more concretely [GKP '01; KKLT '03]:

10d IIB supergravity from IIB string theory:

$$S = \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g}\left\{e^{-2\phi}\left[R+4(\nabla\phi)^2\right] - \frac{1}{2}F_1^2 - \frac{1}{2\cdot 3!}|G_3|^2 - \frac{1}{4\cdot 5!}\tilde{F}_5^2\right\}$$

$$+\frac{1}{8i\kappa_{10}^2}\int e^{\phi}\cdot C_4\wedge G_3\wedge\bar{G}_3 + S_{loc.}$$

$$\tilde{F}_5 = F_5 - \frac{1}{2}C_2\wedge H_3 + \frac{1}{2}B_2\wedge F_3$$

$\tilde{F}_5 = *\tilde{F}_5$ must be imposed on e.o.m.

define: $\tau = C_0 + i\underbrace{e^{-\phi}}_{=1/g_s}$ string coupling

go to 10D Einstein frame:

$$(g_E)_{MN} = (g_S)_{MN}\cdot e^{-\phi/2}$$

$$\Downarrow$$

$$S = \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g}\left\{R - \frac{\partial_M\tau\,\partial^M\bar{\tau}}{2(\mathrm{Im}\,\tau)^2} - \frac{|G_3|^2}{12\cdot\mathrm{Im}\,\tau} - \frac{\tilde{F}_5^2}{4\cdot 5!}\right\} + \dots$$

compactify on :

$$\mathcal{M}_{10} = \mathcal{M}_4 \times_{\text{warped}} CY_3$$

$$ds^2 = e^{2A(y)} \cdot \eta_{\mu\nu}\, dx^\mu dx^\nu$$
$$+ e^{-2A(y)} \cdot \tilde{g}_{mn}\, dy^m dy^n$$

$\mu, \nu = 0, \dots, 3$
$m, n = 4, \dots, 9$

Calabi-Yau 3-manifold (CY_3)

$$g_{mn} = e^{-2A} \cdot \tilde{g}_{mn} \quad : \text{"conformal } CY_3\text{"}$$

CY_3 : complex Kähler manifold with $\dim_{\mathbb{C}} CY_3 = 3$, which has $c_1(CY_3) = 0$

Kähler potential

$\Leftrightarrow$ $\exists$ a metric $g_{i\bar{j}} = \partial_i \partial_{\bar{j}} K$ on CY_3 such that $R_{i\bar{j}} = 0$

pair real coordinates into holomorphic z_i and anti-holomorphic $\bar{z}_i$, $i = 1 \dots 3$

$R_{i\bar{j}} = 0$ allows for certain metric deformations :

δg_{ij} : $(2,1)$-forms $\chi^{(a)}_{ij\bar{k}}$

"complex structure moduli" (change the complex structure, i.e. how real coords. are split into z_i, $\bar{z}_{\bar{j}}$)

$\delta g_{i\bar{j}}$: $(1,1)$-forms $\omega^{(i)}_{j\bar{k}}$

or Poincaré-dual $(2,2)$-forms

"Kähler moduli"

(change volume of CY_3 and 2-dim subspaces = 2-cycles → volume moduli)

such that : $R_{i\bar{j}} = 0$ stays

and :

$$\int d^{10}x \sqrt{-g}\, R \longrightarrow \int d^4x \sqrt{-g} \Big[{}^{(4)}R + \frac{1}{2}\left(\partial_\mu \delta g_{ij}\right)^2 + \frac{1}{2}\left(\partial_\mu \delta g_{i\bar{j}}\right)^2 \Big]$$

moduli are massless scalar fields in 4D :

u^a : complex structure moduli

T^i : Kähler moduli

# of complex structure moduli

$= h^{2,1}$ determined by topology

of 3-cycles (S^3's) in the

CY_3:

for each non-trivial 3-cycle Σ^a

there is a harmonic (2,1)-form

$\chi^{(a)}_{2,1}$ such that:

$$(\ast)\ \triangle \chi^{(a)}_{2,1} \sim \partial_m(\sqrt{-g}\, g^{mn} \partial_n)\, \chi^{(a)}_{2,1} = 0$$

"harmonic form"

and: $d\chi^{(a)}_{2,1} = 0$ but $\chi^{(a)}_{2,1} \neq dA^{(a)}$

$\chi^{(a)}_{2,1}$ is closed, but not exact:

$\rightsquigarrow$ it is <u>not</u> just a local coord.-transf. of $g_{i\bar{j}}$

likewise, every non-trivial 2-cycle Σ^i or Poincaré-dual 4-cycle $\tilde{\Sigma}^i$ has a non-trivial harmonic (1,1)-form $\omega^{(i)}_{1,1}$ associated.

$$\rightsquigarrow \begin{cases} u^a \sim \mathrm{vol}(\Sigma^a) \sim \text{radius}^3 \\ T^i \sim \mathrm{vol}(\tilde{\Sigma}^i) \sim \text{radius}^4 \end{cases}$$

moduli stabilization $\rightarrow$ how to give mass to u^a, T^i ?

$\sim$ 4D Lorentz invariance :

$$\tilde{F}_5 = (1+*)\, d\alpha \wedge dx^0 \wedge \ldots \wedge dx^3$$

$$\alpha = \alpha(y)$$

$\sim$ look at Einstein equations:

$$\tilde{\nabla}^2 e^{4A} = e^{2A} \frac{|G_3|^2}{12 \cdot \mathrm{Im}\,\tau} + e^{-6A}\left[(\partial_m \alpha)^2 + (\partial_m e^{4A})^2\right]$$

$$+ \frac{\kappa_{10}^2}{2} e^{2A} \left(T^m_m - T^\mu_\mu\right)_{loc.}$$

$\Rightarrow$ if I have local objects with

$$T^m_m - T^\mu_\mu < 0$$, can have

flux G_3 and warping ...

(Maldacena-Nuñez 'no-go')

flux on a 3-cycle:

$$\int_{\Sigma^a} F_3 \, , \int_{\Sigma^a} H_3 \in \mathbb{Z}$$

quantized, similar to Dirac magnetic monopole quantizing $\int B_2^{em.}$

local objects:

Dp-branes wrapped on $(p-3)$-cycle in CY_3:

$$S_{Dp} = \underbrace{\frac{1}{\alpha'^{\frac{p+1}{2}}}}_{=T_p} \underbrace{\int d^{p+1}\xi \sqrt{-\det(g+B_2+F_2)}}_{\text{DBI-action}} + \ldots$$

→ brane tension

$$= \frac{1}{\alpha'^2}\int d^4x \int_{\Sigma_{p-3}} d^{p-3}y \sqrt{-g} + \ldots$$

$$\Rightarrow \quad T^m{}_m - T^\mu{}_\mu = (7-p)\,T_p \,, \quad T_p > 0$$

have D3 & D7-branes in IIB

$$\Rightarrow \left(T^m{}_m - T^\mu{}_\mu\right)_{\substack{D3\\D7}} \geq 0$$

but also orientifold planes O3/O7 — needed to break 4D $\mathcal{N}=2$ from IIB on a CY_3 to 4D $\mathcal{N}=1$:

$$T_p^{O3/O7} < 0 \quad !$$

$\Rightarrow$ can have G_3-flux and warping, if O3/O7 is there.

now what does G_3-flux do to the moduli?

→ G_3-flux must sit on 3-cycles Σ^a of CY_3

⇒ specified by (2,1)-forms $\chi^{(a)}_{2,1}$ and holomorphic (3,0)-form Ω of CY_3

→ reduce $|G_3|^2$ kinetic term on the CY_3

⇒ leads to a scalar potential for the complex structure moduli U^a, and for τ

⇝ takes form of a 4D $\mathcal{N}=1$ supergravity scalar potential:

$$V = e^K \left(K^{a\bar b} D_a W \overline{D_b W} + K^{i\bar j} D_i W \overline{D_j W} + K^{\tau\bar\tau} |D_\tau W|^2 - 3|W|^2 \right)$$

with:

$$K = -2 \cdot \ln \mathcal{V}_{CY_3} - \ln\left(-i(\tau - \bar{\tau})\right)$$

$$- \ln\left(-i \int_{CY_3} \Omega(u^a) \wedge \bar{\Omega}(u^a)\right)$$

"moduli Kähler potential"

$\mathcal{V}_{CY_3} = f(T_i, \bar{T}_i)$ e.g. one T: $\mathcal{V}_{CY_3} \sim T^{3/2}$

and a flux-induced superpotential:

$$W_0 = \int_{CY_3} G_3 \wedge \Omega(u^a)$$

$\Rightarrow$ can fix the u^a and τ supersymmetrically:

$$D_{u^a} W = 0 = D_\tau W$$

at isolated point $(\langle u^a \rangle, \langle \tau \rangle)$ if fluxes $\int_{\Sigma^a} G_3$ chosen sufficiently generic.

Kähler moduli unfixed:

$$K^{i\bar{j}} D_i W \overline{D_j W} = 3|W_0|^2 \quad \forall \, W_0$$

cancels $-3|W_0|^2$ piece

$\to$ no-scale vacuum

$\rightsquigarrow$ <u>idea:</u> put stacks of N_i coincident D7-branes on 4-cycles $\tilde{\Sigma}^i$

$\Rightarrow$ gives 4D $\mathcal{N}=1$ super-Yang-Mills theories with gauge couplings: $\frac{1}{g_i^2} \sim T^i$

gauginos in these gauge theories condense, and give non-perturbative contribution W_{np} to W:

$$W_{np} \sim \sum_i e^{-\frac{8\pi^2}{g_i^2}} = \sum_i e^{-\frac{2\pi}{N_i} T^i}$$

now: if $W = W_0 + W_{np}$, $W_0 \neq 0$

$\Rightarrow$ $D_{T_i} W = 0$ has solutions $\langle T^i \rangle \neq 0$

$\to$ stabilizes Kähler moduli (KKLT '03) all moduli are now fixed, <u>but</u>:

$$\langle V \rangle = -3e^{K}|W|^2 < 0$$

$\rightsquigarrow$ anti-de Sitter (AdS)

but need c.c. > 0 !

$\rightsquigarrow$ introduce source of SUSY-breaking, e.g. (KKLT'03) an anti-D3-brane:

$\rightsquigarrow$ $\overline{D3}$ preserves different SUSY generator than CY_3 + O-planes + 7-branes

$\rightsquigarrow$ gives potential energy:

$$\delta V \sim \frac{1}{\alpha'^2}\int d^4x\sqrt{-g} \quad > 0$$

$\rightsquigarrow$ after converting into 4D Einstein frame:

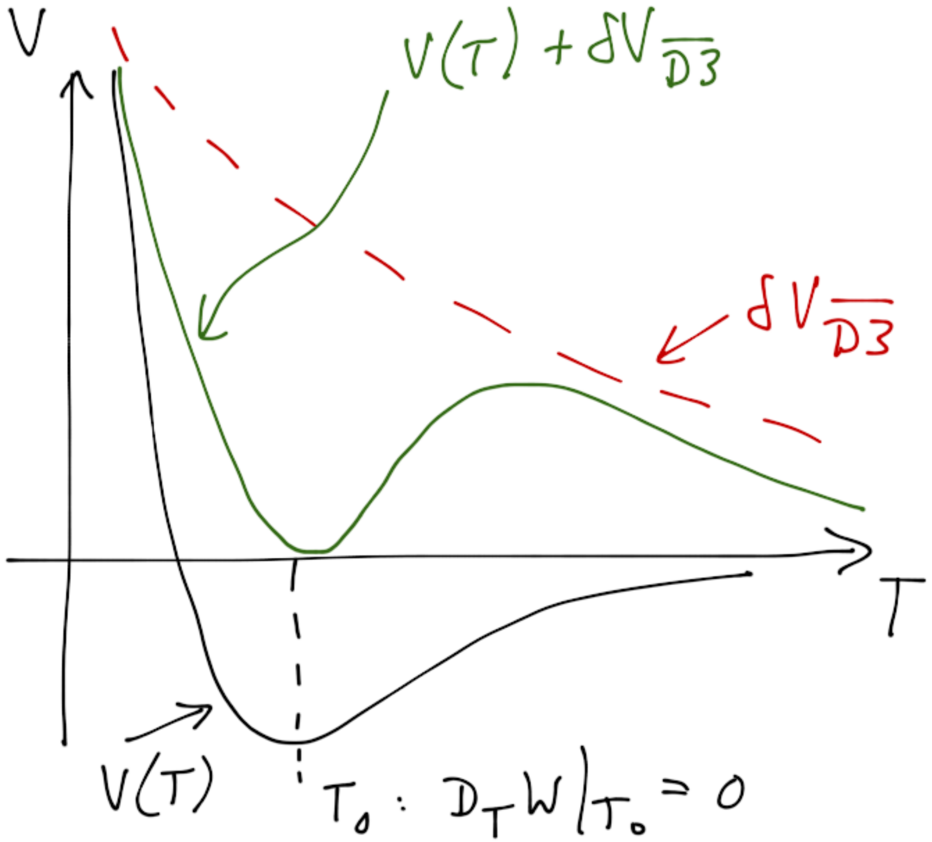
$$\frac{1}{\alpha'^2} \sim \frac{M_P^4}{\mathcal{V}_{CY_3}^2}$$

$$\Rightarrow \quad \delta V_{\overline{D3}} \sim \frac{T_3}{\mathcal{V}^2}$$

example of a single Kähler modulus T:

$$\mathcal{V} \sim T^{3/2} \Rightarrow \delta V_{\overline{D3}} \sim \frac{T_3}{T^3}$$

gives stabilized dS vacuum



$\rightsquigarrow$ now, on typical CY_3:

$$h^{2,1} = \#(\Sigma^a) = \mathcal{O}(100)$$

if $\int_{\Sigma^a} F_3, \int_{\Sigma^a} H_3 \in [-10, 10]$

on each 3-cycle Σ^a

$\Rightarrow 10^{100} \ldots 10^{1000}$ <u>different</u> flux vacua with <u>different</u> c.c. on <u>each</u> CY_3:

"string vacuum landscape"

$\rightsquigarrow$ can give anthropical explanation for our small & pos. c.c. (see Weinberg '87)