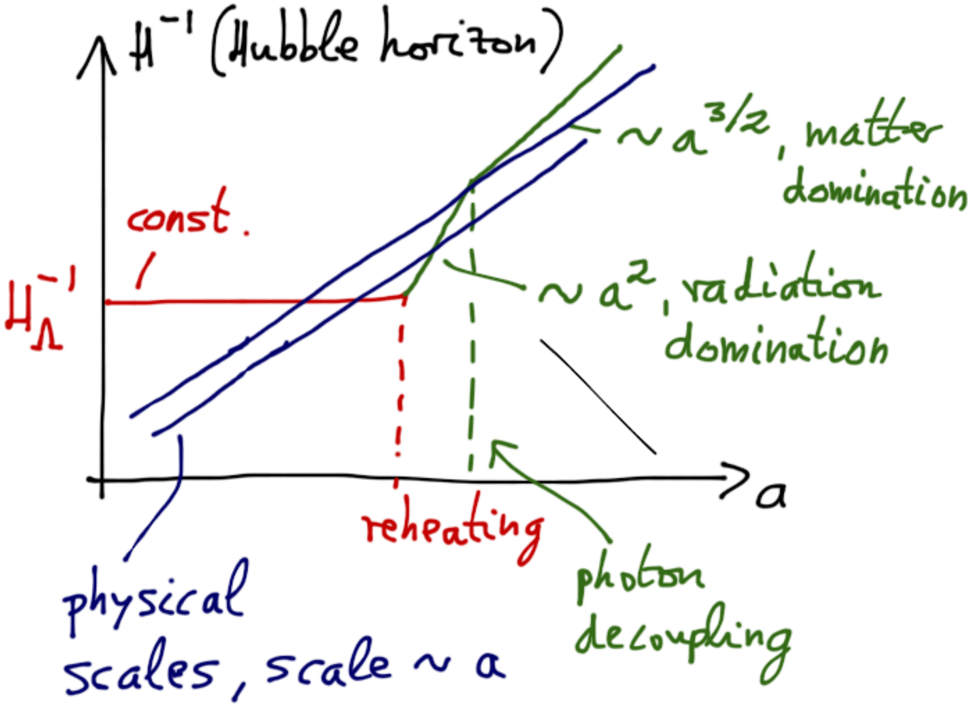


↳ modes left the horizon during inflation & re-entered afterwards → were in causal contact early on

→ solves horizon problem & some others (flatness, ...)

horizon during inflation:

↝ calcute distance, that light can reach at time $t_e > 0$ after emission at $t_i < t_e$:

$$d_{inf.} = a(t_i) \cdot \int_{t_i}^{t_e} \frac{dt}{a(t)}$$

$$= e^{H_\Lambda t_i} \frac{e^{-H_\Lambda t_e} - e^{-H_\Lambda t_i}}{-H_\Lambda}$$

$$= \frac{1}{H_\Lambda}\left(1 - e^{-H_\Lambda (t_e - t_i)}\right)$$

$$\longrightarrow \frac{1}{H_\Lambda} \text{ for } t_e - t_i \to \infty$$

while during matter- or radiation domination:

$$d_{hor.} \sim t_e - t_i \to \infty$$

for $t_e - t_i \to \infty$

$\rightsquigarrow$ totally different behavior!

$\longrightarrow$ during inflation there is a <u>true event horizon</u> at H_Λ^{-1} distance around us, similar to a black hole event horizon, but turned 'inside-out'...

solves flatness problem:

$$\Omega_\Lambda \simeq \text{const.}$$

$$|\Omega_k| \sim \left|\frac{k}{a^2}\right| \lesssim 10^{-60}$$

if $\underbrace{H_\Lambda \cdot \Delta t_{inf.}}_{\equiv N_e} \gtrsim 60$ $\boxed{\text{"e-folds"}}$

also:

$$\frac{a(t+\Delta t_{inf.})}{a(t)} = e^{H_\Lambda \cdot \Delta t_{inf.}} \equiv e^{N_e} \gtrsim e^{60}$$

would have all visible CMB scales today originating from a tiny causal region early on ...

iv) How to get inflation

A. historically 1st: Guth '80

→ supercool Universe into a meta-stable state of positive vacuum energy → problems with exit & reheating

↓

use:

1 scalar field ϕ ...

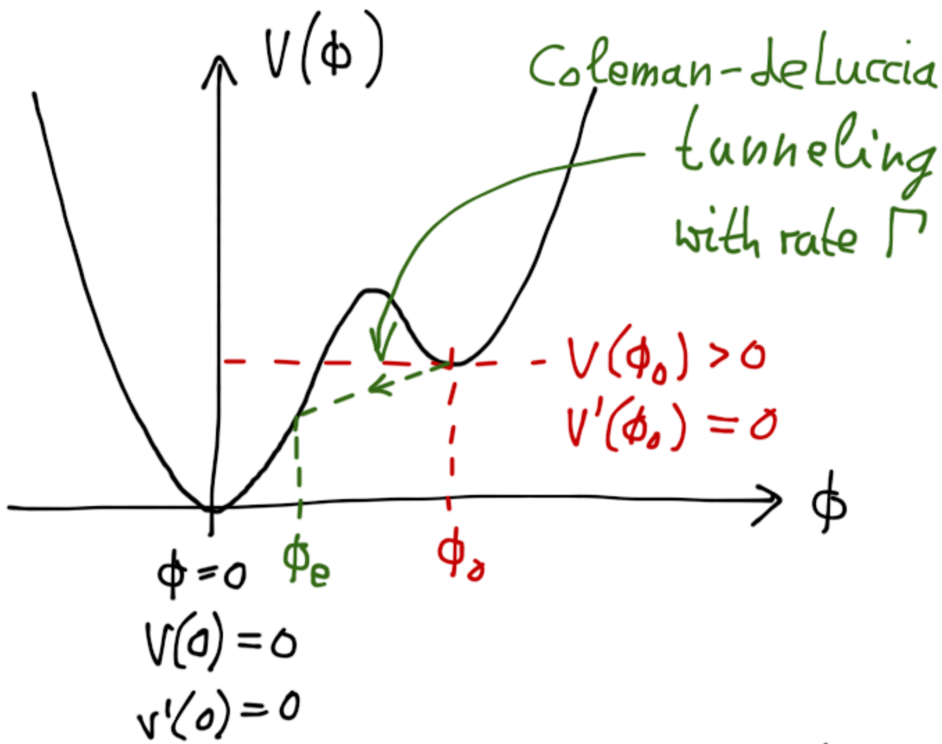
action:

$$S = \int d^4x \sqrt{-g} \left[\tfrac{1}{2} \partial_\mu \phi \partial^\mu \phi - \underbrace{V(\phi)}_{\text{scalar potential}} \right]$$

$\downarrow \delta S / \delta g^{\mu\nu}$

$$T_{\mu\nu} : \rho = \tfrac{1}{2} (\partial\phi)^2 + V, \quad p = \tfrac{1}{2} (\partial\phi)^2 - V$$

metastable vacuum of potential $V(\phi)$ emulates $\Lambda > 0$:



at $\phi = \phi_0$: local minimum, $\dot{\phi} = 0$

$$\Rightarrow \rho = V(\phi_0) = \text{const.} > 0, \quad p = -V(\phi_0) \underline{\underline{= -\rho}}$$

$\Rightarrow$ while ϕ at ϕ_0 : inflation

$\leftrightarrow \; a \sim e^{\sqrt{V(\phi_0)} \cdot t}$

$\Rightarrow$ once ϕ tunnels :

$\rightsquigarrow$ ϕ exits at $\phi_e < \phi_0$, then rolls quickly to $\phi = 0$ & oscillates around $\phi = 0$: non-relativistic matter (ϕ has mass m)

$\leftrightarrow \; a \sim t^{2/3} \Leftrightarrow \ddot{a} < 0$

inflation has ended

$\Rightarrow$ if ϕ coupled to other fields (SM fields) :

$\rightsquigarrow$ decays to SM fields while oscillating around $\phi = 0$, once inflation ended via tunneling to ϕ_e ... decay converts part of $V(\phi_0)$ into new radiation & heat — "reheating" problem:

$\rightsquigarrow$ if tunneling slow, life-time $\tau = \frac{1}{\Gamma}$ of ϕ at ϕ_0 long

$\Rightarrow$ many efolds $N_e = H(\phi_0) \cdot \tau$

but too few regions with exit by tunneling — one such region not big enough for our universe: need many that merge ...

$\rightsquigarrow$ need fast tunneling, short lifetime $\rightarrow$ not enough efolds $\lightning$

the solution:

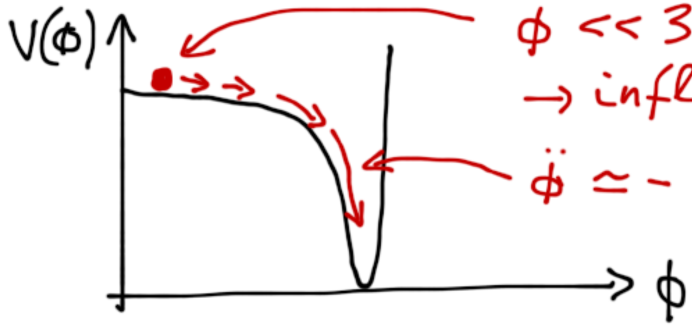
avoid trapping in a metastable minimum of the potential at $V > 0$ and the subsequent tunneling ...

$\downarrow$

B. Slow-roll inflation

(Albrecht & Steinhardt; Linde '82)

$\rightarrow$ slow-roll in potential $V(\phi)$:



$\ddot{\phi} \ll 3H\dot{\phi} = -\frac{\partial V}{\partial \phi}$
$\rightarrow$ inflation

$\ddot{\phi} \simeq -\frac{\partial V}{\partial \phi}$ $\rightarrow$ no inflation consider : $\vec{\nabla}\phi = 0$, only $\dot{\phi}$

redshifts fast, if $a \sim e^{H \cdot t}$

then if : $\boxed{\dot{\phi}^2 << V}$ $\Rightarrow p = -\rho$

$$\Downarrow$$

$$\begin{cases} a \sim e^{H \cdot t} \\ H \simeq \text{const.} \end{cases}$$

motion dominated by potential energy

need this for $N_e \simeq H \cdot t \overset{!}{\simeq} 60$ efolds to solve the horizon etc. problems ...

can ensure this, if slow-roll.

To show this, we follow this logic:

i) assume slow-roll :

$$\ddot{\phi}_0 << 3H\dot{\phi}_0$$

at a given initial time t_0

where, $\phi_0 \equiv \phi(t_0)$, $\dot{\phi}_0$ & $\ddot{\phi}_0$ are specified at $V = V_0 = V(\phi_0)$

ii) Demand vacuum energy domination

$$\frac{p}{\rho} = -\frac{1 - \dot{\phi}^2/2V}{1 + \dot{\phi}^2/2V} \simeq -1$$

$$\Leftrightarrow \dot{\phi}^2 << V$$

$\rightsquigarrow$ i) and ii) imply a condition on V and $V' \equiv \partial V / \partial \phi$

iii) enforce slow-roll:

$$\left|\frac{\ddot{\phi}}{3H\dot{\phi}}\right| \ll 1$$

assuming i) and ii) and their condition on V, V' hold

$\rightsquigarrow$ condition on V and V''.

e.o.m. for ϕ:

$$\ddot{\phi} + 3H\dot{\phi} = -\frac{\partial V}{\partial \phi} \equiv -V'$$

slow-roll: $|\ddot{\phi}| \ll |3H\dot{\phi}|, |V'|$

$$\Rightarrow \quad 3H\dot{\phi} = -V' \qquad \text{slow-roll e.o.m.} \quad (*)$$

then i): $p \simeq -\rho$

$$\Rightarrow \quad 1 \gg \frac{\dot{\phi}^2}{V} = \frac{V'^2}{9VH^2}, \quad H^2 \simeq \frac{V}{3}$$

$$= \frac{1}{3}\left(\frac{V'}{V}\right)^2 \equiv \frac{2}{3}\epsilon, \quad \boxed{\epsilon = \frac{1}{2}\left(\frac{V'}{V}\right)^2}$$

$$\Rightarrow \quad \boxed{\epsilon \ll 1 \text{ ensures } p \simeq -\rho.}$$

<u>1st slow-roll condition</u>

ensure slow-roll for long time:

↳ maintain: $\ddot{\varphi} \ll 3H\dot{\varphi}$

from (*) $\Rightarrow \quad \dot{\varphi}^2 = \frac{V'^2}{3V}$

$$\Rightarrow \quad \dot{\varphi}\ddot{\varphi} = \frac{1}{2}\left(\frac{V'^2}{3V}\right)' \cdot \dot{\varphi}$$

$$\Rightarrow \quad \ddot{\varphi} = V' \cdot \left(\frac{1}{3}\frac{V''}{V} - \frac{V'^2}{V^2}\right)$$

define: $\boxed{\eta \equiv \frac{V''}{V}}$

$$\Rightarrow \quad \frac{\ddot{\varphi}}{3H\dot{\varphi}} = 2\epsilon - \frac{1}{3}\eta \ll 1$$

implies: $\boxed{\eta \ll 1}$ if $\epsilon \ll 1$

2nd slow-roll condition