Freeze out of heavy particles and decoupling

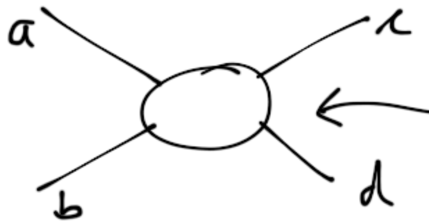
→ 'freeze-out' = decoupling from the thermal bath

⤳ need to know reaction rate Γ:

particle speed $\simeq c$

$$\Gamma_a = n_a \cdot \sigma \cdot v \qquad (43)$$

production of a-particles



cross section σ

2 cases – rule of thumb:

interaction via massless gauge boson (like γ):

$$\text{QFT} \Rightarrow \quad \sigma \sim \frac{\alpha^2}{T^2} \qquad (44)$$

α: gauge coupling

or via massive gauge boson X with mass m_X:

QFT $\Rightarrow$ for $T < m_X$

$$\sigma \sim G_X^2 T^2 \qquad (45)$$

$G_X \sim \frac{\alpha}{m_X^2}$ gauge coupling for relativistic species:

$$n \sim g T^3$$

$\Rightarrow$

$$\Rightarrow \Gamma \sim \begin{cases} g \cdot \alpha^2 T & \text{massless gauge boson} \\ g \cdot \dfrac{\alpha^2}{m_X^4} T^5 & \text{massive gauge boson} \end{cases} \qquad (46)$$

how many particles are produced?

$$N = \int_t^\infty \Gamma(t')\,dt' \quad , \quad \Gamma \sim T^n$$

$$(34): t \sim T^{-2} \Rightarrow dt \sim \frac{dT}{T^3}$$

$$\Rightarrow N \sim \frac{1}{n-2} T^{n-2} \sim \frac{1}{n-2} \frac{\Gamma}{H}$$

$\Rightarrow$ production stops once

$$\boxed{\Gamma < H} \qquad (47)$$

'freeze-out condition'

example: decoupling of neutrinos

$\leadsto$ interact electro-weakly:

$$\Gamma \sim G_F^2 T^5$$

G_F: Fermi constant of weak interaction

and $H \sim \dfrac{T^2}{M_P}$

freeze-out:

$$1 = \frac{\Gamma}{H} \sim M_p G_F^2 T^3$$

$$M_p \sim 2 \cdot 10^{18}\, \mathrm{GeV}$$

$$G_F \sim (200\, \mathrm{GeV})^{-2}$$

$$\sim \frac{10^9}{\mathrm{GeV}^3} \cdot T^3 = \left(\frac{T}{\mathrm{MeV}}\right)^3$$

$\Rightarrow$ ν's decouple at $T \simeq 1$ MeV.

can use this to calculate neutrino temperature today ...

$\downarrow$

after ν's freeze out, at

$T \lesssim m_e = 0.5$ MeV < 1 MeV

the e^-, e^+ annihilate

thus, before $t_{annih.}$

$$g_{*s} = \underbrace{1 \cdot 2}_{\gamma} + \underbrace{\frac{7}{8} \cdot 2 \cdot 2}_{e^+, e^-} + \nu\text{'s}$$

$$= \frac{11}{2}$$

and <u>after</u> $t_{annih.}$:

$$g_{*s} = \underset{\gamma}{2} + \nu's$$

but entropy <u>conserved</u>:

$$S_{t<t_{annih}} = g_{*s}^{before} \cdot (T_\nu a)^3 + S_\nu(T_\nu)$$

$T_\nu = T_\gamma$ still

$$= \frac{11}{2} \cdot (T_\nu a)^3 + S_\nu(T_\nu)$$

$$\overset{!}{=} S_{t>t_{annih.}} = g_{*s}^{after} (T_\gamma a)^3 + S_\nu(T_\nu)$$

$$= 2 (T_\gamma a)^3 + S_\nu(T_\nu)$$

$$\Rightarrow T_\nu = \left(\frac{4}{11}\right)^{1/3} \cdot T_\gamma \simeq 1.91 \text{ K}$$

today

<u>another example</u>: photon decoupling

the process $\gamma + H \to p + e^-$ prevents H from forming if too rapid — Thomson scattering

$$\Gamma_\gamma = n_e \sigma_T, \quad \sigma_T \simeq 6.7 \cdot 10^{-29} \text{ m}^2$$

photon decoupling

$\longleftrightarrow$ mean free path $\sim$ Hubble horizon $\sim \frac{1}{H}$

$$\Rightarrow t_\gamma = \frac{1}{\Gamma_\gamma} = \frac{1}{H} = t_H \quad (*)$$

$$\Gamma_\gamma = n_e \sigma_T = \sigma_T X_e n_p = X_e n_B \sigma_T$$

ionization fraction

$$\begin{cases} n_B = n_B^0 \cdot (1+z)^3 \\ n_B^0 = 0.23/m^3 \end{cases}$$

and matter domination after decoupling:

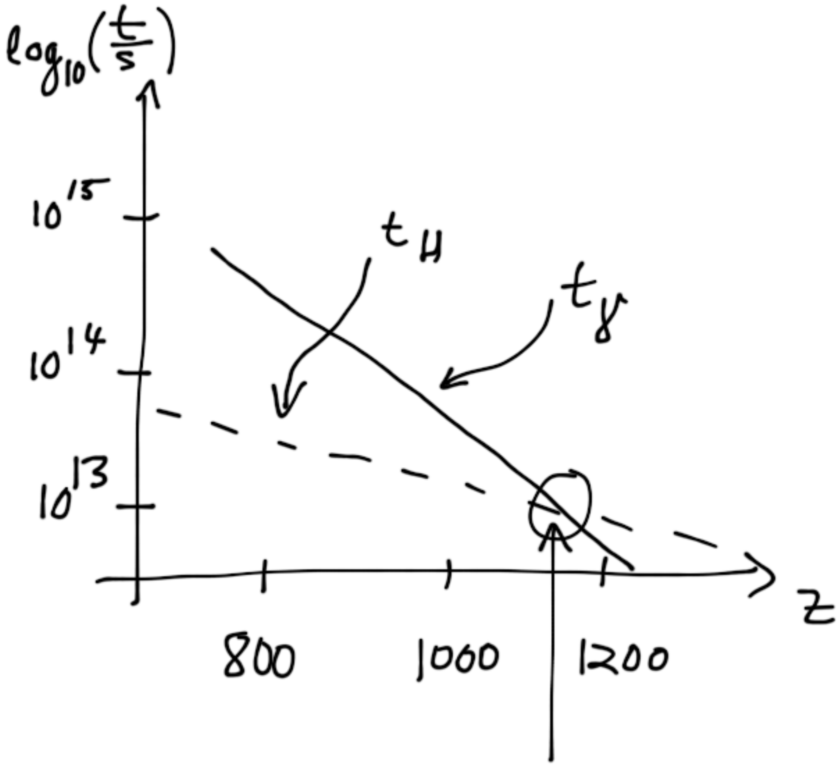
$$\Rightarrow \quad \frac{t}{t_0} = \left(\frac{a}{a_0}\right)^{3/2} = (1+z)^{-3/2}$$

$$t_0 = \frac{2}{3H_0}$$

$$\Rightarrow t_H = \frac{2}{3H_0}\left(1+z_H\right)^{-3/2}$$

$$\simeq 3 \cdot 10^{17} s \cdot (1+z_H)^{-3/2}$$

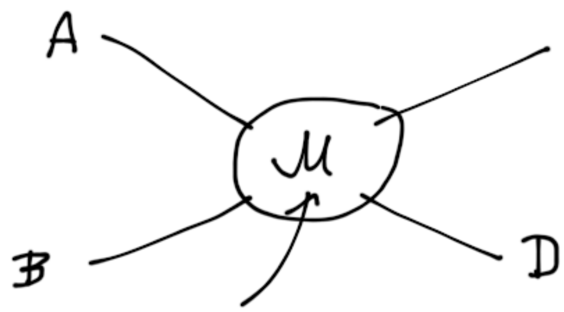
plug into $(*)$, and let's say that decoupling is complete once $X_e \simeq 0.02$ (% level rest ionization) ...



$z_\gamma \simeq 1100$

description of freeze-out

reaction



$A + B \rightarrow C + D$

QFT : matrix element $\mathcal{M}$ of reaction

semi-classically:

$$\frac{d\#}{dt} = \Gamma_{\text{production}} - \Gamma_{\text{annihil./decay}}$$

for e.g. particle type A in more detail:

$$\#(A) = N_A = n_A V \sim n_A a^3$$

and QFT

$$\Rightarrow \frac{1}{a^3}\frac{d}{dt}(n_A a^3) =$$

$$= \int \frac{d^3p_A}{(2\pi)^3 E_A} \int \frac{d^3p_B}{(2\pi)^3 E_B} \int \frac{d^3p_C}{(2\pi)^3 E_C} \int \frac{d^3p_D}{(2\pi)^3 E_D} \quad (48)$$

$$\times (2\pi)^4 \cdot \delta^{(3)}(\vec{p}_A + \vec{p}_B - \vec{p}_C - \vec{p}_D) \cdot \delta(E_A + E_B - E_C - E_D)$$

$$\times \underbrace{|\mathcal{M}|^2}_{\text{Matrix element}} \cdot \Big\{ \underbrace{f_C f_D \cdot (1 \pm f_A)(1 \pm f_B)}_{\text{production of } A} - \underbrace{f_A f_B (1 \pm f_C)(1 \pm f_D)}_{\text{annihilation/decay of } A} \Big\}$$

$+$: boson
$-$: fermion

$f_i(E)$, $i = A, B, C, D$

partition function of particle species i

(48) is the <u>Boltzmann equation</u>.

$\downarrow$

look at two examples ...

1) nucleosynthesis: Helium synthesis

step i): neutron freeze-out

weak interactions

$\rightarrow$ reactions:

$$\left.\begin{aligned} n &\leftrightarrow p + e^- + \bar{\nu} \\ p + e^- &\leftrightarrow n + \nu \\ n + e^+ &\leftrightarrow p + \bar{\nu} \end{aligned}\right\} \quad (49)$$

at high T chemical equilibrium:

$$\mu_e + \mu_p = \mu_n + \mu_\nu \qquad (50)$$

and energy conservation:

$$E_e + E_p = E_n + E_\nu \qquad (51)$$

rate for this process:

$$\Gamma_{ep\to n\nu} = \int \frac{d^3p_e}{(2\pi)^3 E_e} \int \frac{d^3p_\nu}{(2\pi)^3 E_\nu} \int \frac{d^3p_n}{(2\pi)^3 E_n} \qquad (52)$$

$$\times \frac{(2\pi)^4}{8} \cdot \delta^{(4)}(p_e + p_p - p_n - p_\nu)$$

$$\times f_e(E_e) \cdot \left[1 - f_\nu(E_\nu)\right] \cdot |\mathcal{M}|^2$$

$$|\mathcal{M}|^2 \simeq G_F^2 \left(1 + 3g_A^2\right) \qquad (53)$$

G_F: Fermi constant

$$= \frac{G_F^2}{2\pi^3}(1+3g_A^2)$$

$$\times \int_{m_e}^{\infty} dE_e \frac{E_e\sqrt{E_e^2 - m_e^2}\,\big(\overbrace{E_e + E_p - E_n}^{E_\nu}\big)^2}{\left(1 + e^{E_e/T}\right)\cdot\left(1 + e^{-\underbrace{(E_e+E_p-E_n)}_{E_\nu}/T_\nu}\right)}$$

define: $Q = m_n - m_e = 1.293\ \text{MeV}$

"neutron binding energy"

switch to:

$$\epsilon = \frac{E_e}{m_e},\quad q = \frac{Q}{m_e},\quad z = \frac{m_e}{T},\quad z_\nu = \frac{m_e}{T_\nu}$$

$$\Rightarrow \Gamma_{ep\to n\nu} = \frac{G_F^2}{2\pi^3}\left(1+3g_A^2\right) m_e^5 \qquad (54)$$

$$\times \int_q^\infty d\epsilon \, \frac{\epsilon\,(\epsilon-q)^2\sqrt{\epsilon^2-1}}{\left(1+e^{\epsilon z}\right)\left(1+e^{(q-\epsilon)z_\nu}\right)}$$

the prefactor also appears in free neutron decay: no $f_i(E_i)$ for single n

$$\Rightarrow \Gamma_{n\to pe\bar\nu} = \frac{G_F^2}{2\pi^3}\left(1+3g_A^2\right) m_e^5 \cdot \underbrace{\int_1^q d\epsilon \cdot \epsilon\,(\epsilon-q)^2\sqrt{\epsilon^2-1}}_{\lambda_0} \qquad (55)$$