistic & non-relativistic limit:

$$\Gamma_{ep\to n\nu} = \begin{cases} \tau_n^{-1}\left(\frac{T}{m_e}\right)^3 \cdot e^{-Q/T}, & T \ll Q, m_e \\ \frac{7}{30}\pi\,(1+3g_A^2)\, G_F^2 T^5 \simeq G_F^2 T^5 & \text{for } T \gg Q, m_e \end{cases} \quad (59)$$

and: $H = \frac{\pi}{\sqrt{90}} \cdot g_*^{1/2} \cdot \frac{T^2}{M_P}$

and: γ, ν's relativistic

$\Rightarrow g_* \simeq 3.36$

$$\Rightarrow \frac{\Gamma_{ep\to n\nu}}{H} \simeq \left(\frac{T}{0.8\,\text{MeV}}\right)^3 \quad (60)$$

$\Rightarrow$ freeze-out at: $T_F \simeq 0.8$ MeV self-consistent with use of approximation of:

- $\Gamma_{ep\to n\nu}$ for $T > m_e \simeq 0.5\,\mathrm{MeV}$
- only γ, ν's safely relativistic d.o.f in g_* for $T < \mathrm{MeV}$

We can calculate

$$\frac{n_n}{n_p} = e^{-\frac{m_n - m_p}{T} + \frac{\overbrace{\mu_n - \mu_p}^{= \mu_e - \mu_\nu}}{T}} = e^{-\frac{Q}{T}} \cdot e^{-\frac{\mu_e - \mu_\nu}{T}} \qquad (61)$$

by (29) we have:

$$\frac{n_{e^+} - n_{e^-}}{n_\gamma} \sim \frac{\mu_e T^2}{T^3} = \frac{\mu_e}{T} \qquad (62)$$

and: $n_{e^+} - n_{e^-} = n_p - n_{\bar{p}} \simeq n_B$

$$\Rightarrow \frac{n_{e^+} - n_{e^-}}{n_\gamma} \simeq \frac{n_B}{n_\gamma} = \eta_B \sim \frac{\mu_e}{T}$$

$$\eta_B \sim 10^{-9} \quad \Rightarrow \quad \frac{\mu_e}{T} \sim \eta_B \sim 10^{-9}$$

thus we know n_n/n_e at freeze-out:

$$\left.\frac{n_n}{n_p}\right|_{T_F} = e^{-\frac{Q}{T_F}} \simeq \frac{1}{6} \qquad (63)$$

<u>step ii):</u> at $T \lesssim T_F \simeq 0.8\,\mathrm{MeV}$
production of D and ^3He
from n, p slow
$\rightarrow$ small $n_D, n_{^3He}$
$\curvearrowright$ blocks build-up
of ^4He by fusion
until about
$T_{NUC} \simeq 0.1\,\mathrm{MeV}$
"deuterium bottleneck"

thus within:
$$0.1\,\mathrm{MeV} < T < T_F \simeq 0.8\,\mathrm{MeV}$$

$\Rightarrow$ radioactive neutron-decay reduces n_n / n_p somewhat

radiation domination:
$$H = \frac{\pi}{\sqrt{90}}\, g_*^{1/2} \cdot \frac{T^2}{M_P}$$
$$\Rightarrow t = \frac{\sqrt{90}}{2\pi}\, g_*^{-1/2}\, \frac{M_P}{T^2}$$
$$\Rightarrow t(0.1\,\mathrm{MeV}) \simeq t_{NUC} \simeq 100\,\mathrm{s}$$

and : $\tau_n \simeq 900\,\mathrm{s}$
$$\Rightarrow \left.\frac{n_n}{n_p}\right|_{T_{NUC}} = e^{-\frac{Q}{T}} \cdot e^{-\frac{t_{NUC}}{\tau_n}} \simeq \frac{1}{7} \qquad (64)$$

at t_{NUC} n_D and $n_{^3He}$ have grown large, and all n left fuse rapidly into ^4He:

$$\Rightarrow X_{^4He} = \frac{N_{^4He}\, m_{^4He}}{N_N \cdot m_N} = \frac{4\, n_{^4He}}{n_N}$$

He mass fraction

nucleon number $= N_p + N_n$

$$= \frac{2 \cdot n_n}{n_n + n_p} = \frac{2}{1 + \frac{n_p}{n_n}} \qquad (65)$$

$\Rightarrow$ Synthesized ^4He mass fraction:

$$X_{^4He}\Big|_{t_{NUC}} = \frac{2}{1 + \frac{n_p}{n_n}}\Bigg|_{T_{NUC}} \qquad (66)$$

$$= \frac{2}{1+7} = \frac{1}{4} = \underline{\underline{25\%}}\,.$$

2) freeze-out of thermally produced weakly interacting dark matter

Why dark matter (DM)?

- galaxy rotation curves:

$$\frac{mv^2}{r} \sim G\frac{Mm}{r^2} \Rightarrow v^2(r) \sim \frac{1}{r}$$

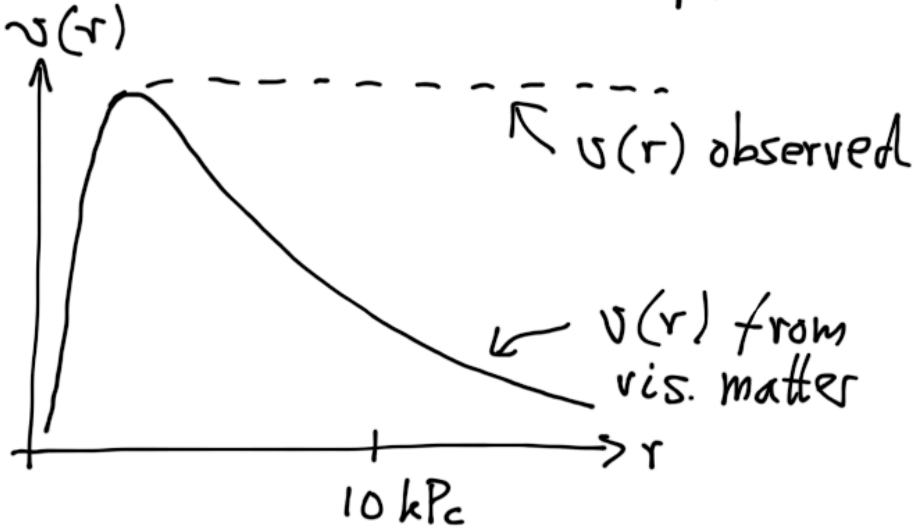
$\rightsquigarrow$ observation:

$$v(r) \simeq \text{const.}$$

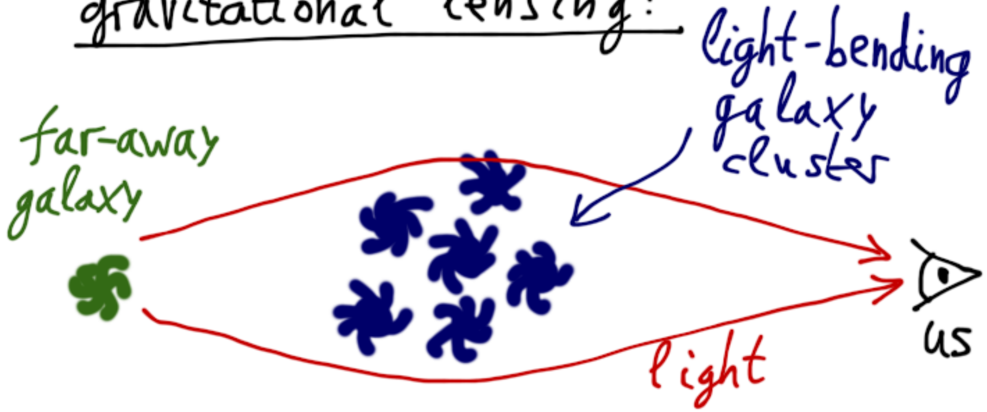
$\rightsquigarrow$ need $M(r) \sim r$ to compensate: invisible/dark ...

galaxy rotation curves:

visible matter: $M(r) \sim \frac{1}{r^{\#}}$, $\# > 0$



gravitational lensing:



- X-ray observations of gas bound in galaxies:

  requires some extra dark gravitating stuff to keep gas inside the galaxy

- WMAP – CMB + BBN:

  BBN fixes $\eta_B \Rightarrow \Omega_B \simeq 0.04$

  WMAP : $\Omega_\Lambda \simeq 0.7$

  $\Omega_0 \simeq 1$

  $\Rightarrow \Omega_{DM} \simeq 0.3$

- gravitational lensing

  $\rightsquigarrow \Omega_m \simeq 0.3$

- structure formation requires some 'extra' gravitational wells: $\Omega_m \simeq 0.3$

$\Rightarrow$ There are many possibilities for DM.

Will focus on WIMP:

new "Weakly Interacting Massive Particle" beyond the SM

$\rightarrow$ has weak-scale interactions

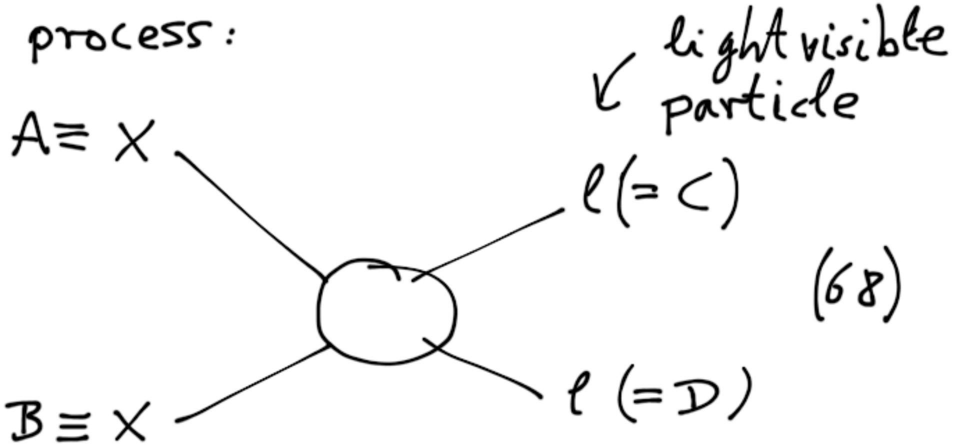
$\rightarrow$ heavy

$\rightsquigarrow$ well-motivated e.g. from SUSY.

single constituent DM:

analysis similar to BBN

process:



(68)

X : heavy DM WIMP

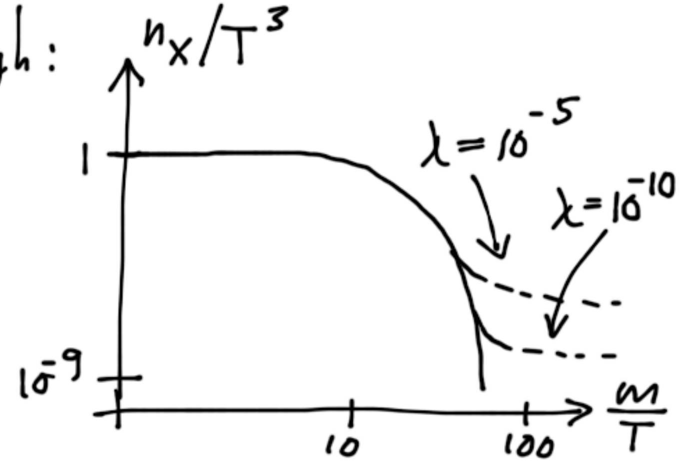
the Boltzmann eq. is again (48).

big picture:

DM particles X start at high T in equilibrium. If they stayed so always, then:

$$n_X \sim e^{-\frac{m_X}{T}} \quad \text{for } T < m_X$$

but, if $\Gamma_X < H$, DM density freezes out & survives, if it lives long enough:



here: $\lambda = \frac{m^2 \langle \sigma v \rangle}{H(m)}$ (69)

$\rightsquigarrow$ will turn out that WIMPS with weak scale masses give $\Omega_{DM} \simeq 0.3$

note: at high T the reaction is in kinetic equilibrium, but not always in chemical equilibrium ...

This implies:

$$(70) \quad \begin{cases} f(E) \sim e^{-\frac{E-\mu}{T}} \\ T \ll E - \mu \end{cases}$$

$\rightsquigarrow$ put differently, μ need not be at its equilibrium value.

$\Rightarrow$ Can rewrite the bracket $\{\dots\}$ of eq. (48) as:

$$\{\dots\} = e^{-\frac{E_A+E_B}{T}} \cdot \left(e^{\frac{\mu_C+\mu_D}{T}} - e^{\frac{\mu_A+\mu_B}{T}} \right) \quad (71)$$

$$(E_A + E_B = E_C + E_D)$$

use the n_i for $T \ll m_i$:

$$(72) \quad \Rightarrow \{\dots\} = e^{-\frac{E_A+E_B}{T}} \cdot \left[\frac{n_C n_D}{n_C^{(0)} n_D^{(0)}} - \frac{n_A n_B}{n_A^{(0)} n_B^{(0)}} \right]$$

with: $\frac{n_i}{n_i^{(0)}} = e^{\mu_i/T}$ equil. n_i at $\mu_i = 0$.

now define the thermally averaged cross section:

$$\langle \sigma v \rangle = \frac{1}{n_A^{(0)} n_B^{(0)}} \cdot \int \frac{d^3 p_i}{(2\pi)^3 E_i} e^{-\frac{E_A + E_B}{T}} \cdot |\mathcal{M}|^2 \cdot (2\pi)^4 \qquad (73)$$

$$\times \delta^{(4)}(p)$$

simplifies the Boltzmann eq. (48):

$$\frac{1}{a^3} \frac{d(n_A a^3)}{dt} = n_A^{(0)} n_B^{(0)} \langle \sigma v \rangle \cdot \left[\frac{n_C n_D}{n_C^{(0)} n_D^{(0)}} - \frac{n_A n_B}{n_A^{(0)} n_B^{(0)}} \right]$$

$$\Downarrow$$

$$\frac{1}{a^3} \frac{d(n_X a^3)}{dt} = \left(n_X^{(0)}\right)^2 \langle \sigma v \rangle \cdot \left[1 - \left(\frac{n_X}{n_X^{(0)}} \right)^2 \right] \qquad (74)$$