if:

$$\epsilon, \eta \ll 1$$

then:

$$\epsilon \simeq \epsilon_H = -\frac{\dot{H}}{H^2} = 1 - \frac{1}{H^2}\frac{\ddot{a}}{a}$$

$$\eta \simeq \eta_H = \frac{\dot{\epsilon}_H}{\epsilon_H H}$$

$\Rightarrow$ $\epsilon \ll 1$ implies $\epsilon_H \ll 1$

$\Rightarrow$ $\ddot{a} > 0$

consistent.

ϵ_H, η_H 'physical' Hubble slow-roll parameters

can do much more:

- multiple fields
- higher derivatives
- non-trivial kinetic terms

but here simple single-field slow-roll will suffice.

<u>General story here:</u>

- need a $V(\phi)$ that satisfies $\epsilon, \eta \ll 1$ at some ϕ_{Ne},
- $\epsilon, \eta < 1$ for $N_e \simeq 60$ efolds at least, then $\epsilon > 1$ must be reached at some ϕ_e

→ slow-roll ends at $\phi = \phi_e$ when $\epsilon = 1$, and ϕ rolls quickly into the next minimum

→ then ϕ oscillates around the minimum, with maximum initial kinetic energy density:

$$\rho_{kin} = \frac{1}{2}\dot{\phi}^2 = V(\phi_e)$$

ρ_{kin} decays with time, as the Hubble friction in the e.o.m.:

$$\ddot{\phi} + 3H\dot{\phi} = -V'$$

causes ϕ to execute <u>damped</u> oscillations around the minimum

→ if ϕ is weakly coupled to other fields, it will slowly decay into other quanta, which will eventually thermalize and '<u>reheat</u>' the Universe. If reheating were instantaneous, then:

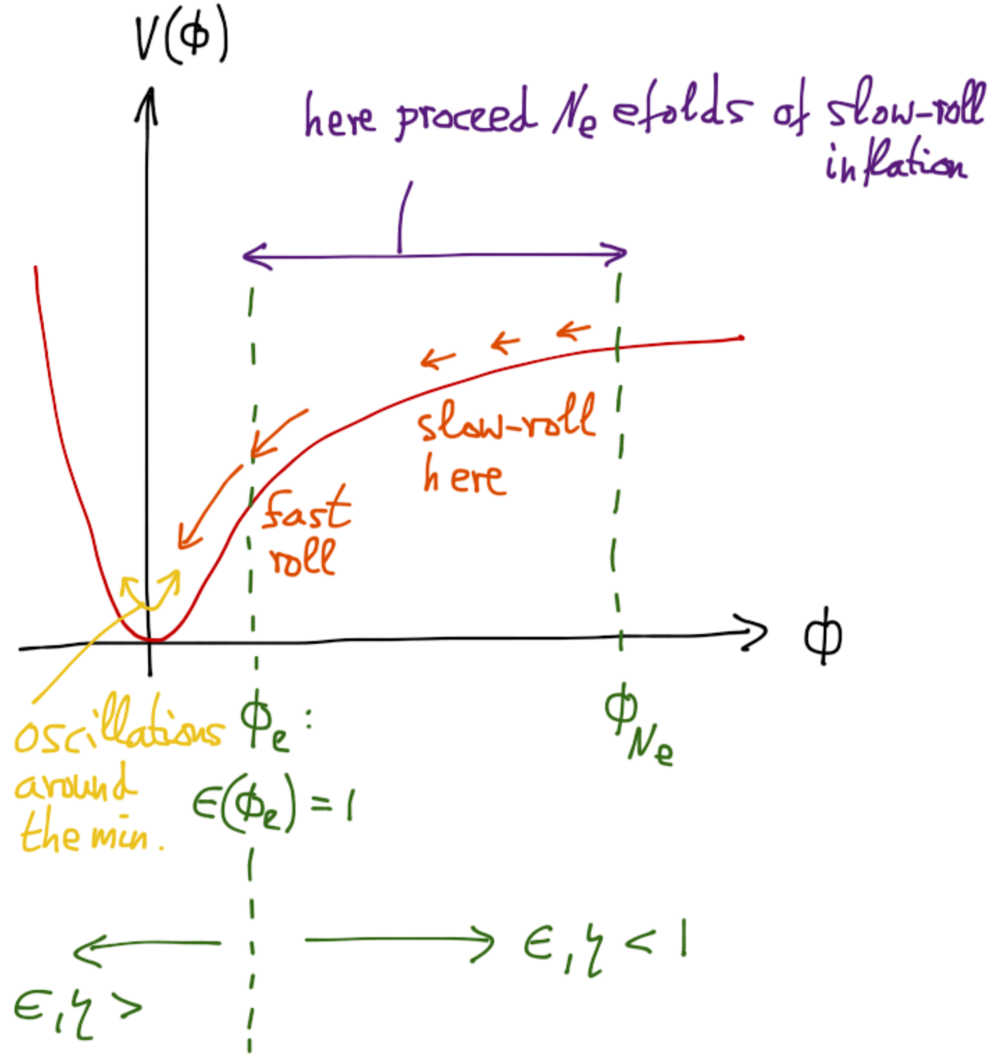
$$\rho_{rad.} = g_* \frac{\pi^2}{30} T^4 = \rho_{kin}(t_e) = V(\phi_e)$$

this defines the maximal reheating temperature of the Universe:

$$T_{reh.}^{max} = \left[\frac{30}{g_* \pi^2} V(\phi_e)\right]^{1/4}$$

since the actual reheating process is slower, the damped oscillations of ϕ during reheating yield:

$$T_{reh.} < T_{reh.}^{max}$$

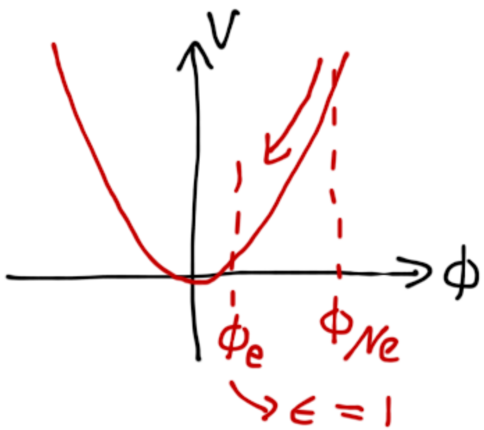


V) simple field theory models of inflation

a) monomial potentials ('chaotic' inflation)
Linde '83

examples : $V(\phi) \sim \phi^p$, $p \gtrsim 2$

e.g. $V(\phi) = \frac{m^2}{2}\phi^2$



inflation ends when $\epsilon \gtrsim 1 \Leftrightarrow \dot\phi^2 \simeq V$,
so, at ϕ_e: $\epsilon(\phi_e) = 1$, and has to be started at a $\phi = \phi_{N_e} > \phi_e$.

slow-roll parameters :

$$\epsilon = \frac{1}{2}\left(\frac{V'}{V}\right)^2 = \frac{p^2}{2\phi^2}\ ,\ \eta = \frac{V''}{V} = \frac{p(p-1)}{\phi^2}$$

$$\Rightarrow \epsilon, \eta < 1 \quad \text{for} \quad \phi \gtrsim 1\ ,\ p = \mathcal{O}(1)$$

$$\epsilon(\phi_e) = \frac{p^2}{2\phi_e^2} \overset{!}{=} 1 \Rightarrow \boxed{\phi_e = \frac{p}{\sqrt{2}} \sim M_p}\ \forall\, p = \mathcal{O}(1)$$

$$N_e = \int_{t_{N_e}}^{t_e} H\,dt = \int_{\phi_e}^{\phi_{N_e}} \frac{H}{\dot\phi}\,d\phi = \int_{\phi_e}^{\phi_{N_e}} \frac{d\phi}{\sqrt{2\epsilon}} = \frac{\phi_{N_e}^2}{2p} - \frac{p}{4}$$

$$\Rightarrow \boxed{\phi_{N_e} = \sqrt{2p\,N_e} \gg M_p}$$

e.g. for $p = 2 \Rightarrow \phi_{60} \simeq 15\, M_p$

do we really have slow-roll at $\phi > M_p$? form of a potential $V \sim \phi^p$ is counter-intuitive shape-wise ...

check slow-roll e.o.m. for $p=2$:

$$3H\dot\phi = -V' = -m^2\phi$$

$\wedge$ in slow-roll: $H \simeq \sqrt{\frac{V}{3}} = \sqrt{\frac{1}{6}}\, m\phi$

$$\Rightarrow -m^2\phi = -V' = 3H\dot\phi = \sqrt{\frac{3}{2}}\, m\phi\dot\phi$$

$$\Leftrightarrow \dot\phi = -\sqrt{\frac{2}{3}}\, m = \text{const.}$$

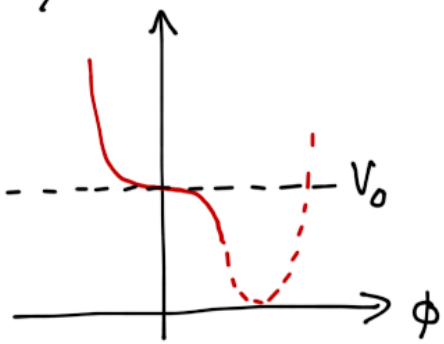
$$\Rightarrow \phi(t) = \phi(t_0) - \sqrt{\frac{3}{2}}\, m\cdot(t-t_0)$$

and: $\ddot\phi = 0 \Rightarrow \frac{\ddot\phi}{3H\dot\phi} = 0$.

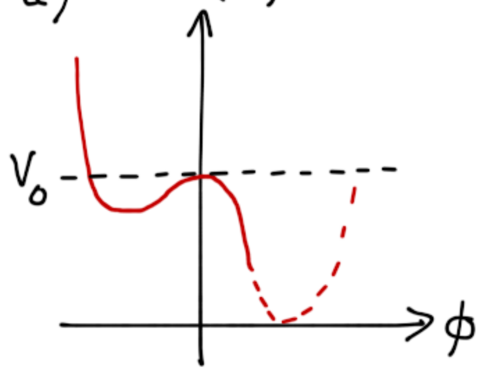
b) inflection/saddle point/hill top inflation

$\rightsquigarrow$ 2 cases

i) $V(\phi)$



ii) $V(\phi)$



the 2 cases can generically written as a Taylor expansion around $\phi = 0$:

i) $V = V_0\cdot\left(1 - \sqrt{2\epsilon_0}\cdot\phi + \frac{\lambda_3}{3}\phi^3 + \dots\right)$

ii) $V = V_0\cdot\left(1 - \frac{1}{2}\eta_0\cdot\phi^2 + \frac{\lambda_3}{3}\phi^3 + \dots\right)$

where: $\epsilon_0 = \epsilon(0) = \frac{1}{2}\left(\frac{V'}{V}\right)^2\Big|_{\phi=0}$

$$\eta_0 = \eta(0) = \frac{V''}{V}\Big|_{\phi=0}$$

by construction we have:

$\eta(0) = 0$ case i) 'inflection' point

$\epsilon(0) = 0$ case ii) 'saddle' point
'hill top'

if $\lambda_3 >> 1$, than $\epsilon(\phi) = 1$

for $\phi = \phi_e << 1$, while still

$\epsilon(\phi), \eta(\phi) << 1$ for $0 \le \phi << \phi_e$

$\Rightarrow$ the whole inflationary process
happens on $\Delta\phi_{N_e} = \phi_e - \phi_{N_e} << M_P$

$\sim$ during inflation the scalar field sits almost fixed in a narrow region around the critical point $V_0' = 0$ or $V_0'' = 0$, kept there by the viscous Hubble 'glue'

$\sim$ inflation ends at $\phi_e << M_P$ by ϕ quickly rolling off into a nearby minimum ...