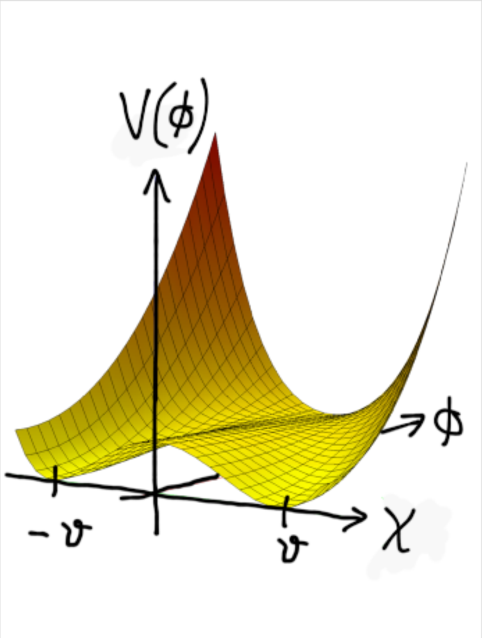
i) hybrid inflation

$$V = \frac{\lambda}{4}\left(\chi^2 - v^2\right)^2 + g \cdot \chi^2 \phi^2 + \frac{1}{2} m^2 \phi^2$$

$\rightsquigarrow$ no longer a pure single-field model



3 stages:

– at $\phi > \phi_c = \sqrt{\frac{\lambda}{2g}}\, v$

χ is massive, $\left.\frac{\partial^2 V}{\partial \chi}\right|_{\chi=0} > 0$

and ϕ drives slow-roll inflation

in $V_{eff.} = V(\chi = 0, \phi) = \frac{\lambda v^2}{4} + \frac{m^2}{2}\phi^2$

if we choose:

$$\lambda \cdot v^2 \ll 1$$

as this enforces:

$$\left| \frac{1}{V} \cdot \left.\frac{\partial^2 V}{\partial \chi^2}\right|_{\chi=0} \right| \gg 1 \quad \forall\, \phi \neq \phi_c$$

so that χ is fast rolling almost everywhere ...

- $\phi = \phi_c \Rightarrow \chi$ is massless there

- $\phi < \phi_c \Rightarrow \left.\frac{\partial^2 V}{\partial \chi^2}\right|_{\chi=0} < 0$

χ turns tachyonic at $\chi = 0$, regaining quickly:

$$\left| \frac{1}{V} \cdot \left.\frac{\partial^2 V}{\partial \chi^2}\right|_{\chi=0} \right| \gg 1$$

$\Rightarrow \chi$ rolls quickly to $\pm v$!

but at $\chi = \pm v$ we have

$$V(\chi = \pm v, \phi < \phi_c) < \underbrace{\tfrac{1}{2} m^2 \phi_c^2}_{= \frac{m^2 \lambda}{4 g}}$$

which can be $\ll \frac{\lambda v^4}{4}$ for

$m^2 \ll g \cdot v^4$

$\Rightarrow \chi$ rolling to $\pm v$ quickly shuts down the big vacuum energy $\frac{1}{4} \lambda v^4$, and so ends inflation in a '<u>waterfall</u>'

$\rightsquigarrow$ <u>waterfall</u> <u>field</u> χ

$\rightsquigarrow$ inflation does not end by the usual violation of slow-roll in ϕ!

$\Rightarrow$ hybridized situation of one field driving slow-roll, and another one driving the vacuum energy of inflation, and ending it quickly by waterfall ...

$\rightsquigarrow$ hybrid inflation

d) an original model by Starobinsky '80:

start with:

$$S = \frac{1}{2} \int d^4x \sqrt{-g} \, \underbrace{\left(R + \frac{1}{M^2} R^2 \right)}_{\equiv f(R)}$$

then Weyl transform the metric:

$$\tilde{g}_{\mu\nu} = \Omega^2 \cdot g_{\mu\nu}$$

$$\Rightarrow R = \Omega^2 \cdot \left(\tilde{R} + 6 \tilde{\Box} \ln \Omega - 6 \tilde{g}^{\mu\nu} \partial_\mu \ln \Omega \, \partial_\nu \ln \Omega \right)$$

$$\sqrt{-g} = \Omega^{-4} \sqrt{-\tilde{g}}$$

then write:

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\left(f'\cdot R - \mathcal{U}\right)$$

$$\mathcal{U}(R) = f'\cdot R - f\,,\quad f' \equiv \frac{\partial f}{\partial R}$$

$$= \frac{1}{2}\int d^4x\sqrt{-\tilde{g}}\Big[f'\cdot\Omega^{-2}\cdot\big(\tilde{R} + \underbrace{\cancel{6\tilde{\Box}\ln\Omega}}_{=0}$$

$$- 6\cdot\tilde{g}^{\mu\nu}\partial_\mu\ln\Omega\,\partial_\nu\ln\Omega\big)$$

$$- \Omega^{-4}\cdot\mathcal{U}\Big]$$

we want now usual Einstein-Hilbert term for $\tilde{R}$ ('Einstein frame'):

$\Rightarrow$ choose: $f' = \Omega^2$

and for a new scalar field variable:

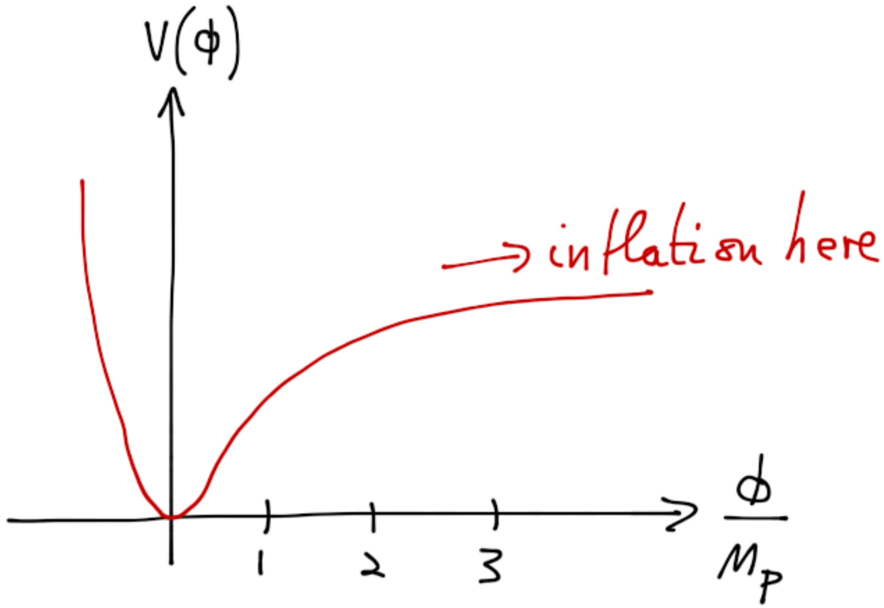
$$\phi \equiv \sqrt{6}\,\ln\Omega = \sqrt{\frac{3}{2}}\,\ln f'$$

we arrive at:

$$S = \frac{1}{2}\int d^4x\sqrt{-\tilde{g}}\left(\tilde{R} - \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - V(\phi)\right)$$

$$\text{with: } V(\phi) = \frac{\mathcal{U}}{f'^2} = \frac{f'\cdot R - f}{f'^2}$$

$$= \frac{3}{4}M^2M_P^2\cdot\left(1 - e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_P}}\right)^2$$


V(ϕ)
inflation here
ϕ/M_P
1
2
3