How do these peaks appear today?

$\rightsquigarrow$ project to sphere in sky:

comoving distance D since decoupling:

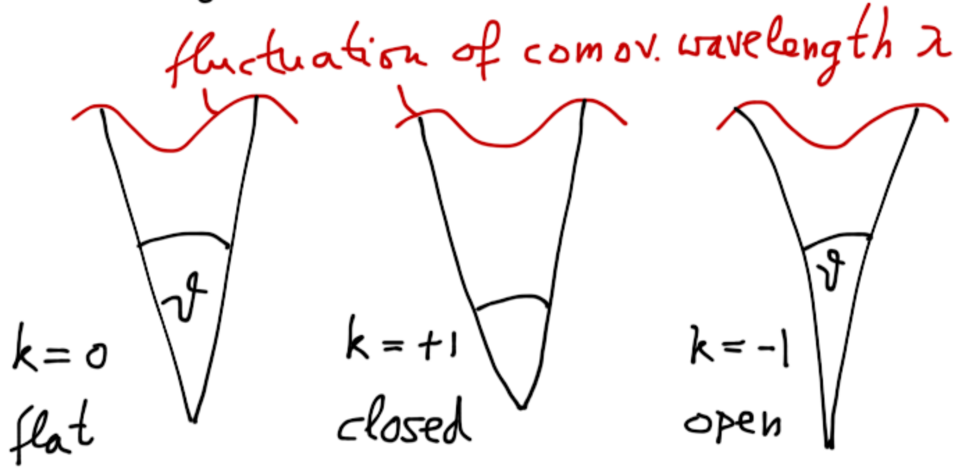
$$D_* = \eta_0 - \eta_* \simeq \eta_0$$

$\Rightarrow$ temperature fluctuation from sound wave with comoving wavelength λ_n of the n^{th} peak appears under angle:

$$\theta_n = \frac{\lambda_n}{D_*} = \frac{2}{n\sqrt{3}} \cdot \frac{\eta_*}{\eta_0}$$

in a spatially flat universe.

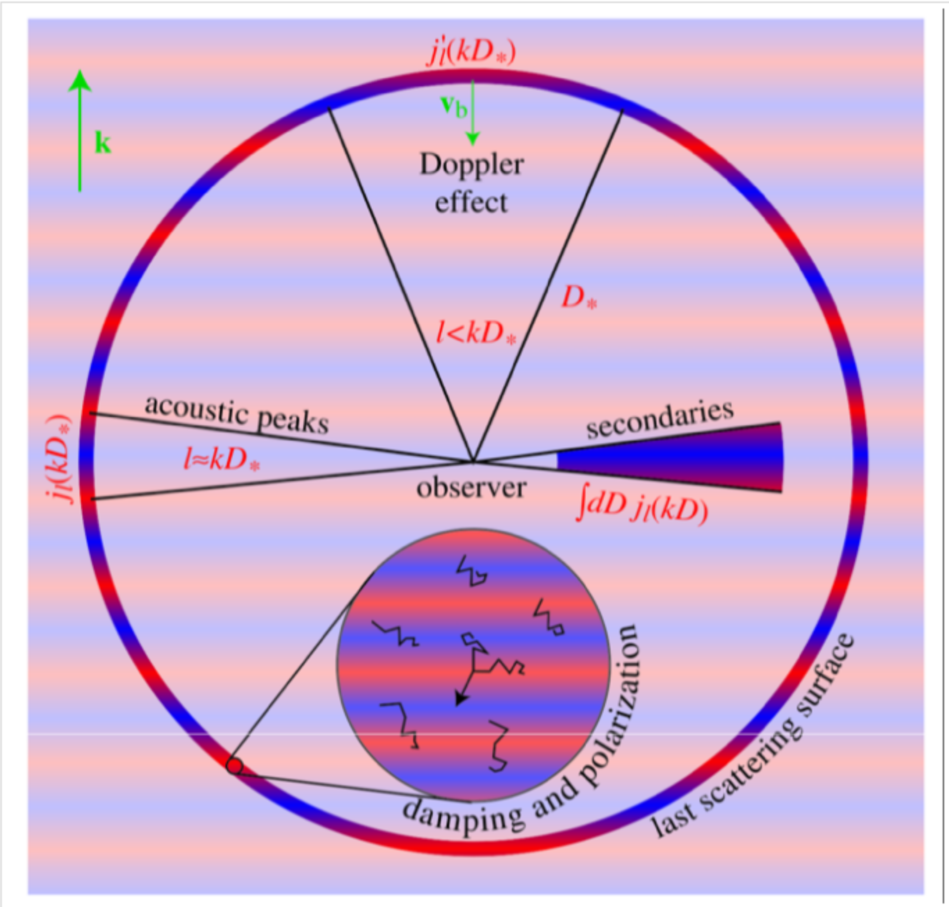
More generally:



$$\Rightarrow \quad \vartheta_{k=-1} < \vartheta = \vartheta_{flat} < \vartheta_{k=+1}$$

↳ measuring e.g. ϑ_1 accurately in the CMB can determine spatial geometry of the visible universe!

but : $\vartheta_1 : \vartheta_2 : \vartheta_3 : \ldots$ peak position ratios do <u>not</u> depend on spatial geometry.



$\Rightarrow$ first maximum of anisotropies in the CMB appears at angular size:

$$\theta_1 = \frac{2}{\sqrt{3}} \frac{\eta_*}{\eta_0} \simeq \frac{1}{30} \hat{=} 2^\circ$$

converting this into multipole moments l of a spherical harmonics decomposition, we get the acoustic peaks in the spherical harmonics decomposition of Δ_θ^2 into C_l's as:

$$l_n = \frac{2\pi}{\theta_n} = n \cdot \pi\sqrt{3} \cdot \frac{\eta_0}{\eta_*}$$

$\rightsquigarrow$ so the first peak is at:

$$\ell_1 \simeq 200$$

$\rightarrow$ dramatic confirmation: COBE, WMAP, ACT, CPT (PLANCK coming) now see 9 peaks!

and that they see:

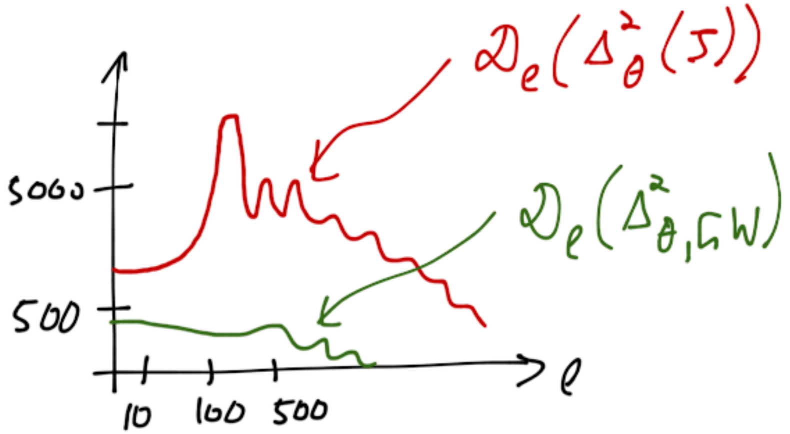
$$\ell_1 : \ell_2 : \ell_3 : \ldots = 1 : 2 : 3 : \ldots$$

is very good evidence of the coheent initial phase $\varphi(k) = 0$ for all k, and thus for super-horizon initial perturbations

$\rightarrow$ strong evidence for inflation!

- precise positions ℓ_n of peaks and their relative height depends on the various effects that baryons, dark matter, a C.C., or spatial curvature have on the sound propagation in the plasma, and the CMB photon propagation after decoupling $\rightarrow$ can measure many cosmological parameters with CMB if I have many peaks!

- tensor perturbations during inflation also source temperature fluctuations in the CMB (gravity waves also red & blue shift the photons):



this way, WMAP, ACT, SPT constrained $r \lesssim 0.11$

- PLANCK, and south pole experiments like BICEP2 or Keck array, look for the B-mode polarization in the CMB, caused by gravity waves alone

$\to$ sensitivity:

$$r \gtrsim 0.01$$

<u>strategy & argument of measuring r:</u>

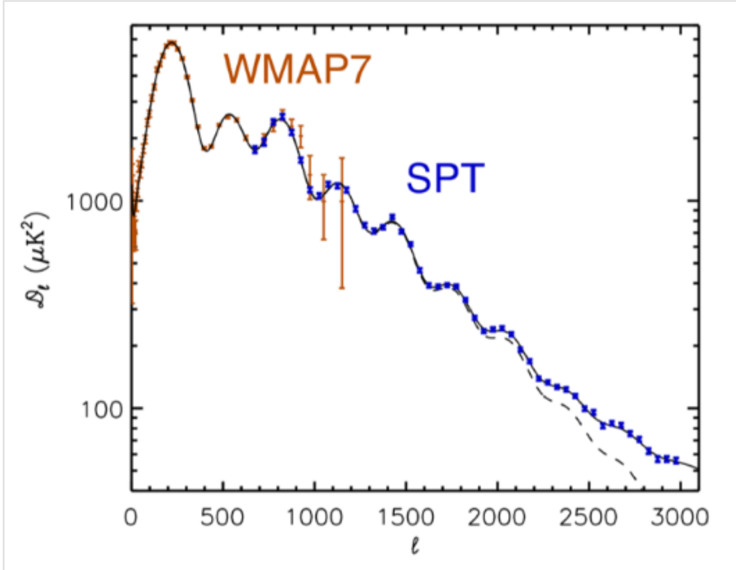
- measuring $\mathcal{D}_\ell\left(\frac{\Delta T}{T}\right) \sim \Delta^2_\zeta$ has determined $\Delta^2_\zeta \sim \frac{H^2}{\epsilon} \sim 10^{-10}$

- measuring $r = 16\epsilon$ would determine ϵ, 5yr reach $r \lesssim 0.01$ upper bound (95%)

$\Rightarrow$ scale of inflation:

$$V^{1/4} \sim H^{1/2} \sim \left(\Delta_s^2 \cdot \epsilon\right)^{1/4} \sim \left(10^{-2} \Delta_s^2 \cdot \frac{r}{0.1}\right)^{1/4} \sim 10^{-3} \cdot \left(\frac{r}{0.1}\right)^{1/4}$$

$\Rightarrow$ detection of $r \gtrsim 0.01$ implies <u>GUT-scale</u> inflation!



WMAP 7-yr data & SPT 2012

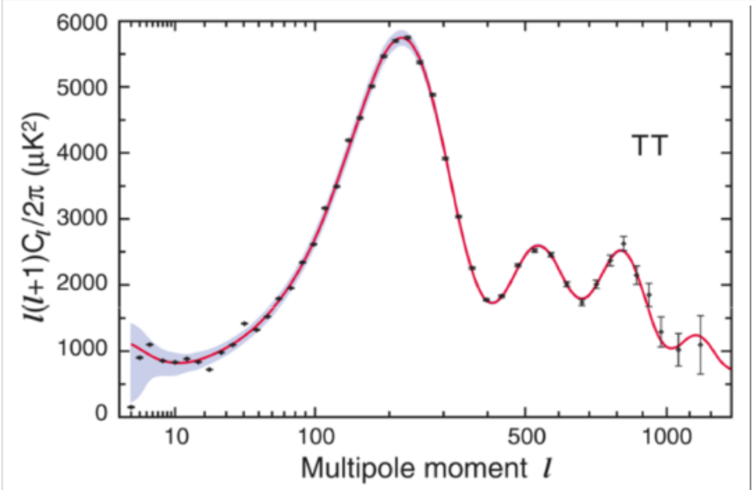
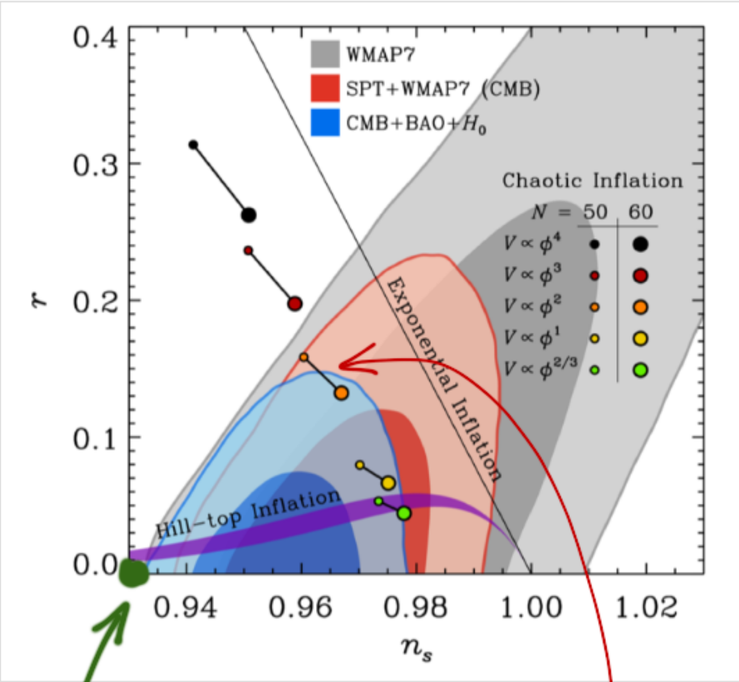


Fig. 32.— The nine-year *WMAP* TT angular power spectrum. The *WMAP* data are in black, with error bars, the best fit model is the red curve, and the smoothed binned cosmic variance curve is the shaded region. The first three acoustic peaks are well-determined.

WMAP 9-yr data (2012)



WMAP 7-yr data & SPT 2012

$V = V_0\left(1 - \frac{\lambda_3}{3}\phi^3\right)$

$V = \frac{1}{2} m^2 \phi^2$

VII Beyond the horizon - eternal inflation

Slow-roll inflation of a single scalar field is governed by a 2-fold dynamics:

- classical slow-roll:
$$3H \cdot \dot{\phi} = -\frac{\partial V}{\partial \phi}$$
in a flat potential ($\epsilon, \eta \ll 1$)

- quantum fluctuations of mean variance:
$$\delta\phi = \frac{H}{2\pi}$$
producing classical waves:
$$\Delta\phi_{q.} = \delta\phi \cdot \cos\left(\vec{k}\cdot\vec{r} - k\eta\right)$$
of average amplitude $\delta\phi$, which at all comoving momenta k freeze once their physical wavelength stretches to horizon scale at:
$$k = aH$$

Views in snapshots of a movie:

$\rightsquigarrow$ time steps $\Delta t = H^{-1}$ per snapshot

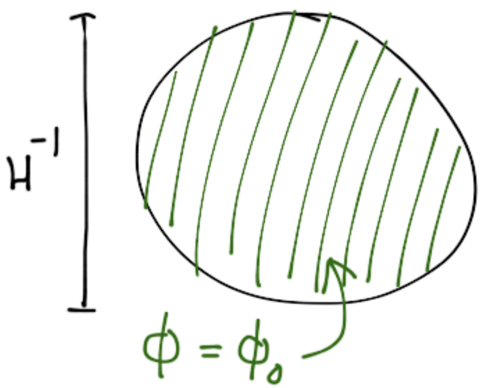
$\rightsquigarrow$ at each timestep a wave profile:
$$\Delta\phi_q\Big|_{k=aH} = \delta\phi \cdot \cos\left(\frac{2\pi}{H^{-1}} \cdot r_{ph.}\right)$$
of a quantum fluctuation turned

classical at $\lambda_{ph.} \sim H^{-1}$ is additively superimposed as a classical field variation on the classical background which has rolled by:

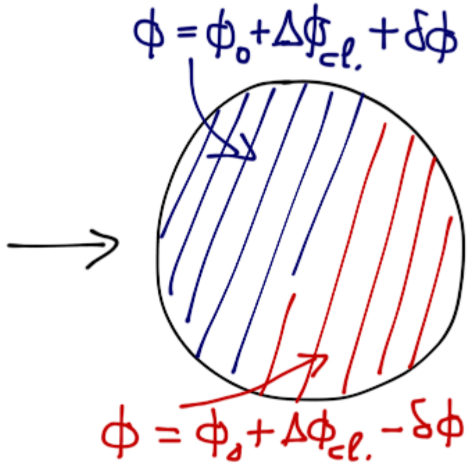
$$\Delta\phi_{cl.} = -\frac{V'}{3H}\cdot\Delta t = -\frac{V'}{3H^2}$$

$\Downarrow$

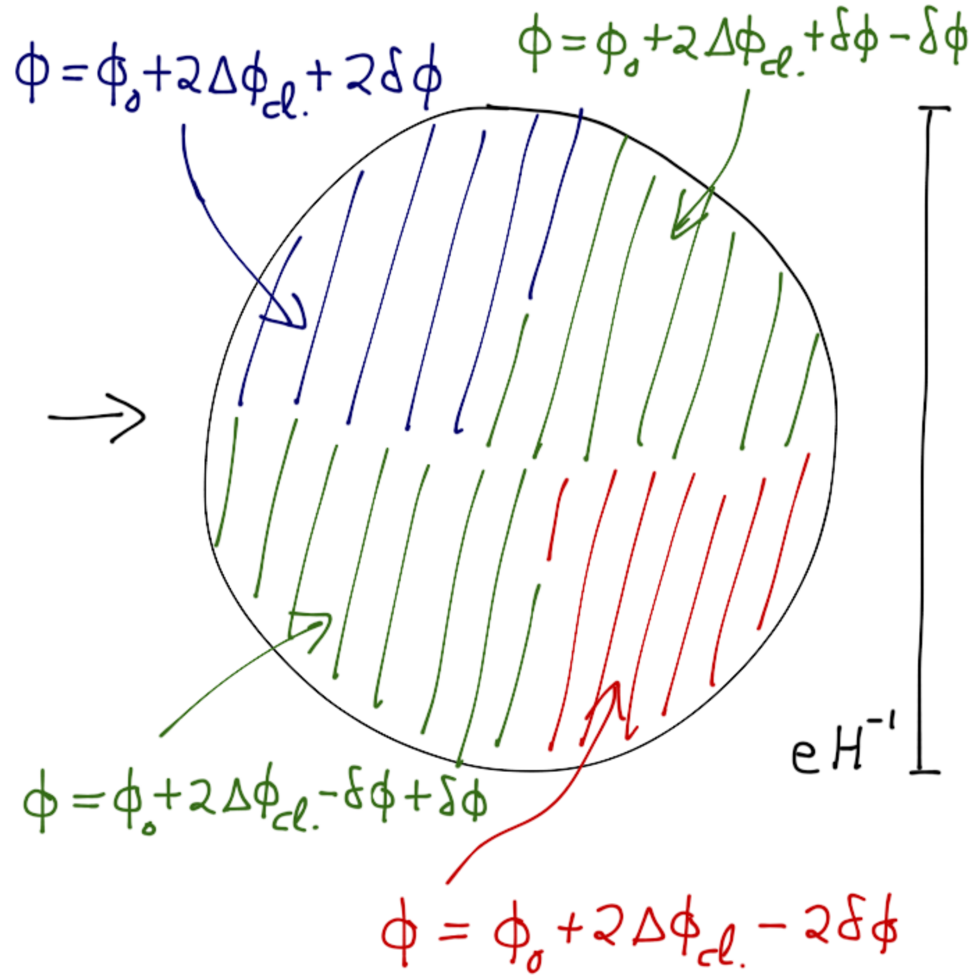
$t = 0:$



$t = \Delta t = H^{-1}:$



$t = 2\cdot\Delta t = 2\cdot H^{-1}:$



and so on ...

At each time step $\Delta t = H^{-1}$ each H^{-1}-size horizon volume expands into $e^3 \simeq 20$ H^{-1}-sized horizon volumes, in half of which the quantum fluctuations cause ϕ to jump upwards on average by $+\delta\phi$, and downwards on average by $-\delta\phi$ in the other half.

$\Rightarrow$ If $\delta\phi \gtrsim |\Delta\phi_{cl.}|$, there will be regions, where ϕ effectively grows, over-coming on average the classical slow-roll down-hill drift, and thus grows <u>without bound!</u>

$\Rightarrow$ Inflation globally never ends, while locally in any given region it will quantum jump to a value ϕ with:

$$\delta\phi < |\Delta\phi_{cl.}|$$

and thus eventually end.

But globally, there will be always regions where ϕ grows and inflation globally never ends:

<u>Eternal inflation!</u>

<u>Criterion for eternal inflation:</u>

$$\boxed{\delta\phi = \frac{H}{2\pi} > |\Delta\phi_{cl.}| = \left| -\frac{V'}{3H^2} \right| \iff \epsilon < \frac{V}{24\pi^2}}$$

<u>example i):</u> $V(\phi) = \frac{1}{2} m^2 \phi^2$

slow-roll inflation ($\epsilon, \eta < 1$) for $\phi > \sqrt{2}$

check condition above:

$$\delta\phi = \frac{H}{2\pi} = \frac{m\phi}{2\sqrt{6}\,\pi} > \left| -\frac{V'}{3H^2} \right| = \frac{2}{\phi}$$

$\Rightarrow$ for $\phi > \phi_* = \frac{2\sqrt{\pi}\,6^{1/4}}{\sqrt{m}}$

eternal inflation ensues.

we know, that $\frac{\Delta T}{T} \sim 10^{-5}$

$$\Rightarrow \quad \Delta_\zeta^2 \Big|_{N_e \simeq 60} \sim \left(\frac{\Delta T}{T}\right)^2 \sim 10^{-10}$$

and $\phi_{N_e \simeq 60} \simeq 15$

$$\Rightarrow \quad m \sim 10^{-5}$$

$$\Rightarrow \quad \phi_* \simeq 500 \gg \phi_{N_e \simeq 60}$$

but : $V(\phi_*) \sim m \ll 1$

still much below Planckian density, so eternal inflation well controlled low-energy effective QFT regime!

<u>example ii)</u> : $V(\phi) = \frac{\lambda}{4}\left(\phi^2 - v^2\right)^2$

'symmetric double well'

$\curvearrowright$ take inflation to start close to the hill-top : $|\phi| \ll v$

$$\Rightarrow \quad \Delta\phi_{cl.} = \left| -\frac{V'}{3H^2} \right| = \left| -\frac{V'}{V} \right| \simeq \frac{\lambda v^2 |\phi|}{\frac{\lambda}{4} v^4} = 4 \cdot \frac{|\phi|}{v^2}$$

$$\delta\phi = \frac{H}{2\pi} \simeq \frac{\sqrt{\lambda}}{4\pi\sqrt{3}} v^2$$

ϕ_* : $\delta\phi > \Delta\phi_{cl.}$

$\Rightarrow$ eternal inflation for

$$\phi < \phi_* \simeq \frac{\sqrt{\lambda}}{16\pi\sqrt{3}} v^4$$

is automatic at $\phi = 0$

however, cannot start at $\phi \lesssim \delta\phi$ as $\delta\phi$ is the size of quantum fluctuations of ϕ :

$\Rightarrow$ $\phi_* > \delta\phi$ for eternal inflation needed as well on a hill-top :

$$\Rightarrow \quad v > 2 \quad \Leftrightarrow \quad \eta\big|_{\phi=0} < 1$$

$\Rightarrow$ double wells with trans-Planckian width support eternal inflation close to the top of the hill, and thus at core $\phi \simeq 0$ of domain wall field profiles interpolating between $\phi = -v$ and $\phi = v$, which are topological defects

$\rightsquigarrow$ eternally inflating topological defects

"topological eternal inflation"

(Linde / Vilenkin '94)

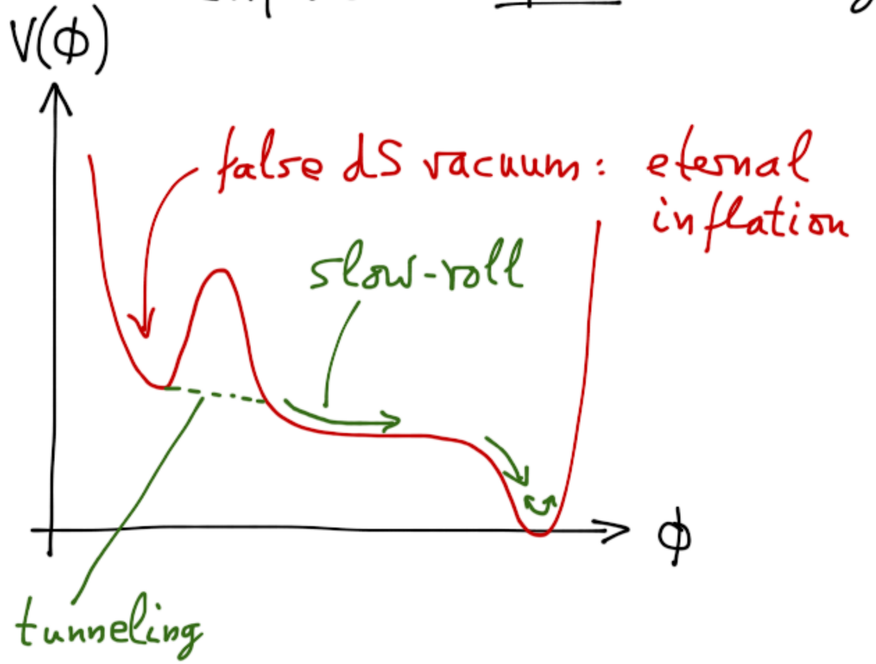
example iii):

- local minima of $V(\phi)$ at $V(\phi_{min.}) > 0$
- exit via tunneling

$\rightsquigarrow$ not viable for observable last 60 e-folds, but false-vacuum eternal inflation

↷ potential for viable false-vacuum eternal inflation:

- false minimum at $V > 0$ followed by slow-roll inflation <u>after</u> tunneling



final project:

can study e.g. $V(\phi) = \frac{1}{2} m^2 \phi^2$ numerically:

↷ recipe:

- evolve in time-steps $\Delta t = H^{-1}$
- at each step add classical drift:

$$\Delta \phi_{cl.} = \sqrt{2\epsilon}$$

- and superimpose average wavelength & amplitude plane-wave field:

$$\Delta\phi_q = \delta\phi \cdot \cos(k_x \cdot x) \cdot \cos(k_y \cdot y)$$

in 2D, where:

$$k = \frac{2\pi}{H^{-1}} \cdot \frac{1}{n}$$

for the n^{th} time-step, to project the expanding (x,y)-space into a finite-size box.

$\sim$ plot $\phi(x,y)$ at each time-step $\to$ movie...

VIII UV completion – inflation in string theory

basics of string theory:

↳ 1-dimensional extended objects

↳ open: or closed:

ℓ_s : string length

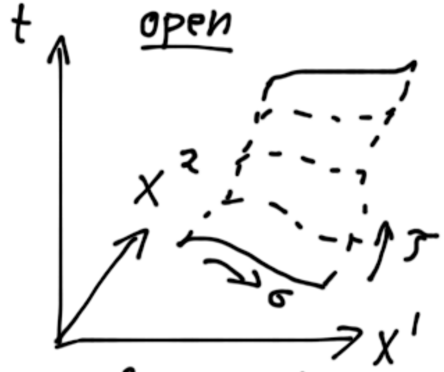
with tension $\sim \frac{1}{\alpha'}$, $\alpha' = \ell_s^2$

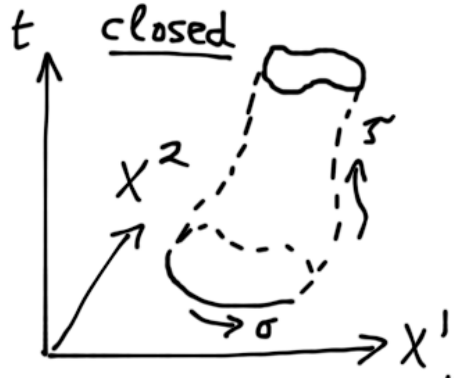
so they vibrate ...

↳ action:

$$S = \frac{1}{\alpha'} \int d^2\sigma \sqrt{-\gamma} = \text{" area of world sheet "}$$

induced metric on world sheet





$d^2\sigma = d\tau\, d\sigma$

X^μ : embedding "target" space-time

equivalent action:

$$S = \frac{1}{\alpha'} \int d^2\sigma \sqrt{-\gamma}\, \gamma^{\alpha\beta} g^{\mu\nu} \partial_\alpha X_\mu \partial_\beta X_\nu$$

$$\text{with} \begin{cases} \alpha, \beta = \tau, \sigma \\ \mu, \nu = 0, \dots, D-1 \end{cases}$$

solutions: standing wave spectrum

→ closed: independent left- & right-moving modes

→ closed: left- & right movers tied by boundary conditions