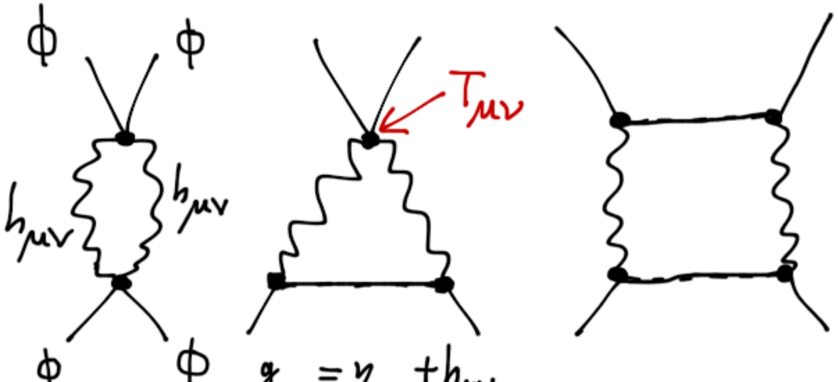
a) vacuum fluctuations induced couplings: $S = \int d^4x \sqrt{-g}\, R$,



$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

b) a change of the spectrum of vacuum fluctuations in presence of external background curvature:

$\longrightarrow$ influences only long-wavelength modes with $\lambda_{ph.} > H^{-1}$ for $V(\phi) \lesssim 1$

$\longrightarrow$ inflationary seeds of density fluct.

$\rightsquigarrow$ look here at corrections a):

consider Einstein equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -T_{\mu\nu}$$

$\Rightarrow$ each vertex in the diagrams of a) generated by $\underline{\underline{T_{\mu\nu}}}$!

$\rightsquigarrow$ $T_{\mu\nu}$ only generated by mass-energy, and thus by $V(\phi)$ or $m^2(\phi) = V''(\phi)$ for scalar fields.

Not generated by field displacements $\Delta\phi$ per se:

$\rightsquigarrow$ pure gravity has a shift symmetry in free scalar fields.

$$\Rightarrow \Delta V(\phi) = c_1 \cdot V''(\phi) \cdot \frac{V}{M_p^2} \cdot \ln\left(\frac{\Lambda^2}{M_p^2}\right) + c_2 \cdot \frac{V^2(\phi)}{M_p^4} \cdot \ln\left(\frac{\Lambda^2}{M_p^2}\right)$$

$$= V \cdot \left(\tilde{c}_1 \cdot \frac{V''}{M_p^2} + \tilde{c}_2 \cdot \frac{V}{M_p^4}\right)$$

as typically: $c_1 \sim c_2 \sim \mathcal{O}(1)$

and $\Lambda \sim M_p$

no terms like:

$$\Delta V \sim \sum_{n>4} c_n \frac{\phi^n}{M_p^{n-4}}$$ are generated.

$\Rightarrow$ as long as $V(\phi), V''(\phi) \ll 1$, gravitational corrections are very small and $\Delta\phi \gg M_p$ no problem $\longrightarrow$ will use this property of pure gravity later for inflation.

iii) thermal effects in field theory

∼ consider again an uncharged scalar field :

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - V(\phi)$$

$$V(\phi) = -\frac{\mu^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4$$

no charge $\Rightarrow$ chemical potential $\mu_\phi = 0$

$\Rightarrow$ thermal state has :

$$n_{\vec{k}} = \frac{1}{e^{\frac{\sqrt{\vec{k}^2+m^2}}{T}} - 1}$$

for $T \to 0$ $\quad n_{\vec{k}}(T) \to 0$

∼ revert to discussion of effective potential in i)

for finite T all relevant quantities from Gibbs averages :

$$\langle \ldots \rangle \equiv \frac{\mathrm{Tr}\left(e^{-H/T}\ldots\right)}{\mathrm{Tr}\left(e^{-H/T}\right)}$$

symmetry breaking VEV will be given by :

$\phi_0(T) = \langle\phi\rangle$, not $\phi_0 = \langle 0|\phi|0\rangle$

$\Rightarrow$ Look at Gibbs average of e.o.m. for ϕ :

$$\left(\partial^2 + \mu^2 - \lambda \phi^2\right)\phi = 0$$

$$\Downarrow \quad \langle \ldots \rangle$$

$$\partial^2 \phi(T) - \left[\lambda \cdot \phi^2(T) - \mu^2\right]\phi(T)$$

$$- 3\lambda \cdot \phi(T) \cdot \langle \phi^2 \rangle - \lambda \langle \phi^3 \rangle = 0$$

with shift of field :

$$\phi \to \phi + \phi(T)$$

such that $\langle \phi \rangle = 0$.

then : $\langle \phi^3 \rangle = O(\lambda^2)$

while :

$$\langle \phi^2 \rangle = \frac{1}{(2\pi)^3} \int \frac{d^3k}{2\sqrt{\vec{k}^2 + m^2}} \left(1 + 2 \langle a^+_{\vec{k}} a^-_{\vec{k}} \rangle\right)$$

$$= \frac{1}{(2\pi)^3} \int \frac{d^3k}{\sqrt{\vec{k}^2 + m^2}} \left(\frac{1}{2} + n_{\vec{k}}\right)$$

vanishes after renormalization $m^2(\phi)$ at $T = 0$.

$$\Rightarrow \langle \phi^2 \rangle = F(T, m(\phi))$$

$$= \frac{1}{2\pi^2} \int_0^\infty \frac{k^2\, dk}{\sqrt{k^2+m^2(\phi)}\left(e^{\sqrt{k^2+m^2(\phi)}/T} - 1\right)}$$

for $T >> m$:

$$\langle \phi^2 \rangle = \frac{T^2}{12}$$

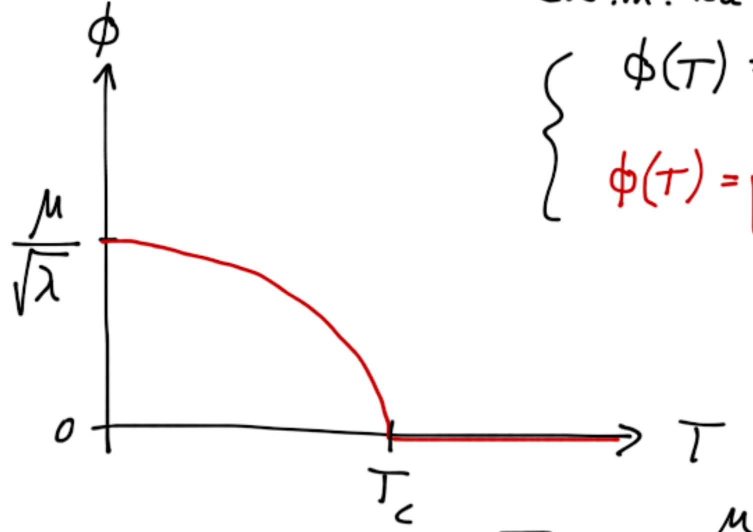
e.o.m. becomes thus:

$$\partial^2 \phi(T) - \left[\lambda \cdot \phi^2(T) - \mu^2 + \frac{\lambda}{4} \cdot T^2\right]\phi(T) = 0$$

$\rightsquigarrow$ phase diagram for constant field $\phi(T)$:

e.o.m. has 2 sols

$$\begin{cases} \phi(T) = 0 \\ \phi(T) = \sqrt{\frac{\mu^2}{\lambda} - \frac{T^2}{4}} \end{cases}$$



$$T_c = 2\frac{\mu}{\sqrt{\lambda}} = 2\phi_0$$

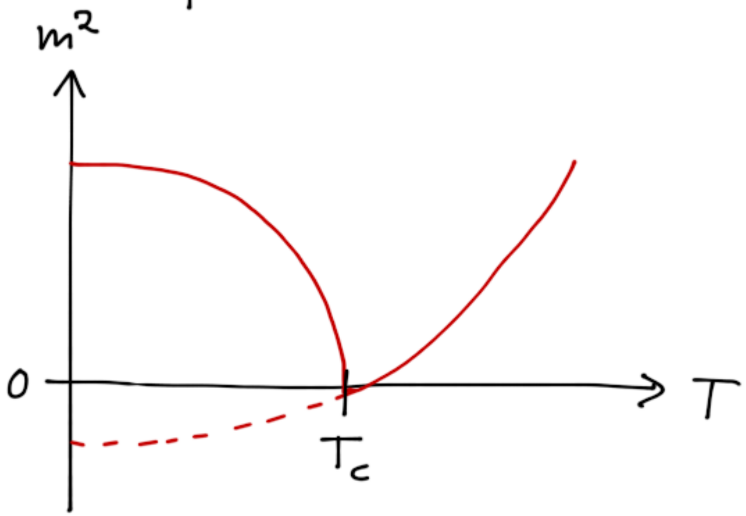
expand : $\phi(T) + \delta\phi$

$\Rightarrow$ effective thermally corrected mass:

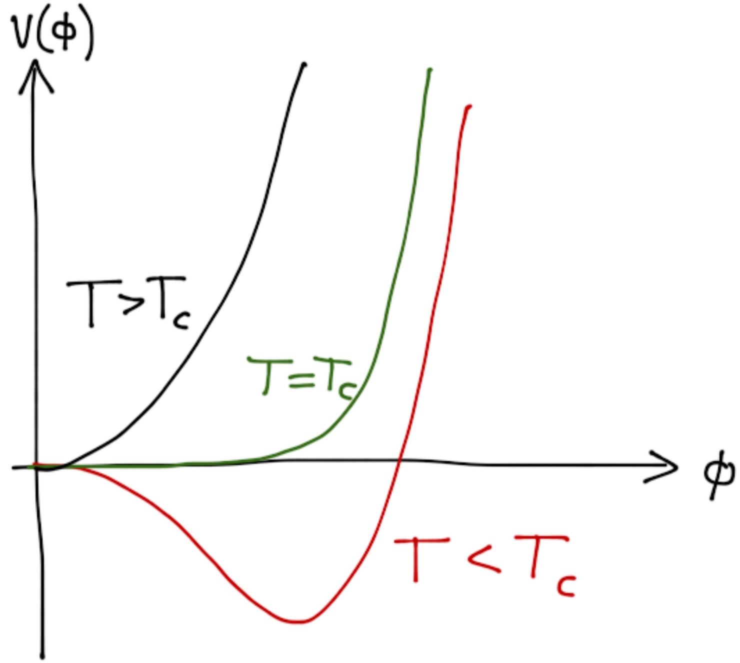
$$m^2(\phi) = -\mu^2 + \frac{\lambda}{4}T^2 + 3\lambda \cdot \phi^2(T)$$

$\Rightarrow$ $\phi(T)$ stable for $T < T_c$, at $T = T_c$ $m = 0$, and $\phi(T) = 0$ at $T > T_c$ when the other solution $\phi(T) = 0$ becomes stable:

phase transition at $T = T_c$



behaviour of eff. potential:



$\rightsquigarrow$ phase transition is 2nd order

The same results can be obtained by using the 1-loop effective potential:

$$V_{eff.}(\phi) = V(\phi) + \frac{1}{2}\int \frac{d^4k}{(2\pi)^4} \ln\left(k^2 + m^2(\phi)\right)$$

and then compactifying the Euclidean time k_4 in this expression on a circle S^1 with radius:

$$R_4 = \beta = \frac{1}{T}$$

Then:

$$\int dk_4 \longrightarrow 2\pi T \sum_{n=-\infty}^{\infty}$$

and:

$$k_4 \rightarrow k_4^{(n)} = 2\pi n \cdot T$$

(Kaluza-Klein compactification of X_4 !)

and thus:

$$V_{eff.}(\phi) = V(\phi) + \frac{T}{2(2\pi)^3} \sum_{n=-\infty}^{\infty} \int d^3k \cdot \ln\left[(2\pi n T)^2 + k^2 + m^2(\phi)\right]$$

$$m^2(\phi) = 3\lambda\phi^2 - \mu^2$$

$$\Rightarrow V_{\text{eff.}}(\phi) = V(\phi) = \frac{\pi^2}{90} T^4 + \frac{m^2(\phi)}{24} \cdot T^2 + \dots$$

same as before

$\rightsquigarrow$ can do this program for fermions and gauge fields as well $\longrightarrow$ <u>thermal effective field theory</u>.