V Cosmological Inflation

i) short excurse on horizons:

horizons → crucial property of FRW expansion

convenient here: conformal time

$\rightsquigarrow \; ds^2 = a^2(\eta)\cdot\left[d\eta^2 - d\rho^2 - f^2(\rho)\cdot d\Omega_2^2\right]$

look at radially outward light rays:

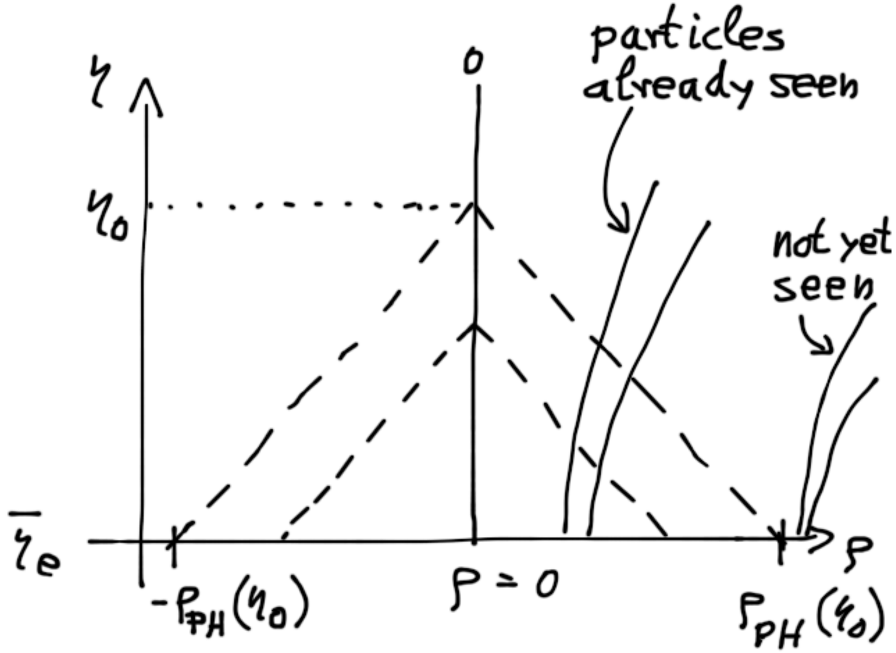
$\rightsquigarrow \; ds^2 = 0$

$\Rightarrow \quad d\rho = \pm d\eta$

$\Rightarrow \quad \rho = \text{const.} \pm \eta$

two horizons:

a) observer O at $\rho = 0$ receives signal at conformal time η_0, an emitter at $\eta_e < \eta_0$ sends the signal...



event at $\eta < \eta_0$ only in causal contact if comoving distance ρ

$$\rho < \rho_e(\eta_e) = \int_{\eta_e}^{\eta_0} d\eta' = \eta_0 - \eta_e$$

↑ maximum visible comoving distance at η_e in the past

if universe begins at $\bar{\eta}_e < \eta_0$, then all visible ρ are bounded:

$$\rho < \rho_{PH}(\eta_0) = \int_{\bar{\eta}_e}^{\eta_0} d\eta' = \eta_0 - \bar{\eta}_e$$

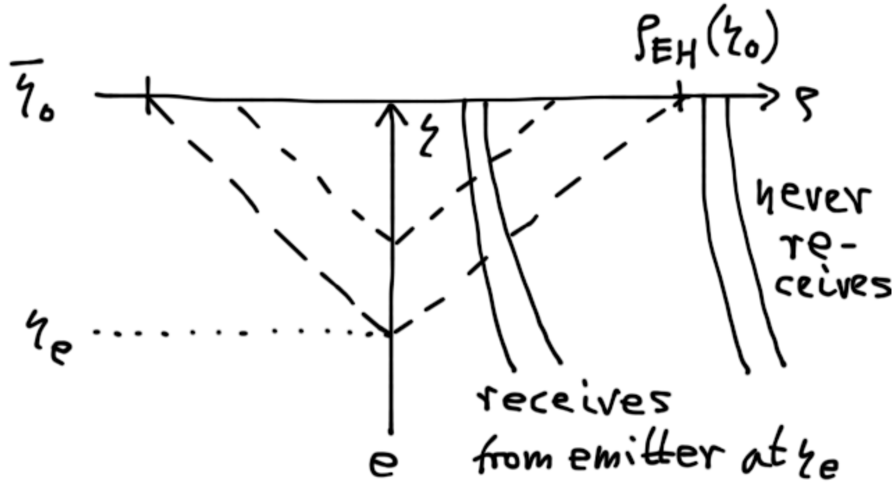
↑ "particle horizon" for observer at η_0

$\rightsquigarrow$ the visible universe at η_0!

b) conversely, if the universe ends at $\bar{\eta}_0 > \eta_e$

$$\rightsquigarrow \rho_{EH}(\eta_e) = \int_{\eta_e}^{\bar{\eta}_0} d\eta' = \bar{\eta}_0 - \eta_e$$

"event horizon", maximum distance of causal influence from now & here



conversion into physical horizon distance:

e.g. $d_{PH}(t_0) = a(t_0) \cdot \rho_{PH}(\eta_0)$

$$= a_0 \cdot \int_{\bar{\eta}_e}^{\eta_0} d\eta' = a_0 \cdot \int_{\bar{t}_e}^{t_0} \frac{dt'}{a(t')}$$

matter/radiation: $a(t) \sim t^p$

$$= t_0^p \cdot \frac{1}{-p+1} \left(t_0^{-p+1} - \bar{t}_e^{-p+1} \right)$$

$t_0 \gg \bar{t}_e$

$$\simeq \frac{1}{1-p} \cdot t_0 \sim H_0^{-1}$$ "Hubble horizon"

$\rightsquigarrow$ $d_{PH} \sim t_0$ grows faster than comoving length scales

$$\lambda = a(t_0)\,\lambda_{com.} \sim t^p$$

with $p = \frac{1}{2}$ or $\frac{2}{3}$ for radiation or matter

iii) initial condition problems of the hot big bang cosmology

1) horizon problem (CMB):

$\rightsquigarrow$ present-day horizon scale H_0^{-1} was smaller by $(1+z_{dec.})^{-1}$ $\simeq 1100$ at CMB decoupling:

$$H_0^{-1} \simeq 10^{10}\, ly$$

$$\Rightarrow \frac{1}{1+z_{dec}} H_0^{-1} \simeq 10^{7}\, ly$$

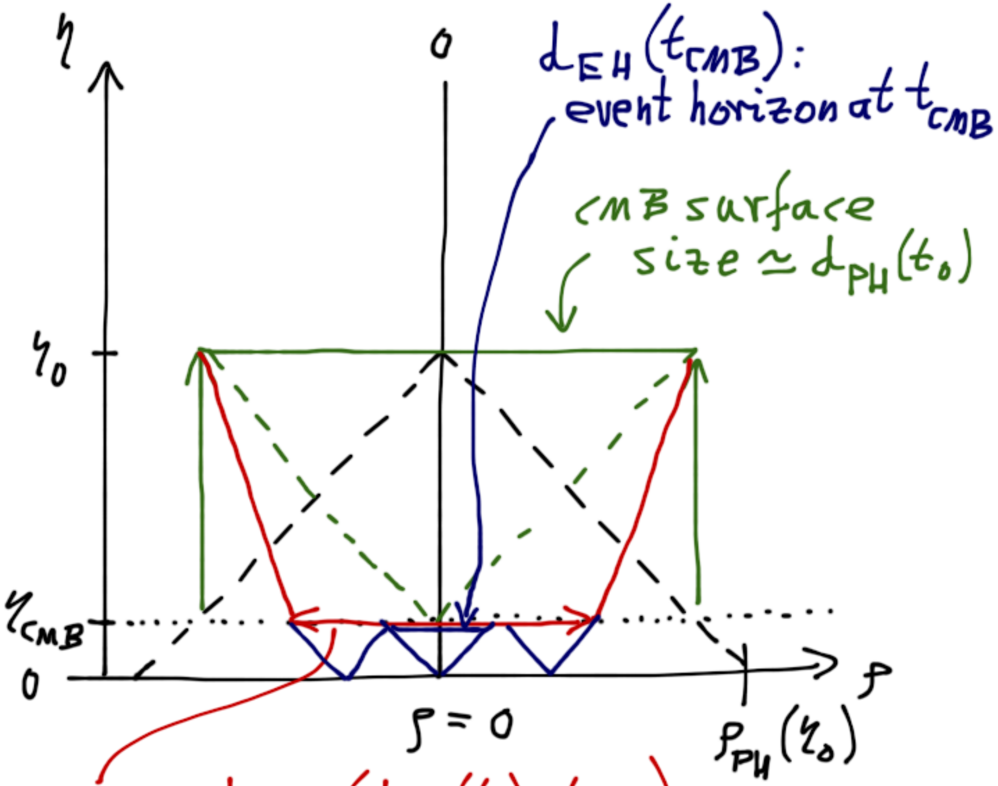
but d_{EH} at decoupling at $t_{dec.} \simeq 400.000\, yr$

$$\Rightarrow d_{EH}(t_{dec.}) \sim 10^{5}\, ly$$

$\rightsquigarrow$ $O(10^4)$ independent horizon-size patches out of causal contact – so why $\Delta T / T \lesssim 10^{-4}$ everywhere ???

$\Downarrow$

"horizon problem"



size $d_{CMB}(d_{PH}(t_0), t_{CMB})$ of the visible CMB region of present-day size $d_{PH}(t_0)$, back at t_{CMB}, shrunken under turned-back matter-dominated expansion $\sim t^{2/3}$

— $\ll$ $\longleftrightarrow$

or :

$$\frac{d_{EH}(t_{CMB})}{d_{CMB}(d_{PH}(t_0))} \simeq \frac{t_{CMB}}{\underbrace{t_0}_{d_{PH}(t_0)} \cdot \left(\frac{t_{CMB}}{t_0}\right)^{2/3}} = \left(\frac{t_{CMB}}{t_0}\right)^{1/3} \simeq (1+z_{CMB})^{-\frac{1}{2}} \simeq 0.03$$

since:

$$1+z_{CMB} = \frac{a_0}{a_{CMB}} = \left(\frac{t_0}{t_{CMB}}\right)^{2/3}$$

$\sim \mathcal{O}(10^3)$ causally disconnected d_{EH}-sized patches at t_{CMB}!

2) size problem:

size of visible universe today

$\rightarrow d_{PH}(t_0) \sim H_0^{-1} \sim 10^{28}\,\mathrm{cm}$

$\rightarrow$ assume radiation domination

$$\Rightarrow d\Big|_{t_P}\left(d_{PH}(t_0)\right) = d_{PH}(t_0)\cdot\left(\frac{t_P}{t_0}\right)^{1/2}$$

$$= 10^{28}\cdot\left(\frac{10^{-43}\,s}{10^{17}\,s}\right)^{1/2}\mathrm{cm}$$

$$= 10^{-2}\,\mathrm{cm} \sim 10^{30}\,\ell_P$$

$\rightsquigarrow$ why <u>so</u> big?

3) another problem: spatial flatness

<u>recall:</u> $\rho_k \sim \frac{k}{a^2},\ \rho_m \sim \frac{1}{a^3},\ \rho_{rad.} \sim \frac{1}{a^4}$

$\Rightarrow \Omega_k/\Omega_{rad.} \sim a^2$ <u>grows!</u>

<u>today:</u> $\langle E_{CMB}\rangle \simeq T_{CMB} \simeq 3K \simeq 10^{-3}\,eV$

<u>at nucleosynthesis:</u> $T_{NUC} \sim 10^6\,eV$

<u>after quantum gravity epoch:</u> $T_P \sim M_P \sim 10^{27}\,eV$

<u>we have:</u> $T \sim \frac{1}{a}$, rad. dom. $\Rightarrow \begin{cases}\Omega_k \sim a^2 \\ \Omega_m \sim a\end{cases}$

<u>and:</u> $|\Omega_k(\text{today})| \lesssim 10^{-2}$

$$\Rightarrow \begin{cases} |\Omega_k(T_{NUC})| \lesssim 10^{-18} \\ |\Omega_k(T_P)| \lesssim 10^{-60} \end{cases}$$

$\lightning$ HOW?

iii) an idea: try getting $\ddot{a} > 0$

$\Rightarrow$ $a(t)$ grows faster than t

$\Rightarrow$ today's CMB sky was smaller than causal patch at some early time

solves horizon problem

$\Downarrow$

try a finite epoch of a quasi-cosmological constant - a large one:

$\Rightarrow \rho_\Lambda = \text{const.} = -p_\Lambda$

$$\Rightarrow H_\Lambda^2 = \frac{\rho_\Lambda}{3M_P^2} = \text{const.} = \frac{\dot{a}^2}{a^2}$$

$$\Rightarrow \boxed{a \sim e^{H_\Lambda \cdot t}}$$ inflation