e) inflation in 4D $\mathcal{N}=1$ supergravity:

SUSY in 4D

$$\to \mathcal{L} = i\psi\sigma^\mu\partial_\mu\bar{\psi} + \partial_\mu\bar{\phi}\partial^\mu\phi + \bar{F}F$$

invariant under:

$$\delta_\epsilon\phi = \sqrt{2}\,\epsilon\psi$$
$$\delta_\epsilon\psi = i\sqrt{2}\,\sigma^\mu\bar{\epsilon}\,\partial_\mu\phi + \sqrt{2}\,\epsilon F$$
$$\delta_\epsilon F = i\sqrt{2}\,\bar{\epsilon}\,\bar{\sigma}^\mu\partial_\mu\psi$$

$\to$ SUSY

can write superfields in superspace

e) inflation in 4D $\mathcal{N}=1$ supergravity

4D $\mathcal{N}=1$ supersymmetry (SUSY):

$\rightsquigarrow$ the Wess-Zumino model with action (complex scalar ϕ, 4D Weyl fermion ψ, auxiliary F):

$$S_{WZ} = \int d^4x\left(i\psi\sigma^\mu\partial_\mu\bar{\psi} + \partial_\mu\bar{\phi}\partial^\mu\phi + \bar{F}F\right)$$

is invariant under a transformation:

$$\delta_\epsilon\phi = \sqrt{2}\,\epsilon\psi$$
$$\delta_\epsilon\psi = i\sqrt{2}\,\sigma^\mu\bar{\epsilon}\,\partial_\mu\phi + \sqrt{2}\,\epsilon F$$
$$\delta_\epsilon F = i\sqrt{2}\,\bar{\epsilon}\,\bar{\sigma}^\mu\partial_\mu\psi$$

ϵ: 1 constant 4D Weyl spinor

$\to$ 1 continuous boson-fermion symmetry

$\rightarrow$ $\mathcal{N}=1$ SUSY in 4D

(ϕ, ψ, F) : 4D $\mathcal{N}=1$ chiral multiplet

more elegant way:

extend $\mathcal{M}_4$ to $\mathcal{M}_4 \times X_\theta$: superspace

X_θ : anticommuting-variable space

θ : θ^α, $\alpha = 1,2$ 4D Weyl spinor is Grassmann-valued

Berezin integral for Grassmann variables:

$$\int d\theta^\alpha \cdot (\theta^\alpha)^n = \begin{cases} 1, & n=1 \\ 0, & n \neq 1 \end{cases}$$

define:

$$\theta^2 = \epsilon_{\alpha\beta}\theta^\alpha\theta^\beta, \quad \theta^4 = \theta^2 \cdot \bar{\theta}^2$$

$$\Rightarrow \quad \int d^2\theta \cdot (\theta^2)^n = \begin{cases} 1, & n=1 \\ 0, & n \neq 1 \end{cases}$$

and correspondingly:

$$\int d^4\theta \cdot \theta^4 = \int d^2\theta \cdot \theta^2 \int d^2\bar{\theta} \cdot \bar{\theta}^2 = 1$$

and zero for different powers of $\theta^2, \bar{\theta}^2$.

$\rightsquigarrow$ define chiral superfield Φ by θ-expansion in terms of components (ϕ, ψ, F) of chiral multiplet:

$$\Phi = \phi + \sqrt{2}\,\theta \cdot \psi + \theta^2 \cdot F$$

$$\Rightarrow \quad S_{WZ} = \int d^4x \int d^4\theta \cdot K(\Phi^\dagger, \Phi)$$

with: $K(\Phi^+, \Phi) = \Phi^+ \Phi$

"Kähler" potential

masses, scalar potential, Yukawa couplings for ϕ and ψ?

$\rightarrow$ holomorphic function

$W(\Phi)$ "superpotential"

generell 4D $N=1$ SUSY effective action for chiral superfield Φ:

$$S^{eff.} = \int d^4x \left\{ \int d^4\theta \cdot K(\Phi^+, \Phi) + \left[\int d^2\theta \cdot W(\Phi) + h.c. \right] \right\}$$

for renormalizable level:

$$K = \Phi^+ \Phi$$

$$W = \sum_{i=0}^{3} c_i \Phi^i$$

in general:

K: real function of Φ^+, Φ

Kähler potential

W: holomorphic function of Φ

superpotential

$$W = \sum_{i=0}^{3} c_i \phi^i + \sum_{j \geq 4} c_j \frac{\phi^j}{M_P^{j-3}}$$

$$\boxed{W_{tree} = W_{all\text{-}loop}}$$ non-renormalization theorem of 4D $N=1$ SUSY

- SUSY transformation parameter ϵ constant $\rightarrow$ global SUSY
- promote $\epsilon \rightarrow \epsilon(x^\mu, \theta^\alpha)$

$\rightarrow$ local SUSY

$\sim$ necessitates supersymmetric version of Einstein gravity

$\sim$ supergravity:

$$S = \int d^4x\, d^2\theta \cdot \mathcal{E} \cdot \left[\frac{3}{4}\left(\bar{\mathcal{D}}^2 - 8R\right) e^{-\frac{K(\bar{\Phi}^\dagger, \Phi)}{3}} + 2 \cdot W(\Phi)\right] + h.c.$$

$\underline{\bar{\mathcal{D}}^2}$: $\mathcal{D}_\alpha$ superspace spinor-valued derivative

relevant property here:

$$\int d^2\theta \cdot \bar{\mathcal{D}}^2 = \int d^4\theta$$

$$\Rightarrow \int d^2\theta \cdot \bar{\mathcal{D}}^2 e^{-\frac{K}{3}} = \int d^4\theta \cdot e^{-\frac{K}{3}} \supset \int d^4\theta \cdot K$$

$\sim$ contains kinetic terms of WZ model for ϕ and ψ

$\underline{\mathcal{E}}$: SUSY version of $\sqrt{-g}$

$\Rightarrow$ since $e^{-K/3} = 1 + \ldots$

S contains a part:

$$S_{\theta^0} \sim \int d^4x \sqrt{-g} \cdot R + \ldots$$

thus includes Einstein gravity

$\sim$ "supergravity"

gives effective action for ϕ in Φ:

$$S_\phi^{\text{eff.}} = \int d^4x \sqrt{-g} \left[K_{\phi\bar{\phi}} \, \partial_\mu \phi \, \partial^\mu \bar{\phi} - V(\phi) \right]$$

with: $K_{\phi\bar{\phi}} \equiv \dfrac{\partial^2 K}{\partial\phi \partial\bar{\phi}} \equiv \left. \dfrac{\partial^2 K}{\partial\Phi \partial\Phi^+} \right|_{\Phi=\phi}$

$K_\phi \equiv \dfrac{\partial K}{\partial \phi}$ in the same sense

and:

$$V(\phi) = e^K \cdot \left(K^{\phi\bar{\phi}} |D_\phi W|^2 - 3|W|^2 \right)$$

$$D_\phi W \equiv \frac{\partial W}{\partial \phi} + W \cdot K_\phi \quad \underline{\text{supercovariant derivative}}$$

$$K^{\phi\bar{\phi}} = \left(K_{\phi\bar{\phi}} \right)^{-1} = \frac{1}{K_{\phi\bar{\phi}}}$$

for single chiral field Φ

$\rightsquigarrow$ for multiple chiral fields

$\Phi^i \rightarrow$ matrix inverse:

$$V = e^K \left(K^{\phi^i \overline{\phi^j}} D_{\phi_i} W \cdot D_{\overline{\phi_j}} \overline{W} - 3|W|^2 \right)$$

$$K^{\phi^i \overline{\phi^j}} = \left(K_{\phi^i \overline{\phi^j}} \right)^{-1}$$

with Planck mass restored:

$$V = e^{\frac{K}{M_P^2}} \left(K^{\phi\bar{\phi}} |D_\phi W|^2 - \frac{3}{M_P^2} |W|^2 \right)$$

$$D_\phi W = \frac{\partial W}{\partial \phi} + \frac{1}{M_P^2} K_\phi \cdot W$$

$\Rightarrow$ for $M_P \rightarrow \infty$ (no gravity):

$$V = K^{\phi\bar{\phi}} \left| \frac{\partial W}{\partial \phi} \right|^2 = V \left(S^{\text{eff.}}_{\substack{4D\ N=1 \\ \text{global}}} \right)$$

correct limit ...

$$\Downarrow$$

inflation in supergravity:

choose K, W such that you get a slow-roll flat potential V.

note that SUSY breaking is controlled by the F-terms:

$$F_\phi = D_\phi W$$

$\Rightarrow$ for unbroken SUSY: $F_\phi = 0$

we have:

$$V(\phi) \begin{cases} = 0, & W = 0 \\ < 0, & W \neq 0 \end{cases}$$

in supergravity

but inflation needs $V > 0$

$\Rightarrow$ during inflation SUSY is always broken!

i) generic case:

- choose $K = \bar{\Phi}\phi$ to give a canonical kinetic term for ϕ
- choose a $W(\phi)$ such that:

$$V^{(0)} = V(\phi \to 0) = K^{\bar{\Phi}\phi} |D_\phi W|^2 - 3|W|^2$$

supports slow-roll:

$$\eta^{(0)} = \frac{V^{(0)''}}{V^{(0)}} \ll 1 \,, \quad \epsilon^{(0)} = \frac{1}{2}\left(\frac{V^{(0)'}}{V^{(0)}}\right)^2 \ll 1$$

then: $V = e^K \cdot V^{(0)} = (1 + \bar{\Phi}\phi + \ldots) \cdot V^{(0)}$

$\Rightarrow \quad \eta = \zeta^{(0)} + 1 + \ldots \simeq \mathcal{O}(1)$

generically

$\rightsquigarrow$ the e^K - dependence in V of 4D $\mathcal{N}=1$ supergravity generically creates a dimension-6 operator:

$$\delta V \sim \frac{\phi^2}{M_P^2} V^{(0)}$$

which destroys slow-roll by giving a large correction to η

$\hookrightarrow$ <u>supergravity "η-problem"</u>

ii) effect of a shift symmetry in supergravity (Kawasaki, Yamaguchi & Yanagida 2000):

- choose 2 fields ϕ, X and K:

$$K = \frac{1}{2}(\phi + \bar{\phi})^2 + \bar{X}X$$

then K is invariant under:

$$\phi \to \phi + i\alpha$$

"shift symmetry"

demanding a full shift symmetry of K implies at all orders:

$$K_{\text{all-loop}} = \frac{1}{2}(\phi+\bar{\phi})^2 + \bar{X}X + \Delta K$$

with $\Delta K = \Delta K(\phi + \bar{\phi}, X, \bar{X})$

write: $\phi = \sigma + i\varphi$

then K and e^K do <u>not</u> depend on φ!

- choose:

$$W = m \cdot \phi X$$

$\Rightarrow$ $\dfrac{\partial W}{\partial \phi}$ independent from ϕ

$$F_\phi = D_\phi W = m \cdot X \cdot (1 + \bar{\phi}\phi)$$

$$F_X = D_X W = m \cdot \phi \cdot (1 + \bar{X} X)$$

now look at regime: $|X| < 1 \lesssim \varphi$

$$\rightsquigarrow \quad |F_X|^2 = m^2 |\phi|^2 \cdot (1 + \mathcal{O}(|X|^2)) \simeq m^2 (\varphi^2 + \sigma^2)$$

$$|F_\phi|^2 = m^2 |X|^2 \cdot (1 + |\phi|^2)^2 = \mathcal{O}(|X|^2)$$

and: $e^K, K_\phi, K^{\phi\bar{\phi}}$ depend on σ but <u>not</u> on φ!

($e^K, K_X, K^{X\bar{X}}$ depend all on $X, \bar{X}$)

$$\Rightarrow \quad V = e^{K} \cdot \left(K^{\bar{\Phi}\Phi} |D_\phi W|^2 + K^{\bar{X}X} |D_X W|^2 - 3|W|^2 \right)$$

$$= e^{\sigma^2 + |X|^2} \cdot \Big[m^2 (\varphi^2 + \sigma^2) \cdot \left(1 + \mathcal{O}(|X|^2)\right)$$

$$+ m^2 |X|^2 \cdot \left(1 + \varphi^2 + \sigma^2\right)^2$$

$$- 3 m^2 |X|^2 (\varphi^2 + \sigma^2) \Big]$$

$|X| < 1, \varphi$

$$= e^{\sigma^2} \cdot m^2 \cdot (\varphi^2 + \sigma^2) + \mathcal{O}(|X|^2)$$

$$\Rightarrow \quad \eta_\sigma = \frac{1}{V} \frac{\partial^2 V}{\partial \sigma^2} = \mathcal{O}(1) \text{ as in i)}$$

<u>but</u>: $\eta_\varphi = \frac{1}{V} \frac{\partial^2 V}{\partial \varphi^2} = \frac{2}{\varphi^2} \ll 1$ for $\varphi \gg 1$

$\leadsto$ we can do large-field $\Delta\varphi_{60} > M_p$ slow-roll inflation with φ, because φ is <u>protected</u> by a <u>shift symmetry</u> from getting the dangerous dimension-6 corrections in V!

$\downarrow$

chaotic inflation with large fields is <u>natural</u> in supergravity if <u>protection by a shift symmetry</u> is realized ...

2 classes of $V(\phi)$:

a) large-field models

examples: • $V(\phi) \sim \phi^p$, $p \geq 2$

• hybrid models for suitable parameter choices

b) small-field models

examples: • inflection & saddle point models

• hybrid models for suitable parameters

Why $\phi \sim M_p$ as discriminator?

→ so far this was classical

→ consider dim-6 operators correcting V ...

⇒ generically we get among them:

$$\Delta V_6 \sim V \cdot \frac{\phi^2}{M_p^2}$$

$$\Rightarrow \Delta\eta = \frac{\Delta V_6''}{V} \sim \mathcal{O}(1)$$

$$\Rightarrow \Delta\epsilon \sim \left(\frac{\phi}{M_p}\right)^2 \to 1 \text{ at } \phi \sim M_p$$

→ destroys slow-roll: "eta-problem"

2 ways out:

- keep $\phi \ll M_P$ "small-field" and look for UV-theory to enumerate finite # of dim-6 corrections — and tune
- find a symmetry, that forbids all dim-6 terms $\rightarrow$ then $\phi \gg M_P$ possible...

$\rightsquigarrow$ example: $V(\phi) \sim \phi^p$ + gravity

graviton vertex $\sim T_{\mu\nu} \sim V, V''$

$\Rightarrow \Delta V \sim V^2, V''V$ not $V \cdot \frac{\phi^2}{M_P^2}$

$\longrightarrow$ shift symmetry of gravity...

$\rightarrow$ $\boxed{\text{Clearly need UV completion!}}$