n-point correlation functions:

$$\langle \delta T(\vec{n}_1)\,\delta T(\vec{n}_2)\rangle = D(\vec{n}_1,\vec{n}_2) = f(\vec{n}_1\cdot\vec{n}_2)$$

$$\langle \delta T(\vec{n}_1)\,\delta T(\vec{n}_2)\,\delta T(\vec{n}_3)\rangle = 0$$

$$\langle \delta T(\vec{n}_1)\,\delta T(\vec{n}_2)\,\delta T(\vec{n}_3)\,\delta T(\vec{n}_4)\rangle =$$

$$= D(\vec{n}_1,\vec{n}_2)\,D(\vec{n}_3,\vec{n}_4) + D(\vec{n}_1,\vec{n}_3)\,D(\vec{n}_2,\vec{n}_4)$$

$$+ D(\vec{n}_2\,\vec{n}_3)\,D(\vec{n}_1,\vec{n}_4)$$

$$\vdots$$

$*$: all independent real & imaginary parts

then obvious:

$$\langle a_{\ell m}\, a^*_{\ell' m'}\rangle =$$

$$= \int da_{\ell m}\, da^*_{\ell' m'}\, a_{\ell m}\, a^*_{\ell' m'}\, P(a_{\ell m})\, P^*(a^*_{\ell' m'})$$

$$= \delta_{\ell\ell'}\,\delta_{mm'}\,C_\ell$$

$$\curvearrowright \langle \delta T(\vec{n}_1)\,\delta T(\vec{n}_2)\rangle = T_0^2 \sum_\ell C_\ell \underbrace{\sum_{m=-\ell}^{\ell} Y_{\ell m}(\vec{n}_1)\,Y^*_{\ell m}(\vec{n}_2)}_{\frac{2\ell+1}{4\pi} P_\ell(\vec{n}_1\cdot\vec{n}_2)}$$

$$= f(\vec{n}_1\cdot\vec{n}_2)$$

as expected based on rotational invariance when averaged over ensemble of universes ... more general than 'Gaussian random variables'

C_ℓ is determined from sum over $2\ell+1$ independent random variables:

$$C_\ell = \frac{1}{2\ell+1} \sum_{m=-\ell}^{\ell} \langle |a_{\ell m}|^2 \rangle$$

$\rightsquigarrow$ statistical uncertainty ($\chi^2_{2\ell+1}$-distribution):

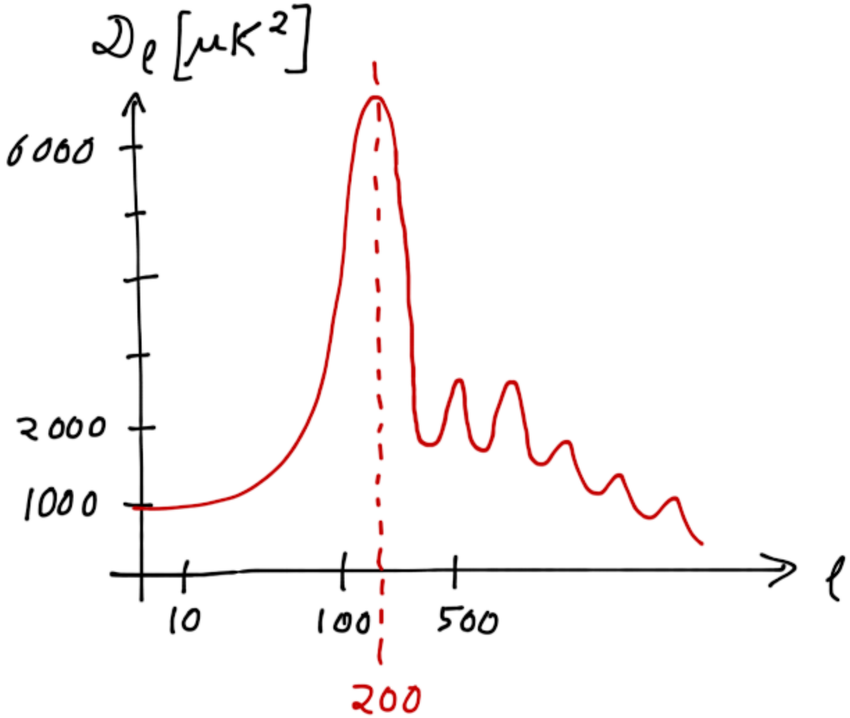
$$\frac{\delta C_\ell^2}{C_\ell^2} = \frac{2}{2\ell+1}$$

irreducible statistical uncertainty, large at low ℓ, called 'cosmic variance'

$$\rightsquigarrow \langle \delta T(\vec{n})^2 \rangle \simeq T_0^2 \cdot \int d\ln\ell \cdot \frac{\ell(\ell+1)}{2\pi} C_\ell$$

$$\equiv \int d\ln\ell \cdot \mathcal{D}_\ell$$

$$\Rightarrow \mathcal{D}_\ell = T_0^2 \cdot \frac{\ell(\ell+1)}{2\pi} C_\ell$$



$$\rightsquigarrow \quad \frac{\delta T}{T} \sim 3\cdot 10^{-5}$$

$\delta T/T$ from the curvature perturbation

ζ

$\rightsquigarrow$ recall : $\zeta = \delta N_e$

during inflation: $a = e^{N_e}$

$$\Rightarrow \delta N_e = \delta \ln a = \frac{\delta a}{a} = -\frac{\delta T}{T}$$

because : $T \sim \frac{1}{a}$

$$\Rightarrow \quad \zeta = -\frac{\delta T}{T} \equiv \Theta \text{ initial}$$

temperature perturbation at the end of inflation $\simeq$ reheating at η_{rh}.

thereafter: radiation/matter domination

$$\Rightarrow a \sim t^p \quad \Rightarrow \quad \Theta^{(0)}_\eta = -\mathcal{O}(1)\cdot\zeta$$

initial temperature perturbation at conformal time $\eta > \eta_{rh}$ after the end of inflation.

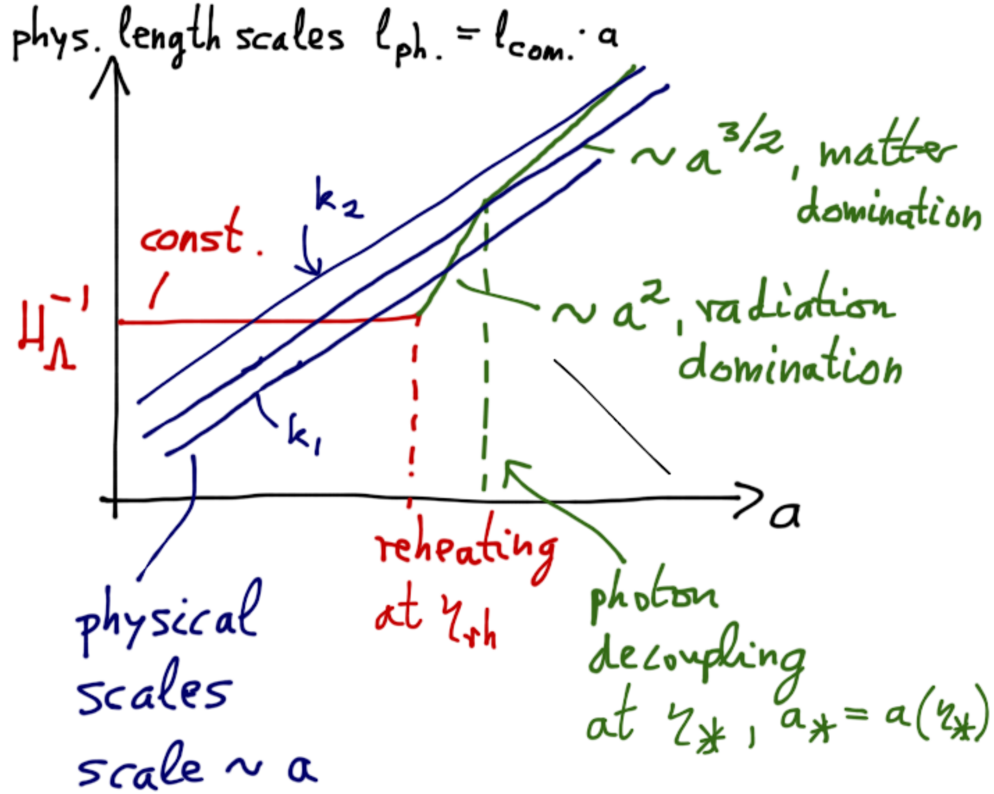
perturbation $\theta_k^{(0)}$ of comoving wave-length $\lambda = \frac{2\pi}{k}$ causes sound wave temperature fluctuation in the plasma after reheating at η_{rh}:

$$\theta_k(\eta) = \theta_k^{(0)} \cdot \cos(k \cdot s) = -\mathcal{O}(1) \cdot \zeta_k \cdot \cos(k \cdot s)$$

$s = \int c_s \, d\eta \simeq \eta/\sqrt{3}$ $\quad c_s = \frac{1}{\sqrt{3}}$: speed of sound in plasma

for evolution to $\eta \gg \eta_{rh}$

note that ζ_k was frozen-in until horizon re-entry



k_1 : horizon re-entry $\eta_{H^{-1}} < \eta_*$ before decoupling

$\rightarrow \theta_k^{(0)}$ causes sound wave with comoving wavelength

$\lambda_1 = \frac{2\pi}{k_1}$ and frequency

$\omega_1 = c_s k_1 = \frac{k_1}{\sqrt{3}}$ $\rightarrow$ oscillates until recombination to phase

$k_1 \cdot s_* \simeq \omega_1 \cdot \eta_* \gtrsim 1$

k_2 : horizon re-entry $\eta_{H^{-1}} \gg \eta_*$

$\rightarrow \theta_k^{(0)}$ causes sound wave with comoving wavelength

$\lambda_2 = \frac{2\pi}{k_2}$ and frequency

$\omega_2 = c_s k_2 = \frac{k_2}{\sqrt{3}}$ $\rightarrow$ oscillates until recombination to phase

$k_2 \cdot s_* \simeq \omega_2 \cdot \eta_* \ll 1$

$\Rightarrow$ sound wave stays at $\theta_k^{(0)}$, large-scale temperature fluctuations measure directly the primordial initial temperature and curvature perturbation, since

$\theta_k(\eta_*) = \theta_k^{(0)} \sim -\zeta_k$

for $k \cdot s_* \simeq \omega \cdot \eta_* \ll 1$.

$\Rightarrow$ time evolution of ζ_k perturbation starts with ζ_k given, and $\frac{\partial \zeta_k}{\partial \eta} = 0$ everywhere, as $\cos(k \int c_s d\eta)$ with zero phase everywhere, for all k!

$\rightsquigarrow$ inflationary perturbations define coherent initial phase conditions for sound waves in plasma produced!

$\rightsquigarrow$ Then, for each k the generated sound waves $\theta_k(\eta)$ evolve until decoupling at $\eta = \eta_*$:

- define $\eta_0 = 1$, then during matter domination:

$$\eta_* = \frac{\eta_*}{\eta_0} = \sqrt{\frac{a_*}{a_0}} = \frac{1}{\sqrt{1+z_*}} \simeq \frac{1}{30}$$

- for each $k = \frac{2\pi}{\lambda}$:

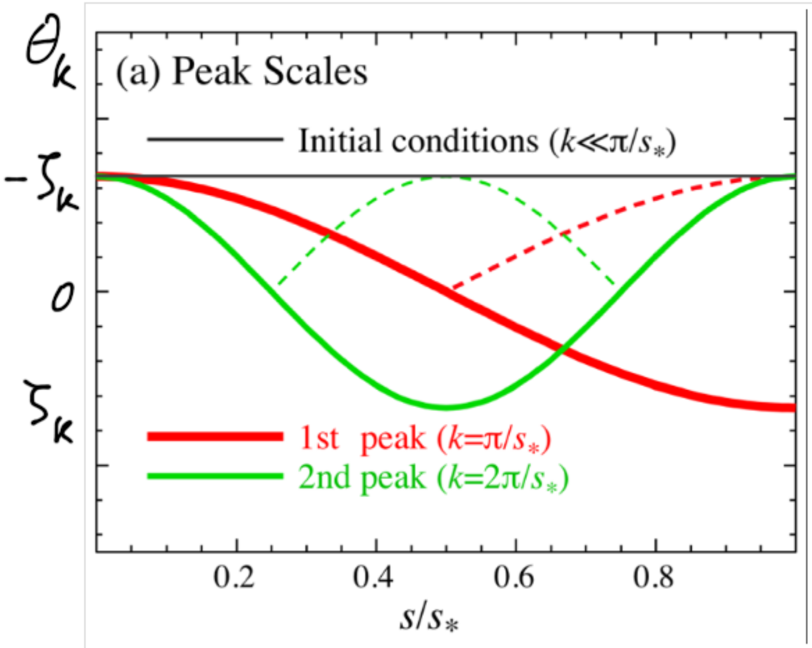
$$\theta_k(\eta_*) = -\zeta_k \cdot \cos(k \cdot s_*)$$

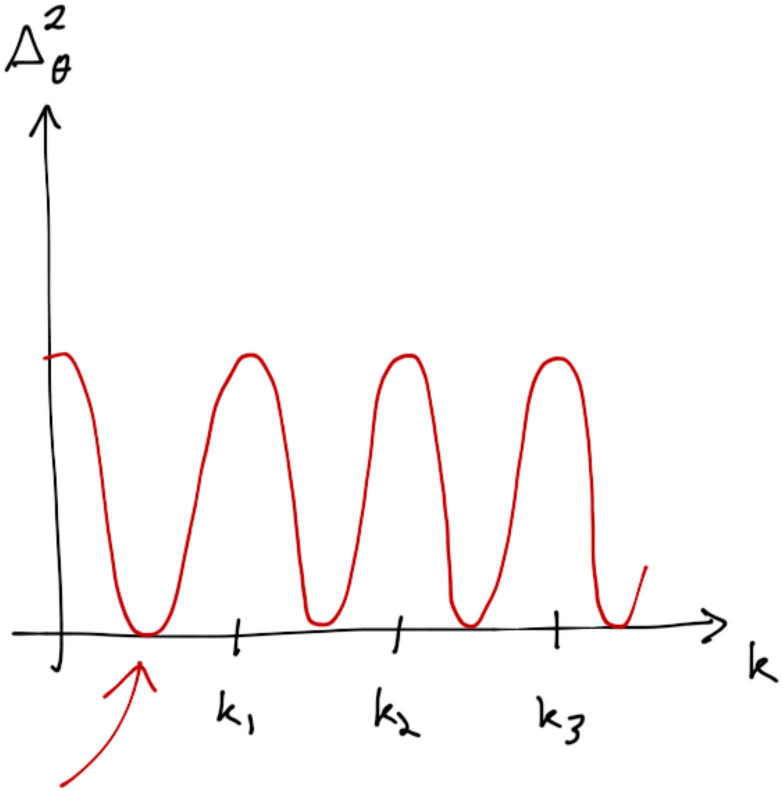
thus for all:

$$k_n = \frac{n \cdot \pi}{s_*} = \frac{n\pi\sqrt{3}}{\eta_*} \Leftrightarrow \lambda_n = \frac{2 s_*}{n} = \frac{2\eta_*}{n\sqrt{3}}$$

$\theta_{k_n}(\eta_*)$ is either at a maximum or minimum, contributing a maximum in the 2-point function power spectrum Δ^2_θ at this k_n

$\Rightarrow$ set of coherent peaks at: $\lambda_n = \frac{2\eta_*}{n\sqrt{3}}$ in the temperature power spectrum Δ^2_θ !





set of <u>coherent</u> <u>acoustic</u> <u>peaks</u>