quantization: modes $\rightarrow$ creation & annihilation operators ...

$\downarrow$

massless states:

open: $\alpha^i_{-1}|0\rangle$, $i=2,\ldots,D-1$; $m^2 = \frac{D-26}{\alpha'}$

$\nwarrow$ $SO(1,D-1)$ vector: gauge field A_μ

closed: $\alpha^i_{-1}(L)\alpha^j_{-1}(R)|0\rangle$, $m^2 = \frac{D-26}{\alpha'}$

$\nwarrow$ $SO(1,D-1)$ 2-tensor:

$g_{\mu\nu} \rightarrow$ gravity

$B_{\mu\nu} \rightarrow$ antisymm. 2-form potential

$\phi \rightarrow$ dilaton scalar

$\rightsquigarrow$ $\underline{D=26}$

add supersymmetry:

$\rightarrow$ removes tachyon of $|0\rangle$

$\rightarrow$ 16 X^μ become fermions $\Rightarrow$ $\boxed{D=10}$

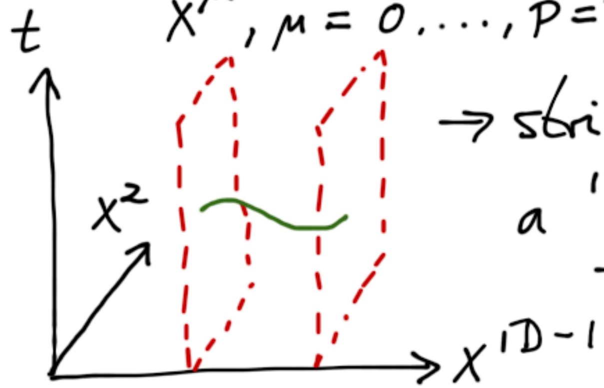
<u>crucial feature:</u> not only strings ...

$\rightsquigarrow$ turn X^{D-1} into S^1 with radius R

$\rightsquigarrow$ now shrink $R \rightarrow 0$

$\rightsquigarrow$ there is a T-dual string theory with dual coordinate X'^{D-1} on dual S^1 with radius $1/R$ — <u>and</u> strings are <u>fixed at point</u> on dual S^1:

$\rightarrow$ can only move on hyperplane X^μ, $\mu = 0,\ldots,p=D-2$



$\rightarrow$ strings <u>end</u> on a <u>'Dp-brane'</u>!

our world is 3+1 dimensional down to lengths $\sim \mathrm{TeV}^{-1} \sim 10^{-19}\,m$

→ however, $l_s = \sqrt{\alpha'} \sim L_p \sim 10^{-35}\,m$

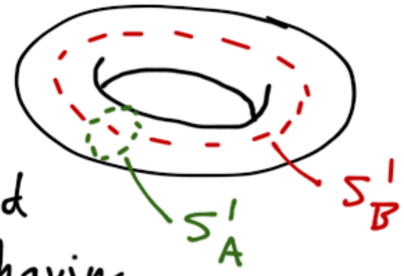
→ compactify 6 of 9 spatial dimensions

→ if you want 4d $\mathcal{N}=1$ SUSY: compact 6D-space is Calabi-Yau

CY_3: 6D space with specific form of metric & pairing of the 6 X^i into 3 complex coord. z^a such that Ricci curvature $R_{ab} = 0$ ('Ricci-flat')

produces problem: CY_3 have sub spaces, like cheese holes...

think of 2-torus:



→ has 2 non-shrinkable S^1-s "cheese-holes" related to its topology of having a 'handle' ...

↝ CY_3 has non-trivial 2- and 3-spheres within ↔ topology of CY_3:

↝ size of each S^2 and S^3 a massless scalar field in 4D:

→ 'moduli': $\# = \mathcal{O}(100)$

S^2 sizes: volume deformations of the CY_3	S^3 sizes: shape deformations → how to group X^i into complex z^a...

→ desaster for inflation:
want <u>one</u> controlled scalar field
with: mass $< H$
<u>but</u> not $\mathcal{O}(100)$ massless!

↓

<u>a):</u> if we could wean ourselves from 4D $\mathcal{N}=1$ SUSY … many more 6D spaces than CY_3's, with <u>few</u> or <u>no</u> moduli …

<u>b):</u> if we insist on 4D $\mathcal{N}=1$ …
→ need to stabilize moduli first

↓

look at d.o.f. of 10D effective action from string theory …

string theory best understood for this purpose:

type IIB: – gravity with $\mathcal{N}=2$ in 10D on closed strings
– matter & gauge fields from open strings ending on D3- or D7-branes, $\mathcal{N}=1$ in 10D

↓

effective action: 10D IIB supergravity

$$S \sim \int d^{10}x\sqrt{-g}\left\{e^{-2\phi}\left[R+(\partial\phi)^2\right]-|F_1|^2-|G_3|^2 - |F_5|^2\right\} + \dots$$

$$\left\{\begin{array}{l} F_p = dC_{p-1} \\ G_3 = F_3 - (C_0 + ie^{-\phi})\cdot \underbrace{H_3}_{= dB_2} \end{array}\right.$$

'p-form gauge field strengths'

flux of field strengths F_p can be quantized:

→ compactify on $\mathcal{M}_4 \times C_6$ ← compact 6D space

→ if C_6 contains non-trivial p-sphere S^p, $0 < p \leq 6$

$$\Rightarrow \int_{S^p} F_p = N_p \in \mathbb{Z}$$

~ analog to quantized magnetic flux $\int_{S^2} B_2$ for a Dirac monopole

now say, S^p has radius R:

→ then: $\int_{S^p} F_p = N_p \in \mathbb{Z} \Rightarrow F_p \sim \frac{N_p}{R^p}$

and S^p has curvature:

$$^{(p)}R = \frac{p(p-1)}{R^2}$$

thus an action:

$$S \sim \int d^{10}x \sqrt{-g}\left[R - |F_p|^2\right]$$

$$\supset \int d^4x \int_0^R dr\, r^5 \cdot \left[{}^{(p)}R - |F_p|^2\right]$$

$$\sim \int d^4x \cdot \underbrace{\left[p(p-1)R^4 - \frac{N_p^2}{R^{2p-6}}\right]}_{\text{scalar potential } V(R)}$$

$$\frac{\partial V}{\partial R} = 0 \quad \Rightarrow \quad \boxed{\langle R\rangle^{p-1} \sim N_p}$$

→ the size modulus R of the p-sphere was <u>stabilized</u> by <u>flux</u> & <u>curvature</u> and is now heavy!

--→ similar arguments (technically more involved) show that:

$\rightsquigarrow$ quantized G_3-flux in IIB stabilizes <u>all</u> the S^3-s of a CY_3 compactification to 4D $\mathcal{N}=1$

$\rightsquigarrow$ "<u>moduli stabilization by flux compactification</u>"

size of the S^2-s of a CY_3?

$\rightsquigarrow$ use non-perturbative effects of the gauge theories on the D7-branes of IIB:

→ D7-brane fills 3+1 and 'wraps' around an "$S^2 \times S^2$"

$$\Rightarrow \left.\frac{1}{g_{YM}^2}\right|_{D7} \sim (\text{size of } S^2)^2$$

→ nonperturbative effects in g_{YM}:

$$\Rightarrow V \supset e^{-\left.\frac{1}{g_{YM}^2}\right|_{D7}}$$

depends on size of S^2

$\Rightarrow$ $\boxed{\text{Can stabilize } \underline{\text{all}} \ CY_3\text{-moduli in IIB.}}$

$\rightsquigarrow$ now, on typical CY_3:

$$h^{2,1} = \#(\Sigma^a) = \mathcal{O}(100)$$

if $\int_{\Sigma^a} F_3, \int_{\Sigma^a} H_3 \in [-10, 10]$

on each 3-cycle Σ^a

$\Rightarrow$ $10^{100} \ldots 10^{1000}$ <u>different</u> flux vacua with <u>different</u> c.c. on <u>each</u> CY_3:

"string vacuum landscape"

$\rightsquigarrow$ can give anthropical explanation for our small & pos. c.c. (see Weinberg '87, Linde '80s):

$\Rightarrow$ look at the $\mathcal{O}(10^{h^{2,1}})$ SUSY flux vacua if $h^{2,1}$ 3-cycles with flux are present

$\Rightarrow$ 4D $\mathcal{N}=1$ effective supergravity determined by <u>flux superpotential</u>

$$W_0 = \int_{CY_3} G_3 \wedge \Omega$$

$$\int_{\Sigma^a} G_3 \supset \begin{cases} \int_{\Sigma^a} F_3 \\ \int_{\Sigma^a} H_3 \end{cases}$$ 3-form flux on 3-cycle Σ^a

Ω : holomorphic 3-form characteristic for every CY_3

$W_0 = W_0(\Sigma^a\text{-radii})$ depends on 3-cycle moduli $z^a(\Sigma^a)$

$\Rightarrow$ SUSY flux vacua satisfy: $D_{z^a} W_0 = 0$

in effective 4D $\mathcal{N}=1$ supergravity.

$\rightsquigarrow$ 7-brane super-YM gauge theories contribute an exponential term to W. This allows to stabilize the S^2-volume moduli T^i supersymmetrically as well:

$$D_{T^i} W = 0$$

$$W = W_0(\text{flux}) + W_{7\text{-branes}}$$

~ full SUSY flux vacua have:

$$D_{z^a} W = D_{T^i} W = 0$$

but $\langle W \rangle \neq 0$

$\Rightarrow$ they are AdS vacua, because in 4D $\mathcal{N}=1$ supergravity:

$$\langle V \rangle = -3 \cdot \langle e^K |W|^2 \rangle$$

as all $D_{\phi^\alpha} W = 0$

~ add SUSY-breaking effect:

$\Rightarrow$ total vacuum energy is

$$V = \langle V \rangle + \Delta V_{\cancel{SUSY}}$$

positive for some fraction of all flux vacua, negative for the rest

$\Rightarrow$ vacuum energy distributes randomly between $\mathcal{O}(-M_p^4)$ and $\mathcal{O}(+M_p^4)$

$\Rightarrow$ with $\mathcal{O}(10^{h^{2,1}})$ flux vacua, smallest expectable positive vacuum energy is:

$$\mathcal{O}(+10^{-h^{2,1}} \cdot M_P^4)$$

$\Rightarrow$ for typically CY_3:

$$h^{2,1} = \mathcal{O}(100)$$

explains our small c.c. 'environmentally/anthropically':

$\rightsquigarrow$ we live in the very small fraction of vacua with $\mathcal{O}(10^{-h^{2,1}} M_P^4) \lesssim 10^{-120} M_P^4$ for $h^{2,1} \gtrsim 120$

simply, because for an $\mathcal{O}(10)$ times larger c.c. large-scale structure formation already fails (Weinberg '87) ...

$\rightsquigarrow$ eternal inflation & tunneling can cosmologically realize <u>all</u> these flux vacua!

↝ eternally inflating landscape multiverse with each of the $\mathcal{O}(10^{h^{2,1}})$ flux vacua realized infinitely often throughout space-time !

↝ many questions:

... most deep/pressing – how to count/weight the infinite realizations for each vacuum ('measure problem') ?